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THE

TRUE SCIENCE OF MUSIC:

BEING

A NEW EXPOSITION OF THE LAWS

OF

MELODY AND HARMONY.

"I call that experimental science which neglects argumentation—for the strongest arguments prove nothing as long as the conclusions are not verified by experience. Experimental science does not receive truth at the hands of superior sciences. It is itself mistress, and other sciences are its servants. It has, in truth, the right to command all sciences, since it alone certifies and sanctions their results. Experimental science is therefore the queen of science, and the limit of all speculation."—ROGER BACON.

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PREFACE.

A VERY slight attention will render it obvious that this treatise *has not* been written in consequence of being in a position to sell such a work—and that its existence is the result of circumstances, adapted to lead to the discovery of the principles and laws therein set forth and exemplified.

Perhaps it is as well that I should here state that in the year 1828, I published a treatise upon the same subject, entitled "New Analysis of Music." But, though the principles therein set forth are the same as in the present work, the space allotted for its development was far too limited for its full and clear elucidation. Moreover, as far the greater number of the copies were feloniously conveyed to America, by one of the parties concerned in getting up the book, who found it expedient to quit this country—only a few copies were left for sale in London. But, though a copy of this work is not now to be had, it can, nevertheless, be seen at the British Museum, and the ten other places where, at that time, a copy had to be deposited.

The preliminary statements put forth in the Introduction are intended to prepare the way for the orderly development of a scientific treatise, amenable throughout all its details to the rules of logic and arithmetic, and copiously verified by music examples from all the

great composers. The student is therefore earnestly requested to pay attention to the general views therein presented to his notice, that he may not commence the study of this work with an idea of science entirely at variance with that entertained by the Author.

5, *John Street, Pentonville,*
London, N., March 14, 1864.

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DEFINITION OF TERMS.

According to the sense in which they are used in this work.

The words in brackets are the synonymes commonly used in other works—a star being put where no synonyme exists, and consequently where the term is presumed or known to be introduced for the first time.

BASIS. [Base or Basis. Radix. Root. Fundamental. Generator.]
That sound, which in respect to any particular string or pipe, is the lowest or gravest such string or pipe is capable of producing, without altering the length or tension of the string, or length of the pipe.
That sound, whence, from the same string or pipe an indefinite series of harmonic sounds may be produced.
That sound which is generated by the coincident vibrations of two or more harmonic sounds.
That sound, the ratio of whose vibrations in respect to the vibrations of some other sound or sounds, when expressed in its lowest terms, is the unit.

Primary Basis. [*]
That sound which, in reference to any of its primary harmonics, is the unit.
That sound which is the first in degree as a generated basis.

Sub-basis. [Subdominant fundamental.]
The primary basis to the sub-system, of the triune system.
A basis to a primary basis.

Central-basis. [Tonic. Key-note.]
The primary basis to the central system of the triune system.
The key-note.
The tonic.

Super-basis. [Dominant fundamental.]
The primary basis to the super system of the triune system.
That sound which in reference to the triune system is always a primary basis.

Medial basis. [*]
That harmonic of one basis, which in the triune system stands in the relation of a basis to the same harmonic of another basis.
A basal mediant.

Medial primary-basis [*]
That medial harmonic which stands in the relation of a primary-basis to the corresponding medial harmonic of another basis.

Medial sub-basis [*]
That medial harmonic which stands in the relation of a sub-basis to the corresponding medial harmonic of another basis.

Secondary basis. [*]
That sound which is the second in degree as a generated basis.
The basis to a harmonic which is the square of one, or product of two primary harmonics.

Tertiary basis. [*]
That sound which is the third in degree as a generated basis.
The basis to a harmonic which is the cube of one—the product of the square of one multiplied by another—or the product of three primary harmonics.
Central basis of the great system [*]
The central-basis of the 19 triune systems which form the great system.

CHORD. [Chord.]
Two or more sounds heard at the same time.
Simple chord. [*]
Any combination of sounds by which only one basis is generated.
A chord, the elements of which are the primary harmonics of *one* basis with or without that basis.
A chord, as above, with the addition of *one* super harmonic.
Compound chord. [*]
The opposite of a simple chord.
A chord by which *two* or *more* bases or basal sounds are generated.
Diatonic chord. [Diatonic chord.]
A chord composed entirely of diatonic harmonics.
Chromatic chord. [Chromatic chord.]
A chord containing one or more chromatic harmonics.
Simple diatonic chord. [*]
A diatonic chord whereby only one basal sound is generated.
Compound diatonic chord. [*]
A diatonic chord whereby two or more basal sounds are generated.
Simple chromatic chord. [*]
A chromatic chord whereby only one basal sound is generated.
Compound chromatic chord. [*]
A chromatic chord whereby two or more basal sounds are generated.

CHROMATIC. [Chromatic.] See *harmonics—fixed and serial.*

DIATONIC. [Diatonic.] See *harmonics—diatonic.*

HARMONICS. [Harmonics.]
All sounds except basal sounds.
All the sounds which any string or pipe is capable of yielding, except its lowest or gravest sound.
All sounds, by the union of two or more of which, basal sounds are generated.
All sounds from the ratio of whose vibrations the unit only is excepted.
All the sounds of the triune system for which, in this work, its three basal sounds are made to stand as representatives—and whereby is obtained the formulæ pertaining to every possible variety of harmonic combinations and progressions.
Primary harmonic. [*]
An harmonic to a primary basis.
An harmonic, or harmonic ratio, the unit to which is a primary basis.
Sub harmonic. [*]
An harmonic which is also a medial basis.
Super harmonic. [*]
An harmonic to a sub basis.
An harmonic, or harmonic ratio, the unit to which is a sub-basis.

Diatonic harmonics. [Diatonic notes or sounds.]
The sounds pertaining to the first development of the triune system.
Those harmonics which, in the triune system, constitute the initial elements of the 1st, 2nd, and 3rd mode.
All sounds which do not require to be resolved.
The resolvents to chromatic sounds.

Fixed chromatic harmonics. [Chromatic notes or sounds.]
The sounds pertaining to the second development of the triune system.
All sounds which require to be resolved.

Serial chromatic harmonics. [Slides.]
Any indefinite series of sounds, commencing with the diatonic relative, and terminating with the resolvent to that fixed chromatic which must, upon keyed instruments, be substituted instead of the series.
The sounds especially pertaining to the 3rd, but which also include those of the 2nd and 1st, development of the triune-system.
All harmonic ratios, *ad infinitum.*

Medial harmonic. [*]
An harmonic to a medial basis, which basis is also an harmonic to a basis which is not medial.

Medial-primary harmonic. [*]
An harmonic to a medial-primary basis.

Medial-super harmonic. [*]
An harmonic to a medial-sub basis.

Ascending harmonic. [*]
A chromatic harmonic whose diatonic relative is below, and resolvent above.

Descending harmonic. [*]
A chromatic harmonic whose diatonic relative is above, and resolvent below.

MODE. [Mode.]
That which relates to the manner of treating the elements of the triune system.
That which in reference to any particular composition relates to the question as to which of the mediants and terminants are the dominant, and which the subordinate powers.

1st Mode. [Major mode.]
That mode in which the harmonic 5 is one of the dominant, and 19 one of the subordinate powers of the triune system.

2nd Mode. [Minor mode]
That mode in which the harmonic 19 is one of the dominant, and 5 one of the subordinate powers of the triune system.

3rd Mode. [*]
That mode in which the harmonic 11 in conjunction with 19, is the dominant, and 5 one of the subordinate powers of the triune system.

MEDIANTS. [Mediants.]
The harmonic 5 or 19 when treated diatonically in subservieney to the mode.

Sub, Central, and Super Mediants. [*]
The mediants to their respective bases in the triune system.

RELATIVE. [*]
That sound which when diatonic is referred to a certain chromatic sound, and when chromatic to a certain diatonic sound, as having a peculiar relation the one to the other.
Two sounds, the one diatonic and the other chromatic, are reciprocally relatives, *the primary basis being the same to both*, and the chromatic harmonic more immediately connected with its diatonic relative than with that of any other diatonic harmonic.
That diatonic sound which might best be substituted instead of its relative chromatic.

RESOLVENT. [*]
That sound to which any particular chromatic sound naturally tends, by which it is said to be resolved, and by means of which a basal transition is effected.
That sound which the ear is led to desire and to anticipate while listening to any chromatic sound.
A resolvent is always a diatonic sound.

SYSTEM. [Scale.]
That system of sounds the vibrations of which stand in a fixed relation the one to the other, and whose derivation from nature is demonstrated by the fact *that musical sounds exist in a state of consociation, or membership, one with another, because numbers themselves stand thus consociated.*

Simple system. [*]
A system composed of one sound only as its basis, and the series of primary harmonics appertaining to that basis.

Sub system. [*]
The lowest or gravest of the three simple systems of the triune system.

Central system. [*]
The central system of the three simple systems of the triune system.
That system the basis to which is the key-note.

Super system. [*]
The highest or most acute of the three simple systems of the triune system.

Triune system. [The scale.]
The above three simple systems combined into one.
All the sounds appertaining to any given key.

Great system. [*]
A system compounded of 19 triune systems.

TERMINANTS. [*]
The harmonics 3 or 11 when treated diatonically in subserviency to the mode.

Sub terminant. [*]
The terminant to the sub basis.

Central terminant. [*]
The terminant to the central basis.

Super terminant. [*]
The terminant to the super basis.

TRANSITION. [Modulation. Change of key.]
The act of passing from one sound to another.
The act of passing from two or more sounds to another or to others—*i.e.* from one chord to another.
Modulation in its widest, and also in its most restricted sense.

Tonic transition. [Change of key. Modulation.]
The transition from one tonic to another.
The transition from the sounds of one tonic or key-note to the sounds of another—consequently from one triune system to another.
A change of key.

Basal transition [*]
The transition from one basis to another.
The transition from a basis or some harmonic of that basis to another basis, or some harmonic of that basis.

Harmonic transition. [*]
The transition from one harmonic to another the primary basis being the same to both.

Principal transitions. [*]
Those basal transitions resulting from the harmony of two or more parts, which being slower constitute the outline to others which are quicker.

Subordinate transitions. [*]
Those basal transitions resulting from the harmony of two or more parts which are quicker than others, and by means of which the principal basal transitions are intersected or subdivided.

Direct transitions. [*]
A transition is direct, when in reference to the two bases concerned therein, the latter stand in the relation of an harmonic to the former—and also when in reference to the great system the newly introduced triune system constitutes one of its super systems.

Retrograde transitions [*]
A transition is retrograde when in reference to the two bases concerned therein, the former stands in the relation of an harmonic to the latter—and also when in reference to the great system the newly introduced triune system constitutes one of its sub-systems.

Enharmonic transitions. [Enharmonic interval and modulation.]
The transition from an ascending to a relative descending chromatic harmonic, or *vice versa*, the resolvent being the same to both.
The transition from an ascending to a descending chromatic harmonic, the resolvent to the former being the basis to the latter.

ERRATA.

Page	15,	line	5,	*for* severed	*read*	several.
„	92,	„	9,	„ triune	„	true.
„	13,	„	13,	„ been	„	being.
„	18,	„	20,	„ Fig. 18	„	Fig. 15.
„	34,	„	20,	„ § 90	„	§ 93.

INTRODUCTION.

NOTWITHSTANDING the immense number of treatises that have been written upon music, the immediate cause of its effects is nearly as great a mystery at the present time as it was three thousand years ago. The fact is, we have hitherto been in utter darkness in respect to the truths which constitute its scientific basis; and, what is worse than all, the two or three initiatory facts, which experiment has made known to us, have been so perverted and misapplied as to add to the bewilderment of everyone who has been induced to place any reliance upon the flickering light they, in their isolated condition, are competent to afford. For, be it observed, utter darkness deceives us not—if it affords no clue to the right course it does not set us upon a wrong one. In what, by a misnomer, is called the science of music, the why and wherefore, except in one solitary instance (the major common chord), is never really answered; nevertheless, there appears ever to have existed in the minds of persons of reflection, a strong conviction that the details of practice are all referable to *some* hidden principle; but, in their efforts to discover this principle, they have uniformly pursued the same path. Each successive writer has hoped to derive something from the labour of his predecessors; the very idea of trusting solely to his own efforts, seems to have operated with each individual, as a check to his exertions. No one has commenced from the foundation, or ventured to explore its depths. Few experiments have been tried, and hence the paucity of facts with which we have been made acquainted—hence, also, the inducement to substitute sophistical arguments in the place of true logical induction—and the propensity to dream of analogies which have no existence.

It is a point of great importance to a student entering upon a scientific work like the present to feel thoroughly satisfied, either that there is, or is not, a real connexion between music and numbers, and consequently not only with arithmetic and the ratios of numbers, but also with the qualities of numbers *per se* as primes, powers, and products. For, if there be none, then is this book worse than useless; but, if there be, then, as that connexion must necessarily exist in subordination to the will of the Supreme Being, it must, in its way, represent the wisdom of that Being.

In this sense science is not an invention but a discovery, and the so-called theories which are merely the offspring of invention and speculation are the hallucinations of a disorderly mind, which having no foundation in the nature of things, are nonentities, and therefore unworthy of any serious attention. An intimate and extensive acquaintance with indisputable facts being, then, the basis of the science of music, as well as of every other science, it would be a work of supererogation to insist upon the importance of entering into an ample and rigid inves-

tigation of these facts were it not that, for various reasons, such knowledge is, at present, rejected, and also because there exists in the opinion of many an irreconcilable difference between theory and practice, and hence a cold disregard for the truths of science. But it may here be asked, of what can the basis of the science of music, or any other science, consist otherwise than *a statement of the facts* which relate to it? For if statements be made which do not describe or show forth any fact or combination of facts, *there can be nothing in them of any value;* if they describe, as facts, matters which are not facts, *they are false statements;* and if they describe facts which do not relate to the subject, *they are misapplied statements.* Now, if the results hitherto obtained from the arithmetical or mathematical operations which affect to relate to music, fall under the censure of *misapplied statements,* or in other words of being based upon *false data,* how can theory and practice be otherwise than at variance the one with the other? Wherefore, if, upon reflection, these persons can be made to perceive that science, if true, can relate to nothing but what is good in practice; this preconceived erroneous opinion must give place to more just and worthy ideas concerning it. However, with the view to put an end, if possible, to this foolish controversy, respecting the relative value of science and practice, let us admit at once the superiority of the latter over the former, by which we mean the superior importance of the end to that of the means, seeing that the means must needs be for the sake of the end, and not the end for the sake of the means. But if, as is unquestionably the case, the attainment of the end is in all cases dependent upon the possession and right application of the means, this objection has no real force. Now, though the understanding, considered in itself, is deaf—and the ear considered in itself, irrational—nevertheless, since we are so constituted that the deficiencies of one faculty are supplied by another, this is no reason why we should despise or even depreciate either faculty because it wholly differs from the other, more especially where both are so evidently wanted. Let the understanding, then, be permitted to perform the task adapted to its peculiar function—that is, let it be engaged occasionally in the exploration of the profound and inexhaustible store of wonders pertaining to this subject—that it may therbey become qualified to administer to the wants and desires of the ear.

Though music can be thought of as existing potentially, in subserviency to the laws which govern the universe—the actuality of its existence must necessarily be coeval with the first song, or first emotional phrase. But as a song or phrase thus spontaneously created, expires the instant after its birth, mankind have for centuries been incessantly at work to embody and fix these evanescent effects, so as to enable us to recall them at pleasure. But, notwithstanding the great length of time that has elapsed since the commencement of these efforts, it is only in times comparatively quite recent that anything worthy of note has been accomplished. What the future may bring forth bearing upon these mixed departments of art and science of course no one can foresee—but we may now safely lay claim to having done something preparatory to the end in view. That is by means of the notation, the instruments now

in use, and the compositions of the great masters, we can do something in addition to that of pouring forth sounds with the voice, which none could in any way fix, and render the subject of future examination. A notation that did not in some measure serve as a guide for the use of the performer, would soon be laid aside for others that did—and these again for others that answered that purpose still better. Musical instruments, moreover, have, in far the majority of instances, been wrought into their present shapes, through many gradations—those whose tones were less pure, whose construction was awkward, and execution difficult, being of course progressively laid aside for others less objectionable. But while the majority of the individuals concerned in these matters confined their attention chiefly to the more immediate objects of pleasure, traffic, or to the rectification of some glaring defect—others, though comparatively few, not content with things as they happened to be, nor with the empirical, piecemeal, and dubious emendations of the mere mechanic, have labored hard with the sincere desire to get at first principles, and by their means to obtain a more comprehensive and clear view of the subject. But in the total absence of materials (good compositions) whereon to lay the foundation, such an undertaking, though in itself laudable, must necessarily prove a failure. There are many checks to the progress of truth, but, generally speaking, they proceed not so much from indifference in regard to it as from the general proneness to turn aside from the contemplation of first principles, to something originating in ourselves; in the present instance, from the contemplation of *sounds themselves* to the notation by which they are very imperfectly represented, or to such portions of the mechanism of certain instruments as enable us, in some measure, to realize their effects. Now the evil effects resulting from this species of idolatry, in which preference is so decidedly shown to the work of our own hands, are much more extensive and serious than the world in general has any idea, or is willing to admit.

The reason the science of music has not progressed for nearly 3000 years is clearly the want of music, properly so called—for it is impossible to conceive anything more deplorable than the music of the ancients, if we may judge from the examples and the instruments transmitted, or in any other way made known to us. As to their own statements concerning the wonderful effects produced thereby, even if they could be relied upon, they prove nothing more than that semi-barbarous people, in all ages, and in all countries, can easily be *excited*, even to madness, and thereby driven, like Alexander the Great, to the commission of the most flagrant acts of cruelty. But let us give them credit for the force and violence of the sensations they lay claim to—this does not place them in the category of angelic emotions, or remove them from those which are of a demoniacal origin. Upon this, as well as many other subjects, speculators have labored hard to no purpose. But, though they cannot recall the past, they might perceive that in reference to excitement, the men to be acted upon, ought to be taken into consideration, as well as the instrument by means of which the operation is to be performed. As for instance, that the excitement, or state of mind, which one of the sublime chorusses of Handel, or the many inimitable compositions of other composers are calculated to produce is not the same *in kind* as those relating

to the music of ancient times, any more than it now is to the excitement occasioned by a bull fight. On such a barren subject as the music of the ancients, the only rational course to be pursued is to let it remain in *statu quo*. To all outward appearance music is a modern phenomenon—almost a thing of yesterday—for setting aside the puerile productions of the middle ages, as of little or no value—all that is truly noble has been put forth during the last and present century. Now, when we consider that these reflections relate to the affections—that is to the inmost state of the human soul—it becomes a matter of great interest and importance to account for the fact of so great a change having so recently been effected—with the view, if possible, to ascertain the end to which it is to be rendered subservient. But as this enquiry is too extensive and too profound for the objects in view at present, I merely allude to the circumstance with the intent to call forth attention and reflection.

The first dawn of science, in reference to music, is referred to Pythagoras, who discovered the connexion between the octave and the number 2, and of the 5th with the number 3, with respect to the relative length of the string required to produce these intervals. But being impressed with the idea that the numbers 1, 2, 3, and these only, were sacred, all others were, by him, rejected as unfit for musical purposes. Sometime afterwards, it was discovered by Aristoxenus, that the major 3rd, in the ratio $3^4 = 81$, was not so agreeable to the ear as the ratio 80, the 4th octave to the prime 5, which has ever since been permitted to hold its place. From that time to the present the ratios derived from the primes 1, 2, 3, 5, are the only ones admitted into the composition of musical intervals—all the higher primes being banished from the society of these adopted favorites. While crippled with these notions, three especial circumstances conspired to render it desirable to stop short at this point. For, with the vista of primes, *ad-infinitum*, staring them in the face, the momentous question, *Where shall we stop?* is sure to start up, and equally sure to create a panic. During this perplexing state of things it was perceived that by the aid of the primes 1, 2, 3, 5, it would be easy to concoct a series of ratios that might seem to do for every interval practical upon keyed instruments, or incidental to the notation—and, if not too severely put to the test of experiment, with these primes only, and some few of their products, a considerable variety of phantasmagoria tricks could be got up, and by this means ward off the disgrace of being utterly defeated, and driven to the humiliating confession that music was to them quite inexplicable, and consequently either sheer nonsense, or something above their intellectual powers to discover.

Such is the state of things that gave birth to that stagnating and benumbing influence which has had the effect of checking the progress of music, as a science, from its very infancy—and in no small degree also to check its free development even in composition.

In reply to the assertion that the primes 7, 11, 13, 17, 19, are all out of tune, I maintain that it is false—and that, on the contrary, *in their right place*, they are all perfectly in tune, and in the highest degree effective. That 11 is frightfully out of tune as the fourth of the scale, and 13, as the sixth of the scale, is true, simply because they are not intended to represent these intervals. As to the primes 7, 17, 19, as it was im-

possible to attach them to wrong practical intervals, the acousticians thought proper to adopt the wretched expedient of attempting to push them aside under the pretence that these primes also were out of tune—or, in other words, that nature—that is the Creator—not knowing what to do with the higher primes, has linked music with numbers in a botchy sort of way—and consequently that falsifications in respect to the ratio of its vibrations are inherent, and that He is the author of them. Of the primes 23, 29, 31, 37, 41, &c., the acousticians have been studiously silent, being utterly at a loss how to dispose of them in any way. By some all the primes from 7 upwards are stigmatized as being "anomalous." I, however, deny that they are so, or that we have any right to speak of them as being "irregular," or as "deviations from the general method or analogy of things." From what has been said, I think it will plainly appear that the acousticians, instead of throwing light upon the science of music, have, by restricting themselves to the use of a few tools—crippled by a scanty acquaintance with the works of the great masters—and the habit of revolving in the same narrow circle, and harping upon the two unimportant ratios, the *comma* (80 to 81), and the *diesis* (125 to 128)—really done nothing but set up idols of their own creation, the worship of which has diverted the attention of the world at large from those forms and ideas which represent the divine mind, to a variety of grotesque and hideous images, with which it is utterly at variance.

Now, since the true science of every real thing appertaining to nature, and in any way subject to the laws of nature, must needs be a discovery, and the fruit of long and careful investigation—it must be evident all invented, artificial, planned, or contrived theories, must represent monstrosities more or less deformed in proportion as they differ from the divine type or pattern impressed upon them. It has been well observed "that we have never so much to say as when we are ignorant of the subject we are speaking about," and the truth of this remark is verified by the prodigious number of treatises upon the theory, written by persons in this condition. This being the real state in which we of the nineteenth century are placed in regard to it—the student whose head has been stuffed with the nonsense called thorough base—who has read and studied volumes concerning the so-called origin of the minor third, some to prove that it is, and others that it is not, derived from nature—volumes to determine what are, and what are not, consonances, without first settling the precise meaning of the term—volumes concerning chords and their bases,* and especially the chord of the added sixth and extreme sharp sixth, the precise meaning of the term basis being also left for conjecture—and volumes about Ecclesiastic, Greek, Chinese, &c., scales or modes—has been ignorantly or basely led astray, and robbed, if not of money, of his time. The result of this confusion is, that he has now to retrace his steps, in order to begin at the beginning—and should he have the courage to do so, he must not be surprised or discouraged at being occasionally called upon to extirpate a stealthy enemy, in the shape of a false dogma, which it has been his mifortune to imbibe and cherish.

* For instance, there is a Dictionary of Chords, by Berton, in eight volumes quarto.

He who has had the good fortune not to have been thus tampered with, has, of course, none of these obstacles to contend with—his course lies straight before him—and so far from being called upon to yield an implicit and blind faith in what is herein taught, he is earnestly solicited never to assent to anything while it appears to him false or even doubtful.

Intervals afford a useful and convenient mode of speaking of and referring to sounds for merely practically purposes, and also in assisting us in the discovery of the practical sounds or musical notes by which the vibrations of any particular ratio may be illustrated. Intervals and ratios are synonymes in so far that both refer to the same idea, that is to the differences of sounds in respect to pitch, but intervals afford only a rude and imperfect mode of pointing out such differences, whereas by ratios they are expressed with perfect accuracy. Intervals are adapted for ordinary practical purposes, where no nicety is required beyond that of determining the note or notes concerned, but ratios are the exponent of truth itself.

As the intervals of the scale, and the series of intervals of any air or tune, are the same in one key as in another, an idea is apt to take possession of the mind, to the effect that intervals constitute the true generalizing principle. But, while the theory of no one scale or key is explained, the mere knowledge of the fact that one is the same as another, affords no clue to the theory of any. Thus, a picture may be executed upon a larger or smaller scale, in which case all its parts ought to have the same relative proportion and bearing; but, the mere discovery of this fact has nothing to do with questions relating to the relative proportion and bearing of the parts in the same picture.

The number of intervals is, in a certain sense (that is practically and within the compass of the octave) no more than 12; but the number of ratios, in the same compass, is indefinite, and hence may be seen the insufficiency of the former as a substitute for the latter. Furthermore, though the number of actual or practical intervals upon keyed instruments, such as the piano and organ, &c., is, in the sense already pointed out limited to 12; the nominal intervals, referring to these 12 practical intervals, amounts at the very lowest reckoning to 17; for, the interval of the ♯1st and ♭2nd, though theoretically distinct and different, are practically the same, as is also that of the ♯2nd and ♭3rd; the ♯4th and ♭5th; the ♯5th and ♭6th, and the ♯6th and ♭7th. In the notation all these distinctions are carefully attended to by correct writers, although disregarded in practice. Though the system of intervals is made to run parallel with that of the notation, and to be exactly commensurate therewith; while on the one hand the varieties of both exceed the capabilities of actual practice, on the other hand their sphere of operation is far too contracted ever to become the substitute for ratios. Nor can any valid reason be assigned for attempting to evade having recourse to the exact sciences in reference to music, any more than in other affairs; neither would this attempt ever have been made, or any reluctance shown thereto, had there not hitherto existed a conscious feeling of inability to grapple with the subject upon grounds so severe and unyielding, and at the same time so antithetical to the very idea of evasion and mystification.

The relation in which intervals stand to ratios in one direction, and to

facts in actual practice in another, being now pointed out, we should endeavour, when using this word, to think of it; first, in reference to the number of letters or notes between its two terms, and then, where greater nicety is required, to the number of semitones; and, though these degrees or points of separation are purely arbitrary, and in most instances of equivocal value, they are nevertheless useful adjuncts in the study of intervals.

It will be found that the addition of intervals answers to the multiplication of ratios, the subtraction of intervals to the division of ratios, and consequently the multiplication of intervals to the powers of ratios, and the division of intervals to the roots of ratios. And it will be necessary to bear in mind these facts in all our calculations relating to the ratios of intervals. Since the number of sounds on keyed instruments is, as before observed, only 12, and since, by taking that sound which is nearest to the sound required, these 12 sounds are, in practice, substituted in the place of an indefinite series: it evidently follows that this artificially constructed scale or system of sounds must be very imperfect. These imperfections, when considered in the aggregate, are expressed by the word *temperament*, but before it can be clearly seen to what degree any particular interval stands affected by temperament, or, in other words, how far, or to what extent, the true sounds required are falsified by the substitution of this artificial scale, denominatated the tempered scale, it is evident we must not only know what truth demands, but also what we have to substitute in its place, that by comparing the one with the other the difference may in all cases be discovered. But since neither mathematicians nor musicians have been thus qualified for investigating this subject, their notions and statements have been extremely incorrect both as to degree and direction. In other words, where, in their opinion, the true interval is *plus* the tempered interval, it is in exactly 10 cases out of 13 *minus*;—and where in their opinion the true interval is *minus* the tempered interval, it is in exactly 10 cases out of 13 *plus*. Mathematicians, for example, in the total absence of anything like proof, that is of proof derived from facts, but simply because certain favorite calculations are true in themselves, though quite irrelevant to the point under consideration, and in the face of all experience to the contrary, and in utter contempt and defiance of their own and other people's ears, persist in maintaining that the interval of the ♯1st is *less* than that of the ♭2nd,—the ♯2nd *less* than the ♭3rd, &c.—and when it has been urged against them that all good singers, and good performers upon instruments capable of making any distinction in regard to these intervals invariably do exactly the reverse—they endeavour to evade the difficulty by pretending that this is the result of a vitiated taste, which by continued indulgence has at length engendered a sort of craving for what they call seasoning—that is for seasoning those intervals sharper which ought to be flatter, and those flatter which ought to be sharper.

As the subject of temperament is best understood, and with infinitely less trouble, by a careful examination of certain diagrams which I have designed and adapted for this purpose, it will be unnecessary to add more at present, and still less to go into the minutiæ of particular cases.

The term scale in its popular sense is used to signify a series of sounds rising or falling by degrees from any given pitch or tone to that of any

other. If these degrees are diatonic, or supposed to be diatonic, the scale is called diatonic; and if chromatic, the scale is called chromatic. But there is a fundamental error involved in this idea of a scale, because in this sense it is made to stand in the place of melody. A melody may ascend or descend diatonically or chromatically, consistently with the laws of harmony, but not so a scale, for a scale or ladder constitutes the represented or conceived relation, one to another, of such sounds as are adapted to the requirements of melody and harmony, both universally and specifically. In other words, a scale is an orderly arrangement of sounds appertaining to all melodies without exception, when put in the same key; but the exercises erroneously called diatonic scales, constitute one particular form of melody, and the chromatic another; and the only reason why the ascending and descending series differ more or less the one from the other, is simply because all melodies are amenable to the laws of harmony, and a melody which is good in itself is not necessarily so upon being played backwards. Scales without end may be constructed upon an artificicial and fanciful plan, all of which are not only useless, but worse than useless, because they divert the mind from truth and nature to that which is absurd and inconsistent. To such conceits are to be ascribed what are called National, Ecclesiastic, &c., scales or modes, and also the Enharmonic scale, none of which have any existence except in the dreams of pseudo philosophers.

As the tendency to the formation of scales originates in what at first is only an obscure perception relating to form and order, it ought not to create surprise that herein, as in other things, the objects first presented to view should only represent distorted images of the true type or types appertaining to nature. What these types are will be distinctly and elaborately portrayed hereafter; but, in order to convey a general idea of the subject, it may be observed that the system of the least magnitude, and which I therefore call the simple system, relates to a unity of particulars only, which though of indefinite extent as to length, is nevertheless practically useless for want of breadth. The system next in degree to this is that which truly represents what is commonly but vaguely understood by the sounds belonging to any given key or scale; and as this system is built up of the elements of three simple systems, I call it the triune system. The relation in which the three simple systems stand to one another in the triune system may be familiarly described as follows:—

Let *a*, *b*, *c*, represent the three simple systems of the triune system—then as *a* is to *b*, so is *b* to *c*, and in both these instances the relation or degree of affinity is the simplest possible, and hence it follows that the relation or degree of affinity answering to *a*, *c*, is of necessity more remote than that of *a*, *b*, or *b*, *c*.

As the triune system is built or compounded of the elements of three simple systems, like a thread of three finer threads, and not of distinct elements peculiar to itself; so, in like manner, there also appertains to nature a scale built up or compounded of nineteen triune systems. Of these nineteen triune systems that which constitutes the supreme governing power occupies the centre, in reference to which the others are all arranged in order according to the degrees of affinity with that in the centre—nine on the left answering to a descending, and nine on the

right to a corresponding ascending series. The elements of these three systems are homogeneous in their nature, while on the contrary in all previously contrived systems the elements are heterogeneous. It is important that it should be distinctly perceived that the "true science of music" is based upon mathematics, or rather numbers, and the ratios of numbers—and that the terms vibration, bases, and harmonics are always to be understood as referring to, and resting upon the same, and not as constituting another basis. To prevent misapprehension concerning what is to be understood by a mathematical basis in respect to music, it will be necessary to bear in mind:—(1) That in reference to such matters the use of the mathematics is to furnish a code of abstract laws, showing the relation which one truth bears to another, and consequently of truths in general, one with another. (2) That these truths are of two kinds, or rather of two degrees—those which relate to material things, and to the ideas thence derived—and those which relate to sensations, and to the affections thereby excited. (3) That the truths of the first kind, or lower degree, relate entirely to ideas derived from time and space, and hence to commensurable quantities or magnitudes. (4) That upon the foundation or basis of the first degree there exists a superstructure, which transcends the sphere of nature, and in respect to which magnitude, and the laws relating to magnitude, cannot be predicated. (5) That by means of this second step we become elevated to the threshold of things above the sphere of nature—that is to things pertaining to sensation and the affections of the will—and though no link can be discerned between these two spheres, while looking from the lower to the higher, this is not the case in looking from the higher to the lower. And (6) That there is a link between the things within, and those above the sphere of nature, is unquestionably true—and that in music the laws relating to this connexion come perceptibly into operation is equally true.

Among the many other particulars peculiar to this work the utility of my symbolic method of representing sounds will be found too great and too various to be duly appreciated at once. For besides the uses to which it is applied for the purpose of designating, unequivocally, the exact sounds in melody, and of chords in harmony, and of thereby pointing to the bases and key—by reference to the large diagram at the end of this work, they also render the true pitch of these sounds familiar to the eye. These symbols, will, moreover, be found of incalcalable utility both in speaking and writing upon the sounds in melody and the chords in harmony, not only on account of their claims as the exponents of truth, but also on account of the great facilities they afford of so doing with the utmost possible brevity and perspicuity, and being the same in all keys. That the reader may not be put to the trouble and inconvenience of hunting about for the music example or figure to which any section relates, I have uniformly referred from the one to the other, and also, in numerous instances, from one section to another, where the subject matter was either the same or somewhat similar.

I could here introduce a great many remarks from various authors, and in so doing fill very many pages in verification of the facts alluded to in the course of this work, all tending to prove that, up to the present

time, music, as a science has no existence; but as this is neither desirable nor necessary, I shall merely quote a few of the observations which have from time to time dropped from their pens, viz. :—

"Strange to say, theory and practice are at variance as to which is the larger interval, the chromatic or diatonic semitone; and, by the same rule, which note should be more acute in pitch, Sol♭ or Fa♯. It is universally allowed that the intervals in question are not equal sized, and that the notes composing them should be slightly * different; yet while arithmetical calculation proves † that Sol♭ should be more acute than Fa♯, performers on instruments capable of different intonation (such as the violin) and very finished singers invariably consider, and strive to make, Fa♯ more acute than Sol♭!"

"Modern composers, in their instrumental pieces, introduce trancient chromatics with great freedom, and frequently so as to produce combinations altogether unaccountable in harmony."

"It is only by closely examining all the evident data, by comparing them with each other, and by discovering in one or in a few of them an origin common to all others, that professors and philosophers may be brought to reach the end, so much wished for, of giving the science of music a complete, precise, and luminous theory."

"The theory is reduced to certain mystical subtleties, of which the application and truth have nothing either constant or certain."

"I have presumed that the suggestions of the ear depended on principles extremely simple; that these principles, once thoroughly understood, could not only account for everything practiced in point of harmony, but also indicate the course followed by the ear, as it were instinctively, in the combination of its best productions. I have thought, in consequence, that we ought not to despair of the attainment of a radical and fruitful theory of music, and particularly of harmony, which would enable us not only to explain and appreciate, but even to anticipate the operations of the ear."

"In this subject, therefore, it would be in vain to expect what is called demonstration." . . . "Nor do we conceal the fact, however mortifying, that there are certain points which appear still in some degree unaccountable from our principles." . . . "Harmony has, perhaps, some unknown principle."

In additon to the above, it is to be noted that it is now openly admitted that all the existing rules of harmony have been violated with impunity and good effect—and surely no one will adopt the insane idea that the pleasures of hearing, taste, and feeling, can only be fully enjoyed while the understanding is immersed in darkness in respect to such things as relate thereto? Or that the Creator has given us more faculties than one, not with the view to highten and diversify our enjoyments, but that by reciprocally neutralizing each other, they may be toned down to a low standard, and thereby be rendered sufficiently dull and commonplace to suit our condition in this world.

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* In speaking generally these differences are not slight.

† But calculations, though correct in themselves, prove nothing when misapplied.

THE TRUE SCIENCE OF MUSIC.

PART 1.

SOUND—MOTION—VIBRATION—INTERVAL.

1. *Sound*, and consequently music, is the result of *motion*—and the motion which gives birth to sound is expressed by the term *vibration*, *undulation*, or *pulsation*.

Any one willing to take the trouble to ascertain the means by which sound is produced, may satisfy himself that it is an effect of which *motion* is the cause—for, although motion is not the cause of sound in a metaphysical point of view, nevertheless, since it is impossible to produce a single instance in which sound is obtained unaccompanied by motion, it is useful and expedient, in matters relating to physical science, to compare these effects the one with the other, under the supposition that this is the case, that science may thereby become fixed, and accommodated to ideas connected with time and space—motion and sound being in reality effects corresponding to the different sensories by which the cause in operation is perceived.

That sound cannot be propogated through a vacuum—that the intensity of sound is increased in proportion to the density of the atmosphere—and that the atmosphere is the grand medium of communication between the moving body and the sound thence communicated to the ear, are admitted facts.

2. The term vibration is used to denote that tremulous motion produced in bodies by striking, or otherwise acting upon them—and also to express the time of one complete vibration—that is, the time which passes between the departure of the vibrating body from any assigned place, and its arrival at the same place again.

3. Vibrations commencing with elastic bodies, communicate corresponding vibrations to liquid and fluid bodies, and thereby to the ear.

Now, although from peculiar circumstances, vibrations which are visible, may not at the same time be audible—and vibrations which are audible, may not at the same time be visible—it is expedient to think of them as correlatives, and inseparable. And, since science is more immediately connected with ideas derived from visible than audible sensations, it is therefore expedient to contemplate the audible vibrations of music through the medium of its correlative visible vibrations.

If, moreover, vibration be the sole cause of sound—and if effects, both generally and specifically, are to be explained only by reference to their causes—then are musical effects to be explained only by reference to vibrations.

4. The principal medium by which the vibrations of elastic bodies are communicated to the ear, is air—that is, the atmosphere.

That the atmosphere is the general medium by which vibrations are conveyed to and fro, from one object to another, and to the ear, can be proved by a variety of experiments, among which may be mentioned the following. Tune two strings to the same pitch, one upon one instrument, and another upon another—two violins for instance—then, if a piece of paper be laid across the string of one instrument, while bowing upon the other, the paper will immediately be jerked off. If the two strings be not of the same pitch, or nearly the same, the effect will not take place—but, if the two strings be tuned to ratios that are very simple, the effect will be the same, but weaker, as the ratio of the interval becomes higher, and consequently less simple. These effects are commonly termed sympathetic—but it is evident that, in reference to the cause, they are purely mechanical.

5. In this work I wish to restrict the idea of vibration to the motion of a distended string, the representation of which can in all cases be rendered clear and distinct. Part 3 is therefore devoted to represent and explain how these aerial undulations are effected, by describing their movements under a series of modifications, each with the view to some special object.

6. The vibrations of a pipe or distended string produce the sensation called sound, the sounds being more grave, lower, flatter, or deeper, as the vibrations become slower—and more acute, sharper, or higher, as the vibrations become quicker. With pipes, the absolute number of vibrations performed in any given time depends upon the length and diameter—with distended strings, it depends upon the length, thickness, specific gravity, and tension.

But as the great object of importance, in a scientific point of view, is not the *absolute*, but the *relative* number of vibrations performed in a given time—it is to this point I am anxious to fix the attention.

7. An interval, as represented by musical notation and keyed instruments, is a rude method of expressing the difference between any two sounds. It is a *rude* method, because, by intervals, the *exact* difference between sounds can never be expressed with anything approaching to accuracy. The terms relating to intervals are, nevertheless, useful as far as they go, and when applied to purposes for which they are adapted. But every effort to extend their application beyond this point has always proved a failure.

The truth of this assertion will appear in what follows, wherein is stated a series of facts. The knowledge of these facts is of the highest importance, since, to the want of it is to be ascribed the confusion and ignorance that everywhere prevails in respect to music, as a pure science. By a pure science is here meant a science at all times consistent with itself, susceptible of rigid mathematical demonstration, and experimental tests.

The Unison, or Unit-sound.

8. Vibrations which take place at equal intervals of time are called isochronous, and two or more united sounds, whose vibrations are isochronous, produce the unison, or unit-sound.

Though an increase of power is the only result of a combination of unities, the unison is, nevertheless, classed among intervals—that is, though the unison is evidently *not* an interval, custom has made it so, by assigning to it the number 1, instead of 0—and so irrevocably fixed is this error, that reformation is utterly hopeless, notwithstanding that, by so doing, all the rules of arithmetic become falsified. For example—the interval called a 5th (C, d, e, f, G), added to the interval called a 4th (G, a, b, C), produces the octave, or 8th (C—C), and not a 9th—the interval called a 3rd (C, d, E), added to the 3rd (E, f, G), produces the 5th (C, G), and not a 6th—and lastly the 8th or octave (G—G), minus the 2nd, produces the 7th (G—F), and not a 6th, &c.

9. As the starting point to real intervals, the unison is called the basis, root, generator, radix, fundamental. These, and other terms, being adopted to express a mysterious something, without incurring the danger of distinctly, and unequivocally, pointing out what that something is—in short, a string of dubious names, instead of one, expressing a fact. In this work the term basis will be used in preference to any of the others, meaning thereby the *unit, or unit-sound, strictly, and without prevarication.*

In counting intervals merely by reference to the letters, that is, the names given to musical notes, whole tones and semitones are continually intermixed in the reckoning, as if they were both the same in value. For example, in the diatonic series, C, D, E, F, G, A, B, C, the intervals C, D—D, E—F, G—G, A—A, B, are whole tones, but E, F—B, C, are semitones.

When, however, necessity demands it, this rude method of calculating intervals is made to undergo some sort of correction, by means of the terms Major, Minor, Sharp, Flat, Extreme Sharp, Extreme Flat. Thus, for example, the major 2nd consists of 2, and the minor 2nd of 1, semitone—the major 3rd of 4, and the minor 3rd of 3, semitones—the major 6th of 9, and the minor 6th of 8, semitones—the major 7th of 11, and the minor 7th of 10, semitones. But as the 4th, 5th, and 8th are continually subject to *three* variations, these changes are commonly expressed by the terms sharp or flat—and hence, as the 4th consists of 5 semitones, the sharp 4th consists of $5 + 1 = 6$, and the flat 4th of $5 - 1 = 4$ semitones—as the 5th consists of 7 semitones, the sharp 5th consists of $7 + 1 = 8$, and the flat 5th of $7 - 1 = 6$ semitones—and as the 8th, or octave, consists of 12 semitones, the sharp 8th consists of $12 + 1 = 13$, and the flat 8th of $12 - 1 = 11$ semitones. Furthermore, as the major 3rd consists of 4, the extreme sharp 3rd consists of $4 + 1 = 5$ semitones—and as the minor 3rd consists of 3, the extreme flat 3rd consists of $3 - 1 = 2$ semitones, &c.

The octave, or 8th—double octave, or 15th—triple octave, or 22nd, &c. [ex. 1.]

10. Two sounds, whose vibrations involve the ratio 2 to 1,

produce the interval called the octave, or 8th—and consequently if the ratio of the vibrations be $2^2 = 4$ to 1, the interval produced is the double octave, or 15th—if $2^3 = 8$ to 1, the triple octave or 22nd, &c. Now, if we reflect upon these facts, it will be seen that the numbers 2 to 1 represent as it were the original, or primary ratio—whence the ratios 4 to 1, 8 to 1, &c. derive their existence. In other words, that 2 is a primary number, and that 4, 8, 16, 32, &c., as being the mere products of 2, are derivative numbers. If numbers differ as to their qualities, as primary and derivative, or as primes and products, it is evident that the relation of the unit in respect to these numbers must differ also—and consequently, if the unit in the ratio 2 to 1 represents the proximate, or primary basis, the unit in the ratios 4 to 1,—8 to 1,—16 to 1, &c., must represent a series of derivative, or sub-bases.

All numbers are either *primes* or *products*. Primes are *original*—products *derivative* numbers.

A prime, or primary number, cannot be divided by any number without a remainder, or without involving a fraction. Any number can, of course, be divided by itself, and produce 1,—or by 1, and produce itself. For example, $17 \div 17 = 1$,—and $17 \div 1 = 17$,—but this forms no real exception to the rule. A product may express the powers of one or result from the multiplication of two or more primes. The products arising from the *square*, or 2nd power, of any primary number, or from the multiplication of any *two* primes, I call *secondary* numbers—and those arising from the *cube*, or 3rd power, of any primary number, or from the multiplication of any *three* primes, I call *tertiary* numbers.

The number of primes beyond three, involved in any product, are not specifically adverted to, because of little use in practice. As 2, and all the powers of 2, merely refer to octaves, the involutions arising from this number may be disregarded in matters relating to musical ratios and intervals.

11. As the subject now under investigation is of great importance—and if once clearly perceived, will remove a thousand impediments to our future progress—I introduce [ex. 1] in musical notes, a practical illustration of what has been said in regard to it—observing, at the same time, that all sounds, except the basis, are in this work called *harmonics*.

12. In this example, the lowest note C, in every bar, represents the primary basis to the harmonic 2, its octave above. In the 1st, 2nd, and 3rd bars, these *primary* bases become *sub*-bases to the higher octaves, whose vibrations are as 4 to 1,—8 to 1,—16 to 1. Moreover, by this example, it will be seen that the ratio 2 to 4 in the 1st bar, becomes 1 to 2 in the 2nd—that 2 to 4 in the 2nd bar, becomes 1 to 2 in the 3rd, &c.—that in the 1st bar, the ratio 4 to 8 becomes 2 to 4 in the 2nd, and 1 to 2 in the 3rd—that the ratio 8 to 16 in the 1st bar, becomes 4 to 8 in the 2nd, 2 to 4 in the 3rd, and 1 to 2 in the last—

proving thereby the identity of these ratios, as well as that of the notes by which they are represented.

It must not be supposed, from what is here stated, that this minute attention to circumstances, in regard to octaves, will often be required. The object here, is, to portray the *modus operandi* of a universal principle, the importance of which will be seen hereafter.

The 12*th or octave* 5*th*—19*th or double octave* 5*th, &c.* [ex. 2.]

13. Two sounds, whose vibrations involve the ratio 3 to 1, produce the interval called the 12th, or octave 5th. If the ratio of the vibrations be $3 \times 2 = 6$ to 1, the interval produced is the 19th, or double octave 5th—and if the ratio of the vibrations be as 3 to 2, the interval produced is the 5th itself.

14. In ex. 2, C, the primary basis to the prime 3, becomes a secondary or sub-basis to the numbers 6, 12, &c. For ordinary practical purposes, however, the distinctions which relate to the octave, may, in all cases, be overlooked. By transposition is meant change of position or change of place, without any change in the interval itself, whence it follows that those which are primary, secondary, tertiary, major, minor, sharp, or flat, in any one instance, must always remain the same—[ex. 3].

The 23*rd, triple octave* 2*nd, double octave* 9*th, or* 12*th to the* 12*th.* [ex. 4.]

15. Two sounds, whose vibrations involve the ratio $3^2 = 9$ to 1, produce the interval of the 23rd—that is, the triple octave 2nd, double octave 9th, or 12th to the 12th. That this is true, and but the necessary consequence of what has already been said, is evinced by ex. 4, whereby it is evident, that the quality of the number 9, as being the product of 3×3, differs essentially from that of the primary number 3 itself, whence it is derived—and surely it will be admitted that, precisely as numbers differ in respect to particular bases, the bases must differ in reference to such numbers—since it is evident, that the difference between any two objects, or two effects, must of necessity be reciprocal.

The major 17*th, or double octave major* 3*rd*—24*th, or triple octave major* 3*rd, &c.* [ex. 5].

16. Two sounds, whose vibrations involve the ratio 5 to 1, produce the interval of the major 17th, that is, the double octave to the major 3rd. And if the ratio be $5 \times 2 = 10$ to 1, the interval produced is the major 24th, or triple octave to the

major 3rd. And as $5 \times 2 = 10$, is the octave to the prime 5, so $5 \times 2^2 = 20$, is the octave to 10—and $5 \times 2^3 = 40$, is the octave to 20.

*The ♭21st, or double octave ♭7th, &c.** [ex. 6.]

17. Two sounds, whose vibrations involve the ratio 7 to 1, produce the interval of the ♭21st, that is, the double octave to the ♭7th. As 7 is a prime, the basis to 7 is a primary basis, and the numbers 14, 28, &c., its octaves.

It is here worthy of remark, that the prime 7, after having for ages been struck out of the list of musical ratios, is at present partially restored, and its importance acknowledged. This is encouraging, since it affords reason to hope that a revolution may some day take place in favor of the other outcast primes, 11, 13, 17, 19, &c.

The ♭26th, or triple octave ♭5th—the ×24th (i. e. the extreme sharp 24th), or triple octave ×3rd. [ex. 7.]

18. Two sounds, whose vibrations involve the ratio 11 to 1, produce the interval of the ♭26th, that is, the triple octave to the ♭5th. But in one particular case, the same ratio (11 to 1) produces the interval of the ×24th, or triple octave ×3rd. As 11 is a prime, the basis to 11 is a primary basis, and the numbers 22, 44, &c., its octaves.

That it may be understood how it is that the ratio 11 to 1 is represented in practice by *two* distinct intervals, which differ a semitone the one from the other, let fig. 1 represent the 12 semitones of the chromatic scale on the piano or organ.

We will now suppose a sound to be wanted, lying in the centre between any two of these divisions. Let the point *a*, for instance, be made to represent this intermediate sound, and let us call this the first sound, and the semitone 10 the second sound. In this case it is evident the semitone 11 is the best substitute for the required, but unattainable sound *a*. But if, on the contrary, the semitone 11 must be kept in reserve for the second sound, it is evident, that in this case, the semitone 10 becomes the best substitute for the first sound—that is, for the required, but unattainable sound *a*.

To take the same sound twice, that is, to substitute the semitone 10 for the first sound *a*, and then to take it again for the second sound—or to substitute the semitone 11 for the first sound *a*, and then to take that again for the second sound,

* It will hereafter be shown that this is in reality the ♭♭8th.

would be inadmissible in practice—because, in this case, the required descending progression in the first instance, and ascending progression in the next, would cease to exist. In this, and other similar instances, the notation is of course modified, so as to suit the occasion.

To be told that, upon the the piano or organ, for a single required true sound we become in certain cases, obliged to take sometimes one, and sometimes another, of the sounds appertaining to the semitonic scale, may appear strange, but so it is, and the effect of the imperfections inherent in these instruments, upon our ears, is not removed by our ignorance of the facts relating thereto, or determination not to recognise the truth in regard to them.

The ♯26*th, or triple octave* ♯5*th—the* ♭♭28*th, or triple octave* ♭♭7*th* [ex. 8.]

19. Two sounds, whose vibrations involve the ratio 13 to 1, produce the interval of the ♯26th—that is, the triple octave to the ♯5th. But, on some occasions, the same ratio (13 to 1) produces the interval of the ♭♭28th, or triple octave ♭♭7th. As 13 is a prime, the basis to 13 is a primary basis, and the numbers 26, 52, &c., its octaves.

The reason why the ratio 13 to 1 is represented in practice by *two* distinct intervals, which differ a semitone the one from the other, being precisely the same as for that of the harmonic 11, § 18, and the expedients adopted in practice being also the same, it is unnecessary here to say more upon the subject.

The ♯29*th, or fourth octave* ♯1*st—the* ♭♭31*st, or fourth octave* ♭♭3*rd.* [ex. 9.]

20. Two sounds, whose vibrations involve the ratio 17 to 1, produce the interval of the ♯29th—that is, the fourth octave to the ♯1st. But, on some occasions, the same ratio (17 to 1) produces the interval of the ♭♭31st, or fourth octave to the ♭♭3rd. As 17 is a prime, the basis to 17 is a primary basis, and the numbers 34, 68, &c., its octaves.

The reason why the ratio 17 to 1 is represented in practice by *two* distinct intervals, which differ a semitone the one from the other, is the same as for the harmonics 11 and 13, § 18, 19. In this instance, however, the required sound *a*, fig. 1, is much nearer the semitone 10 than 11. Notwithstanding this, the expedients adopted in practice being also the same, it is unnecessary to say more upon the subject.

The minor 31*st, or fourth octave to the minor* 3*rd.* [ex. 10.]

21. Two sounds, whose vibrations involve the ratio 19 to

1, produce the interval of the minor 31st—that is, the fourth octave to the minor 3rd. As 19 is a prime, the basis to 19 is a primary basis, and the numbers 38, 76, &c., its octaves.

With the prime 19 I close, for the present, what I have to say in regard to primes and products. Let it, however, be distinctly understood that, when *rightly* applied to practice, the intonation resulting from the intervals here assigned them will be found *pure*, and in the highest degree gratifying to the ear.

PART 2.

MEANS BY WHICH INTERVALS MAY BE OBTAINED, AND THEIR EFFECTS PRODUCED, IN AGREEMENT WITH THEIR TRUE RATIOS.

22. There are many modes of obtaining an interval perfectly in tune, that is, in agreement with its required ratio—and since it is desirable not to think of matters relating to this subject through the medium of one class of facts only, I shall here refer to a few, out of the immense variety which nature presents to our observation—in all of which, numbers, and numbers only, are the exponents of the facts referred to, when the precision appertaining to truth itself is required.

23. Of all the means ever thought of for this purpose, there is none so simple and accessible to practice, as that which refers to *length*, in respect to vibrating strings—the law relating to which, is—*their vibrations are in the inverse proportion to the length*—that is, the diameter, density, tension, &c., being the same, the vibrations answering to the length 1, 2, are 2, 1,—the vibrations answering to the length 3, 5, are 5, 3,—or, if expressed by fractions, the vibrations answering to the length $\frac{1}{2}$ are $\frac{2}{1}$ —the vibrations answering to the length $\frac{3}{5}$ are $\frac{5}{3}$, &c.

24. The law in respect to pipes is the same as with strings, provided their diameters, and the pressure of the air, in blowing, be the same. Nevertheless, vibrating strings are to be preferred in experiments of this kind, being much cheaper, and more accurate in their application to practice.

25. The law in respect to the diameter of strings, is the same as that relating to length—that is, *their vibrations are in the inverse proportion to the diameter.* The examples relating to length, § 23, are therefore applicable to the diameter also. But,

though true in itself, this mode of experimenting is objectionable on account of the trouble and difficulties attending its operations.

26. The law in respect to covered strings (that is, one string wound round another, like those for the base notes of the piano), is—*their vibrations are in the inverse proportion to their diameters.* This is true upon the supposition that the specific gravity of the material made use of for the covering is the same as for the inclosed string, and also that the inner string and the covering form together one compact body. Wherefore, if their diameters be as 1 to 2, their vibrations are as 2 to 1—if as 3 to 5, their vibrations are as 5 to 3, &c.

27. The law in respect to covered strings in reference to weight, is—*their vibrations are in the inverse proportion to the square-root of their weights.* Wherefore, if the weights be as 1 to 2, their vibrations are as $\sqrt[2]{2}$ to $\sqrt[2]{1} = 1.4142$ to 1,—if as 3 to 5, their vibrations are as $\sqrt[2]{5}$ to $\sqrt[2]{3} = 2.2360$ to 1.7320.

28. The law in respect to vibrating springs or tongues, is—*their vibrations are in the inverse proportion to the square of their lengths* —that is, their lengths being as 2 to 1, their vibrations are as $1^2 = 1$ to $2^2 = 4$—their lengths being as 3 to 2, their vibrations are as $2^2 = 4$, to $3^2 = 9$, &c.

29. The law in respect to the thickness of springs, is—*their vibrations are in the direct proportion to their thickness*—that is, if the thickness be as 1 to 2, their vibrations are as 1 to 2,—if as 2 to 3, their vibrations are as 2 to 3, &c. Difference in respect to breadth, only affects the volume, not the pitch.

30. The law in respect to revolving wheels having teeth, * is —*their vibrations are in the direct proportion to the velocity*—and also, *in the direct proportion to the number of teeth contained in the periphery of each wheel.* That is, if with the same number of teeth, the velocity of one wheel to another be as 2 to 1, their vibrations will be as 2 to 1—if as 3 to 2, their vibrations will be as 3 to 2, &c. Or if, with the same velocity, the number of teeth be as 2 to 1, their vibrations will be as 2 to 1—if as 3 to 2, their vibrations will be as 3 to 2, &c.

31. The law in respect to the time occupied by an equal number of vibrations, is—*in the direct proportion to the length of*

* To elicit musical sounds by this means, the teeth of the wheel must be made to strike against a card, or pass through a narrow orifice.

the string—also in *the direct proportion to the diameter.* For example, if either the length, or diameter, be as 2 to 1, the time occupied by a given number of vibrations will be as 2 to 1—if as 4 to 5, the time occupied by a given number of vibrations will be as 4 to 5, &c.

32. It may be here expedient to point out some of the circumstances relating to the vibration of springs, with the view to show the perfect consistency that reigns in all the operations of nature. Let 0, fig. 2, represent the fixed end, and 7 the free end of the vibrating portion of a spring of equal thickness throughout, and the odd numbers 1, 3, 5, 7, equal portions of that length—then, if the time occupied by a vibration of a spring of the length of the portion 0, 1 be 1, the time occupied by a vibration of the length of the portion 0, 3, will be $1 + 3 = 4$, the square of 2—by a vibration of the length of the portion 0, 5, it will be $1 + 3 + 5 = 9$, the square of 3—by a vibration of the length of the portion 0, 7, it will be $1 + 3 + 5 + 7 = 16$, the square of 4, &c., § 28.

Here the law relating to length and vibration in respect to springs corresponds with that relating to time and distance in respect to falling bodies. If the spring be made gradually thinner towards the fixed end 0, the pitch will be lower—if it be made gradually thicker towards the fixed end 0, the pitch will be higher.

33. I now proceed to explain the facts presented by vibrating springs in reference to their thickness.

It is well known that the strength of materials is in proportion to the square of their thickness—hence the thickness being 2 to 1, the strength is 4 to 1—the thickness being 3 to 2, the strength is 9 to 4. But, in respect to springs, the ratio of the thickness being 2 to 1, the vibrations are 2 to 1 also, and not 4 to 1,—and the thickness being 3 to 2, the vibrations are also 3 to 2, and not 9 to 4, as might be expected, from jumping to a conclusion at once, without due circumspection. Here then let it be observed that the thickness being 2 to 1, the weight is also 2 to 1,—and that 2 vibrations of the weight 2 are equal to 4 vibrations of the weight 1, is evident. So also the thickness being 3 to 2, and the weight 3 to 2, the vibrations must also be 3 to 2, simply, because 3 vibrations of the weight 3 are equal to 9 vibrations of the weight 1,—and 2 vibrations of the weight 2 equal to 4 vibrations of the weight 1.

34. Various effects also result from holding metallic or glass

plates in a pair of pincers, at some extreme point—sprinkling fine dry sand upon their surface, and operating upon them with a bow. In so doing an immense variety of figures are formed by *lines*,* spontaneously created in the sand, in agreement with the nature of the particular harmonic thus produced. In all these multifarious effects law and order is everywhere found to prevail. But law and order in reference to time, space, magnitude, and motion, is *science*.

35. In the part which follows is described a series of facts, the result of experiment, which partly from their unlooked for effects, and partly from mystification and ignorance in regard to them, have excited more than ordinary attention—and, in short, more than what is actually due to them, when put in the scale with other facts, of a similar or different kind.

PART 3.

DESCRIPTION OF A SERIES OF EXPERIMENTS AND OF THE PHENOMENA RESULTING THEREFROM.

36. When a string vibrates from end to end, that is from A to B, fig. 3, its lowest sound, called the basis or basal sound, is thereby produced. † Those obtained from aliquot parts of the string, in the manner pointed out in what follows, being denominated harmonics. Hence it follows that, in respect to the vibrations of any and all its harmonics, a basis is a unit-sound—*that* is, a basis is a unit-sound indiscriminately to all numbers whatever which stand in the relation of harmonics to that basis —but a primary basis is the unit-sound to primary numbers only—the bases to products being *sub*-bases. This will be more distinctly perceived from what follows.

The straight line A, B, represents a string at rest—the curved line A, *a*, *a*, *a*, B, its elevated position, and the dotted line A, *b*, *b*, *b*, B, its depressed position, § 8.

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* The nodal points in respect to strings become lines with plates.

† The most effective means of eliciting the tones of a string is the bow—although a hammer, a quill, or the finger, may be employed for that purpose.

37. When the unit length, A, B, fig. 4, by which the basis is produced, is divided into two equal parts, by gently touching the centre point 1, * its vibrations, in respect to those of the unit-length, are accelerated in the ratio 2 to 1—the interval produced being the octave. In other words, the length of the portion by which the harmonic sound is produced being as 1 to $2 = \frac{1}{2}$ the unit-length, the ratio of its vibrations is 2 to $1 = \frac{2}{1}$ faster than those of the basis—and so universally in all other ratios. § 10.

In vibrating, the course of the string is always as represented above—that is, its elevated and depressed portions always dispose themselves in alternate series, and thus, by pulling in opposite directions, balance their forces one against another, so as to maintain a perfect equilibrium.

38. When the unit-length A, B, is divided into four equal parts, its vibrations are accelerated in the ratio 4 to 1, consistently with what has been said in § 37. But, as the number 4 is the product of 2×2, and therefore not a prime, it follows that the only interval that can possibly result from this division, is the octave to the octave, or double octave—or that which corresponds with the number 4 as the product of 2×2. That this must be the case, is moreover evident, upon mere inspection, and by comparing fig. 5 with fig. 4—for, except as to magnitude, fig. 5 is the same as fig. 4, repeated twice over. Furthermore, it will be found (fig. 5) that by slightly touching the node 1 only—or 3 only—or both together—or both these in conjunction with that of 2—the double octave to the sound of the unit-length A, B, will in every case be the result. But, if only the node 2 be touched, the octave, and not the double octave to the unit length, will be the result—because the fraction $\frac{2}{4}$, which answers to this node, is $= \frac{1}{2}$, and the effect of any aliquot division, is, in all cases, that answering to the fraction, expressed in its lowest terms.

Furthermore, as the second octave is the result of the subdivision of 2 into 4 equal parts, the third octave must result from the subdivision of 4 into 8 equal parts, &c. § 10.

Great care must be taken not to confound the idea of an *harmonic* with that of a *stopped* note—in the former the string is in all cases vibrating from A to B, except at the nodal points—in the latter the vibration passes entire from the stopped point to the fixed end, without being intersected by nodes. For the production of any harmonic, some one or

* These points, which are at rest, are called nodes, or nodal points.

more of the nodes appertaining to the particular harmonic, must be touched very gently, without forcing the string out of its natural, or rectilinear position, and with extreme accuracy with respect to the nodal point or points. Moreover, the bow must by no means be allowed to act either directly upon, or near any of the nodal points—for it must be evident, in the case of harmonics, that to bow upon a node, is to make an effort to disturb the very point which the finger, by its conjunction with some other node, is endeavoring to render quiescent. In stopped notes, the string, whether pressed out of its rectilinear position or not, must be firmly held down upon the neck or solid part below.

How important the knowledge of these facts, is to violin and violoncello performers, I leave them to judge

39. In figs. 6, 7, 8, 9, are represented the vibrations of the primes 3, 5, 7, 11. In fig. 6, the unit length A, B, is divided into 3 equal parts, its vibrations been accelerated in the ratio 3 to 1, producing the octave 5th, § 13—in fig. 7, into 5 equal parts, producing the double octave major 3rd, § 16—in fig. 8, into 7 equal parts, producing the double octave ♭7th, § 17—and in fig. 9, into 11 equal parts, producing the triple octave ♭5th, § 18. As these divisions are all the result of primary numbers, the harmonic produced by touching any one, any two, or more of the nodes appertaining to any given example is the same. In fig. 6, for instance, the sound is the same whether we touch the node 1, or 2, singly, or both together—and the result is the same universally, with the divisions resulting from any other primary number.

In the above figures the fixed end A, is also marked 0, to represent the starting point—and the fixed end B, by the highest number in the series—or that of the harmonic represented in each particular case.

As might naturally be expected, the higher harmonics require a longer string for their easy practical development—and though a gentle touch upon a single node, even with the harmonic 19, will cause the string to yield that harmonic—it nevertheless comes out clearer, and bolder, in proportion as the efforts of nature to maintain the fixed position of these nodal points, are assisted by touching other nodes also.

40. That by the subdivision of any of these primary divisions into 2, 4, 8, &c., parts, the original harmonic will be changed, respectively, to the octave, double octave, triple octave, &c., has been shown, § 10, &c. And since, in a practical point of view, it is unnecessary, except on special occasions, to distinguish octaves one from another, the products arising from the multiplication of any number by 2, 4, 8, &c., may, therefore, for all ordinary purposes, be looked upon as identical with that number.

41. The intervals produced by the harmonics treated of

above, I here repeat in a tabular form, the better to impress them on the memory:—

The interval between the basis and harmonic represented by, and resulting from the prime.

2	is the 8th or octave	as from	C	to	C
3	„ 12th, or octave 5th ...	„	C	„	G
5	„ major 17th, or double octave major 3rd		C	„	E
7	„ ♭21st, or double octave ♭7th		C	„	♭B
11	„ ♭26th, or triple octave ♭5th .	„	C	„	♭G
	„ ×24th, or triple octave ×3rd	„	F	„	♯A *
13	„ ♯26th, or triple octave ♯5th	„	C	„	♯G
	„ ♭♭28th, or triple octave ♭♭7th	„	C	„	♭♭B †
17	„ ♯29th, or fourth octave ♯1st	„	C	„	♯C
	„ ♭♭31st, or fourth octave ♭♭3rd	„	C	„	♭♭E †
19	„ minor 31st, or fourth octave minor 3rd		C	„	♭E

42. In figs. 10, 11, 12, are represented the vibrations of the harmonics 9 and 15,—as the products 3×3,—3×5,—and 5×3. In fig. 10, the unit-length, A, B, is divided into 3, and also into $3 \times 3 = 9$ equal parts. By its division into 3 equal parts only, it is the same as fig. 6, and the subdivision of each of these portions into 3 smaller portions, which are also equal, is but the operation of the same principle upon a small scale. But, since the laws relating to the science of acoustics are of universal application, if the 3rd division of the unit-length, A, B, produces the 12th or octave 5th to the unit-sound answering to that length, the 3rd division of the short unit-lengths 0, 1,—1, 2,—2, 3, must also produce the 12th or octave 5th to these unit-lengths—or what amounts to the same thing, the 9th division of the unit-length, A, B, must, in obedience to this law, produce the interval of the 12th + 12th = 23rd, § 15, or triple octave to the major 2nd.

43. In fig. 11, the unit-length, A, B, is divided into 3, and also into $3 \times 5 = 15$ equal parts. By its division into 3 equal parts only, it is the same as fig. 6, and by the subdivision of each of these portions into 5 small portions, which are also

* As in the instances here given, every example refers to C as the key-note, the reason the ×3rd is exemplified by the notes F, ♯A, and not by C, ♯E, is, because ♯E does not belong to the key of C.

† The note here expressed as ♭♭B, differs from A, the major 3rd to the basis F, and the note here expressed as ♭♭E, from the note D the fifth to the basis G. It is necessary here to make this appear, although in practice it may be overlooked in the notation, for the sake of simplicity. For, though it is of importance, in respect to the notation, to adhere to rule and consistency, so far as it is useful so to do, it becomes pedantry to carry it farther —otherwise in some keys we should become involved in triple sharps and flats.

equal, the interval produced is the major 17th, or double octave major 3rd to the short unit-lengths, 0, 1,—1, 2,—2, 3, —consequently the 15th division of the unit-length, A, B, must produce the 17th + 12th = the major 28th, or triple octave to the major 7th.

44. In fig. 12 the unit-length, A, B, is first divided into 5, and then into $5 \times 3 = 15$ equal parts. By its first division into 5 equal parts only, it is the same as fig. 7, and by the subdivision of each of these small portions, which are also equal, the interval produced, is the 12th or octave 5th to the short unit-lengths, 0, 1,—1, 2,—2, 3,—3, 4,—4, 5;—consequently the 15th division of the unit-length, A, B, must produce the 12th + 17th = major 28th, or triple octave to the major 7th, the same as in fig. 11.

45. It will now be seen that the connection, or affinity, between a basis, simply as being a unit-length to any of its aliquot portions, and the harmonics produced by, and resulting from, the primes is immediate, and the closest possible—and to products mediate, that is through the intervention of the primes, and consequently more remote. A little attention to figs. 10, 11, 12, will also make it evident that with products, and the basis to any product, the result is, in all cases, in strict accordance with the nature of that product. The product 9, for example, can only be obtained as 3×3—but the product 15 may be obtained either as 3×5, or 5×3—the product 21 either as 3×7—or 7×3—the product 27, only as $3 \times 3 \times 3$, &c.

46. Besides the intervals existing between sounds, one of which is a basis, and the other a harmonic, and the obvious distinction between those which originate in primes, and others which originate in products—it is evident that those which refer to harmonics only, and not to any basis, must also give birth to new varieties—such, for instance, as the minor 3rd between the prime 5 and the product 6—the minor 3rd between the product 6 and the prime 7—the major 3rd between the prime 7 and the product 9—the major 3rd between the prime 19 and the product 24, &c. Wherefore, to distinguish intervals, the one from the other, according to their genera and species, I call those *basal* in which one of the two sounds is the basis—*primary* when the numbers representing the ratio of their vibrations are primes—*secondary* and *tertiary* when the numbers representing the ratio of their vibrations are products—and the

intervals between the harmonics of the same basis, or different bases, I call *intermediate.*

47. To prevent the evil consequences which result from taxing the memory with too many particulars, before proceeding to practice, and with the view to consolidate and establish the facts already advanced, I herewith present the reader (ex. 11—16) with a few preparatory exercises, in all of which the key is C.

In these examples, 1 shows that the note over which it stands is the basis, or some one of the octaves to the basis—3, that the note over which it stands is the harmonic 3 to the basis—5, the harmonic 5 to the basis, &c. And, since these numbers are alike applicable to all keys, the student is recommended to transpose these examples into several, if not into all the keys.

48. Of the almost innumerable actual combinations presented in music, a few, and only a few, have specific names given to them, and they are then called chords. Thus, the combination in ex. 13, bar 1, is called the major common chord, and also the harmonic triad—that in bar 2, is called the minor common chord, but not the harmonic triad, although equally entitled to the appellation. The combination in bar 3 is denominated the chord of the 7th, but more correctly the chord of the flat 7th—that in bar 4 is called the chord of the added 6th—in bar 5, the chord of the extreme flat 7th—in bar 6, the chord of the extreme sharp 6th.

As these chords, except those in bars 5 and 6, are all named by reference to their bases, there is no reason for changing the denomination at present affixed to them. It would, however, be better to call the combination in bar 5 the chord of 17, 7, and that in bar 6 the chord of 19, 17. For, it is to be noted, that any combination or chord, whatever, can be thus classically symbolized and named by reference to two or more of its principal harmonics.

PART 4.

THE UNIT-SOUND, COMMONLY CALLED THE GRAVE HARMONIC, OR GENERATED THIRD SOUND.

49. With the view to form a correct idea of a musical sound, let us imagine a series of knocks, produced at equal intervals of time by a machine, sufficiently slow to be counted, and indivi-

dually distinguished, the one from the other. The noise thus produced no one would call musical, nevertheless, by merely accelerating the time, so that the knocks could no longer be counted, and individually perceived, a musical sound is the result—the pitch being higher, as the knocks are quicker, and lower, when slower—and the sound louder or softer in proportion to their force and number in a given time. It is also evident that if two or more knocking machines were set to work, and so constructed as to unite periodically, these united knocks must needs affect the ear so as to be distinguished from the weaker or single knocks. Let the reader now attach to the idea of the united knocks the term basis, or unit-sound, and to the idea of the single knocks the term harmonic, and in so doing he will lay the foundation of the science of music upon a rock which nothing can overturn. By a little attention to the subject it will also be perceived that the effect of continuity in respect to a series of knocks, a revolving spark, &c., all proceed from a similar cause.

50. The dots in figs. 13—18 represent vibrations, slower or quicker in proportion to the space between them—those coincident with the generated basis being represented by stars.

51. Fig. 13 represents the unison, properly so called, the ratio of which is 1 to 1.

52. Fig. 14 represents an imperfect unison—that is, the unison out of tune, which, in point of fact, is not a unison.

Imperfect unisons are of two kinds, those which involve high, but definite ratios—as 1,000 to 1,001—1,000,000 to 1,000,001, &c., by the coincident vibrations of which a *beat* is periodically generated—and others, the ratio of whose vibrations involve surd quantities, by which no beat can be produced.

53. Fig. 15 represents the unison and octave, the ratio of whose vibrations is as 1 to 2.

54. Fig. 16 represents the octave, the 12th or octave 5th, and the generated basis, the ratios of whose vibrations are 1, 2, 3.

55. Fig. 17 represents the octave, the 12th, the 17th, or double octave major 3rd—and the generated basis, the ratios of whose vibrations are 1, 2, 3, 5. These are the elements of the major common chord.

56. Fig. 18 represents the octave, the 12th, the 17th, the ♭21st, or double octave ♭7th—and the generated basis, the ratios

of whose vibrations are 1, 2, 3, 5, 7. These are the elements of the chord of the ♭7th.

57. Upon examination it will be seen, that if the unit-sounds, represented by the stars, were erased from figs. 16, 17, 18, as the vibrations answering thereto are still retained in the upper parts, the effect ought also to be retained, though not with the same force—and this is what we actually experience. It is, moreover, evident that the generation of spontaneous unit-sounds is not fortuitous, but essential—and also that the volume of tone must keep pace with the the number of harmonics which thus periodically become coincident.

58. Let us now imagine two machines made to produce a series of knocks, in perfect agreement with the order represented by fig. 13—one machine for the upper, and another for the lower row. In this case it is evident that the effect of setting both machines at work simultaneously would be more noise—or, in other words, merely an increase of power. If fig. 14 be thus exemplified, a series of beats may, or may not, be the result; this being dependent upon the ratio involved by the imperfect unison, § 52. If fig. 18 be thus exemplified, the result will be an alternate series of louder and softer knocks—and, as the louder knocks are occasioned by the coincident knocks of the two machines, the attention must necessarily be drawn to this fact, the effect of which is, the basis. In figs. 16, 17, 18, these unit-sounds are represented as being generated by the harmonics above.

59. In fig. 16, if the two machines, after starting together, continue their knockings in perfect agreement with the ratio 2 to 3, it is evident the basis must be generated, whether that basis be superadded or not.

60. To put fig. 17 to the same sort of experimental test, three, and for fig. 18, four, knocking machines would be required—which, after starting together, as before, must continue the knockings in perfect agreement with the ratios therein described. In this case it is evident that the crash occasioned by the simultaneous knocks of three or four machines, as represented by the stars, must be stronger—that is, louder, than where these louder knocks are the result of the united action of only two machines—and that the force of their united knocks must correspond to their number, those resulting from each machine being understood as equally loud.

61. Now, it will, perhaps, be asked, will not the effect produced by these machines be a mere clattering noise—and if so, what has this to do with music? But let the reader have a little patience, and he will find that this idea is not quite correct. It is indeed true that this would be the effect, upon the supposition that the individual knocks were all sufficiently loud and slow to be distinctly and individually perceived, but not otherwise. Let us suppose, for example, the individual knocks to be very soft, and so exceedingly quick that the vibrations represented by the prime 2 take place at the rate of 200 in a second—those by the prime 3 at the rate of 300 in a second—those by the prime 5 at the rate of 500 in a second—and those by the prime 7 at the rate of 700 in a second. In this case, besides the individual and peculiar effect of each particular prime, we shall have, as the common or general result of their combined forces, the effect of the *unit-sound* or *basis*, as represented by the stars in fig. 18—that is, the effect of the sound whose vibrations are 100 in a second. This will be seen in ex. 17.

It is to be observed that the sounds represented by the notes in connexion with these ratios will be perfectly in tune, though somewhat below the modern concert pitch.

62. Having shown the means by which harmonic sounds can be elicited from a vibrating string, and also that by the union of two or more harmonic sounds the addition of the basis is the natural result, it will now, in some measure, be seen that harmonic sounds are by no means an essential accompaniment to basal sounds—that is, those sounds whose vibrations extend from one end of the string to the other, fig. 3. This fact, however, will be more distinctly perceived upon a future occasion, when the nature of compound vibrations comes to be examined. By compound vibrations is meant the vibrations of a string simultaneously producing its basis, and some one or more of its harmonics. It must, however, be evident, that by the union of two or more harmonic sounds, the generation of the basis is essential—that is, the production of the basis is the necessary and inevitable consequence, except where the ratio of the vibrations involve surd numbers.

63. It is further to be remarked, that when the bases thus generated are sufficiently slow to admit of their vibrations being counted, or individually perceived, they are denominated beats

—and, moreover, that where the fact is unknown that what are called fundamentals, roots, bases, and beats, &c., all mean the same thing, namely, unit-sounds, the mind must necessarily be in the utmost state of confusion in regard to them.

64. Bases have hitherto been universally regarded as being all alike as to their nature and quality. This, however, is not the case, for bases are of *three* kinds—namely, *primary*, *sub*, (secondary, tertiary, &c.) and *medial*. The nature of primary bases having been explained, I proceed to the examination of sub-bases. By the division of a string into 9 equal parts, fig. 10, the interval of a wholetone above the third octave is produced—and by its division into 15 equal parts, figs. 11, 12, the interval of the major 7th above the third octave is produced. But as 9 is the product of 3×3, the harmonic resulting from this ratio will not be produced by touching the nodal point 3 or 6, because $\frac{3}{9} = \frac{1}{3}$ and $\frac{6}{9} = \frac{2}{3}$, as represented by the numerals. The effect produced by touching these nodal points will therefore answer to the prime 3, and not to the product 9, as in the case of the other nodal points.

65. From hence it follows that a sub-basis is a basis to a basis, and that the bases to products must in all cases be sub-bases. It is, moreover, evident, that the product by which any basis is expressed may involve two or more primes, and be thereby removed farther and farther from the simplicity of a primary basis. In these examples, for instance, if we touch the nodes $\frac{5}{15}$ or $\frac{10}{15}$ the result will answer to the prime 3, and not to the product 15, because $\frac{5}{15} = \frac{1}{3}$ and $\frac{10}{15} = \frac{2}{3}$. And, again, if we touch the nodes $\frac{3}{15}$, $\frac{6}{15}$, $\frac{9}{15}$, or $\frac{12}{15}$, the result will answer to the prime 5, and not to the product 15. The reason is because $\frac{3}{15} = \frac{1}{5}$, $\frac{6}{15} = \frac{2}{5}$, $\frac{9}{15} = \frac{3}{5}$ and $\frac{12}{15} = \frac{4}{5}$. Hence it follows that though the nodal points 1, 2, 4, 7, 8, 11, 13, and 14, are adapted to realize the effect answering to the product 15, the other nodal points are not so—whereas, in the case of primary harmonics, the effect produced by touching any one or more of the nodes appertaining to any harmonic in particular is throughout the same. It is, moreover, to be observed, in regard to the products 9 and 15, that the result must necessarily be as stated above, from what has been said concerning figs. 6, 7, or otherwise the statements in one place, would contradict what is said in another.

66. If we now proceed to examine the numbers from 1 to

41, with reference to their inherent qualities, the matter will be found to stand thus:—1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, are all primes—of these there are fourteen; 4, 6, 8, 10, 12, 14, 16, 20, 22, 24, 26, 28, 32, 34, 38, 40, are strictly speaking all products—of these there are sixteen. But as these numbers, in respect to music, only differ from primes by referring the unit-sound or basis to one or more octaves below the primary basis itself, these also are classed among the primes. 9, 15, 21, 33, 39, are all products—of these there are but five. But as these numbers are all divisible by 3, and by so doing become resolved into the primes 3, 5, 7, 11, 13—and thereby refer, as primes, to another basis belonging to the same key—these also are classed among the primes;—to which are to be added, 18, 36, the octaves to 9, and 30 the octave to 15. 25, 27, 35, are products, 25 being the product of 5×5;—27 the product of $3 \times 3 \times 3$; and 35 the product of 5×7 or 7×5. From hence it appears that out of the series of numbers from 1 to 41,—38, in respect to music, are primes, and only 3 products. In other words, that out of this series 3 only represent sounds in so far extraneous to the system, as not to constitute any prominent feature therein. But this fact will be much more clearly set forth and explained hereafter.

67. That the reader may not imagine that what has here been said concerning the vibrations of harmonic sounds, and the coincident vibrations or bases thence arising are, as generally supposed, mere matters of curiosity, and in their own nature barren of utility, it is requisite to inform him that the facts hitherto described will hereafter be exhibited in a new and systematic form. THAT IT IS BY REFERENCE TO THESE FACTS, AND THESE ONLY, THAT THE LAWS RELATING TO MELODY AND HARMONY ARE OBTAINED, AND THAT THIS IS ACCOMPLISHED BY THE ADOPTION OF A COURSE OF GENERALIZATION AT ONCE SIMPLE, NATURAL, AND ORDERLY.

PART 5.

SYSTEMS OF SOUNDS.

68. Systems of sounds (improperly denominated scales) derive their origin from the constitution of the human mind, whereby the relation or affinity between a vast number, and in

respect to quality, an immense variety of particulars is perceived as constituting one united whole. In this operation, the past, by the uniting medium of the *memory*, is, as it were, rendered present, and thereby conjoined and compared with that which is actually so.

69. A system of sounds, when portrayed to the eye, gives birth to the formal display of a series of sounds standing in a fixed relation the one to the other. That system upon which practical music is based, being distinguished from those of a merely fanciful and artificial construction, by the homogeneity of its elements, and also by the natural order of its development—which order proceeds through three distinct, and as it were discrete degrees—in each of which is represented an image of what is infinite, or at least of what is indefinite.

If we are debarred from thus referring the evanescent effects, and never-ending varieties, appertaining to the practice of music, to the system of sounds which thus portray its fixed relations, we are thereby deprived of the means of investigating those effects by reference to their causes—and of thus perceiving that the phases presented by things mutable, are ever dependent upon that which is immutable. But the propriety of systematizing the materials obtained from practical music, involves in the very idea of such an undertaking the establishment of fixed and determinable relations.

That various systems are frequently spoken of, and alluded to, in reference to which no fixed and determinable sounds are represented, or understood, is undoubtedly true. But it is evident that systems based upon mere visionary ideas, are no systems at all—and, at the very best, can do no more than shadow forth that which is real. It is moreover to be observed that the laws and principles relating to things evanescent and transitory, must be sought for by reference to things fixed and immutable, to the secret influence of which they owe all their powers, qualities, and effects.

According to this doctrine the whole theory of harmony is, in a certain sense, summed up, and concentrated, in the knowledge of that system to which the practice of music ever points—and of which *nature alone, and not man, is the builder.*

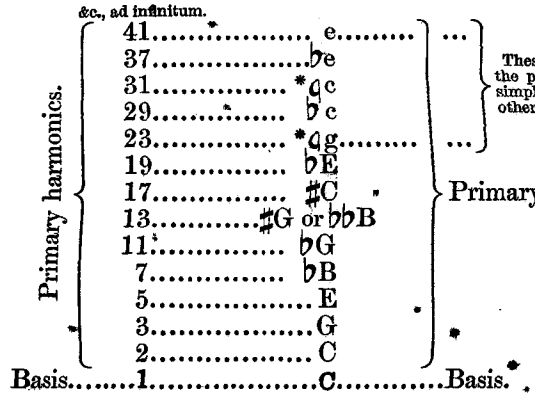
The question therefore in regard to any chord, or combination of sounds, should be what are the elements of which it is composed, in respect to this system of sounds? This being ascertained, the facts relating to the ratio of its vibrations, practical intervals, and name (if any) are also known.

To jump at conclusions concerning things interior, from external and irrelevant matters —the method hitherto adopted in regard to chords—is the fruitful source of fallacies of all kinds. Such a mode of procedure ought therefore to be condemned, and will be condemned, and discountenanced, by those who are rational, the moment its error begetting tendency is perceived

How, for instance, can any one judge accurately concerning the nature and quality of sounds, while the precise pitch, or, in other words, the sounds themselves are unknown? Moreover, is it rational for any one to contend, that to know this (the pitch) within certain limits—that is, the mere thereabouts concerning it, is quite sufficient? And yet this is all the information the notation is calculated to afford. Would such an unphilosophical mode of procedure be admissable in any other science? Would it not, in fact, be instantly rejected with scorn and derision?

70. Consistently with the natural order of development, there is a SIMPLE SYSTEM, a TRIUNE SYSTEM, and a GREAT SYSTEM.

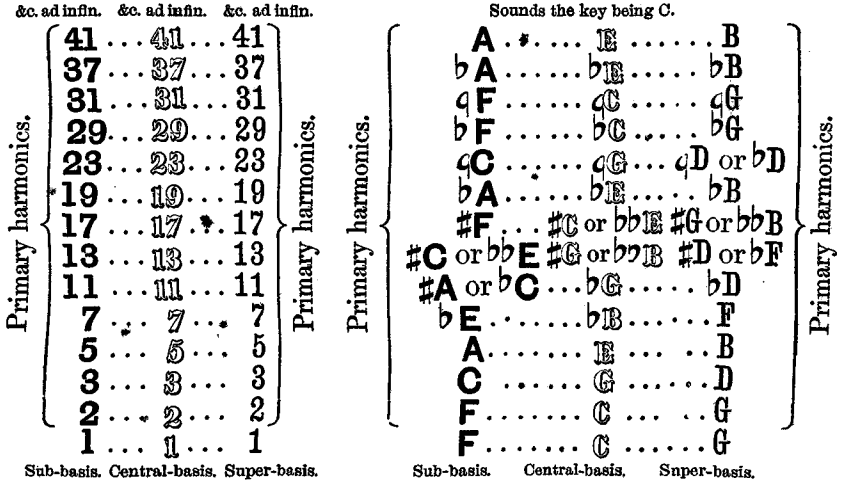
71. The SIMPLE SYSTEM is composed of one basis only, in connection with an indefinite series of primes or primary harmonics appertaining to that basis, as in the following example:—


&c., ad infinitum.
41 ..... e ....
37 ..... ♭e
31 ..... *♳c
29 ..... ♭c
23 ..... *♳g ....
19 ..... ♭E
17 ..... ♯C
13 ..... ♯G or ♭♭B
11 ..... ♭G
7 ..... ♭B
5 ..... E
3 ..... G
2 C
Primary harmonics.
These sounds cannot be distinguished upon the piano from those which answer to the simpler harmonics of the same, or some other basis, appertaining to the same key.
Primary harmonics to the basis C.
Basis..... 1 ..... C ..... Basis.

It is to be observed in regard to this system of sounds that notwithstanding the unlimited extent of its primary harmonics, it is nevertheless almost barren of utility, because the sounds resulting from the ratios of its vibrations, except those of 2, 3, 5, or 2, 3, 19, create a strong desire to flow into, or resolve themselves into, the simpler harmonics of another simple system.

72. The TRIUNE SYSTEM is produced by uniting three simple systems into one. The three simple systems which thus become consolidated being those, and those only, which stand in the simplest possible relation the one to the other.

73. It being necessary to distinguish the basis and harmonics of the sub, central, and super systems, one from another, I have used characteristic numerals and letters for that purpose. Thus **1..9**, for the *sub*—𝟙..𝟡, for the *central*—and **1..9** for the *super* system.


&c. ad infin. &c. ad infin. &c. ad infin.
41 ... 𝟜𝟙 ... 41
37 ... 𝟛𝟟 ... 37
31 ... 𝟛𝟙 ... 31
29 ... 𝟚𝟡 ... 29
23 ... 𝟚𝟛 ... 23
19 ... 𝟙𝟡 ... 19
17 ... 𝟙𝟟 ... 17
13 ... 𝟙𝟛 ... 13
11 ... 𝟙𝟙 ... 11
7 ... 𝟟 ... 7
5 ... 𝟝 ... 5
3 ... 𝟛 ... 3
2 ... 𝟚 ... 2
1 ... 𝟙 ... 1
Primary harmonics.
Primary harmonics.
Sub-basis. Central-basis. Super-basis.
Sounds the key being C.
A ... 𝔼 ..... B
♭A ..... ♭𝔼 ..... ♭B
♮F ..... ♮𝕔 ..... ♮𝔾
♭F ..... ♭𝕔 ..... ♭𝔾
♮C ..... ♮𝔾 ... ♮D or ♭D
♭A ..... ♭𝔼 ..... ♭B
♯F ... ♯𝕔 or ♭♭𝔼 ♯𝔾 or ♭♭B
♯C or ♭♭E ♯𝔾 or ♭♭B ♯D or ♭F
♯A or ♭C ... ♭𝔾 ..... ♭D
♭E ..... ♭𝔹 ..... F
A ..... 𝔼 ..... B
C ..... 𝔾 ..... D
F ..... 𝕔 ..... G
F ..... 𝕔 ..... G
Primary harmonics.
Primary harmonics.
Sub-basis. Central-basis. Super-basis.

* The symbol (♮) signifies *flattish*, but though ♮c is ♮c and ♮g is ♮g on the piano, the sound required is not the same, but in both instances considerably flatter.

Here the letters answer to the figures with which they correspond, as to position. For example, in the key of C, the harmonic **41** is represented by the note **A**—41 by E—and 41 by B &c.

74. In regard to the question whether the ratio of the vibrations of the sounds on the right are correctly portrayed by the numerals on the left—though this, strictly speaking, is a matter for experiment, and for experiment only, to decide—yet abundant evidence will hereafter appear to convince that in reference to every particular, the one is the true exponent of the other. In the mean time I vouch for the facts, one and all, and also for the correct and consistent application of the double notation in every case in which it is found to occur. To attempt to prove this, and other similar matters, would at present be premature.

75. That the student may become fully prepared to go to work himself, and thereby gain strength, and confidence, in the pursuit in which he is now engaged, I here present him with a table showing the ratios which answer to some of the octaves to the primes 1 to 41,—and it must be constantly borne in mind, that these ratios can be carried on to any extent, and still be represented by the prime or primary ratio which stands below.

TABLE

Of the ratios of the octaves to the primes 1 *to* 41.

352	368	384	416	448	464	496	512	544	592	608	640	656	704
176	184	192	208	224	232	248	256	272	296	304	320	328	352
88	92	96	104	112	116	124	128	136	148	152	160	164	176
44	46	48	52	56	58	62	64	68	74	76	80	82	88
22	23	24	26	28	29	31	32	34	37	38	40	41	44
11	.....	12	13	14	.....	.....	16	17	.....	19	20	.....	22
		6	.....	7	.....	.....	8	.....	.....	.....	10	.....	11
		3	.....	.....	.....	.....	4	.....	.....	.....	5		
							{2 1}						

It is to be recollected that though, numerically, the octaves to primes are not primes, they are to be regarded as such in reference to music. § 10, 11, 12, 37.

76. I now proceed to the practical development of the triune system as shown § 72. For the present, however, I pass over the differences involved in the ratios representing the same sounds upon keyed instruments, and in many instances also the same note in respect to the notation. Of course the letters of the simple and triune systems can be transposed to any other key—the numerals on the left remaining the same. It is evident also that the observation relating to the primary ratios 23, 29, 31, 37, 41, in speaking of the simple system, is equally applicable to the triune system.

77. It has already been shown by ex. 11—16, § 47, how much can be done, by the application of a few elementary principles, which have truth for their foundation—since, even here we have chords, and other matters, explained by a method quite new, free from all ambiguity, and presenting at the same time points of importance for future investigation.

78. In the examples which follow, characteristic numerals will constantly be employed (§ 73). This will be found a great improvement—since in this case, the note, or sound itself, in reference to any key, is thereby accurately determined.

By attending to the bases, or to the character of the numerals, in these examples, it will be seen that the basis changes with every note as we ascend alphabetically—but not uniformly so, as we descend. The two consecutive notes referable to the same basis, are pointed out by the slur over the numerals.

79. These particulars, insignificant as they may seem to some, are nevertheless pregnant with important consequences, as will hereafter be seen. But since particulars flow from things more general—these from things universal—and the whole concentred in their effects—and since all this cannot be put forth at once—let us, for the present, test this matter by the ear, and it will thence appear that the substitution of G, instead of F as the basis to the note F in the ascending series—or of F, instead of G in the descending series—is, in both instances, repulsive to the ear. But though ♭B 7, is it be preferred to ♭B 19, this is not a parallel case with that of the note F.

80. By attention to these two examples (18, 19) it will also appear that, in conformity with the order appertaining to this series of sounds, F, and ♭B, are the only notes referable to two distinct bases. For though the note C is a harmonic 3 in reference to the basis F, and the note G a harmonic 3 in reference to the basis C, this does not prevent their being per-

ceived as bases to their own system of harmonics, and still preserving their identity as to pitch. But, between **F 1** and **F 7**, and between **♭B 19** and **♭B 7**, the difference of pitch is considerable. As these examples will shortly become the subjects of further investigation, what is here said must suffice for the present.

81. In ex. 20 it will be seen that the five notes appertaining to the semitonic scale, and which, in the key of C, fall upon the black keys of the piano, are all referable to the three bases **F**, **C**, **G**, taken from the triune system, § 72. And it is to be remarked that though, in every instance, the notation commences with the flat of the higher note, and ends with the sharp of the lower (higher and lower in respect to their position upon the staff), the sounds represented *constantly ascend* from first to last, as will be seen hereafter.

82. The sounds in ex. 18, 19, are, in most instances, *diatonic*, and those in ex. 20, *chromatic*—and hence has arisen the terms diatonic and chromatic, in reference to scales—together with certain obscure ideas respecting what these terms signify. The general notion that the structure of the diatonic scale is simple, and that of the chromatic more complex, is true—but the idea that these scales constitute two really distinct systems of sounds is evidently false, since the diatonic is only the outline, or skeleton, of the chromatic, which is the ONLY SYSTEM OF NATURE.

83. The fact of the triple function, so to speak, of each of the five chromatic intervals practicable upon keyed instruments, to say nothing of the numerous delicate shades, which the primes above 19 are calculated to afford, is quite sufficient to account for the "confusion doubly confounded" into which the science of music has been thrown, by the peddling and tinkering interference of persons utterly unfit for the task with which they though fit to meddle.

84. If ex. 18 be compared with ex. 19, it will be perceived that notwithstanding a general resemblance, there exists a characteristic distinction between them—and that this proceeds from the introduction of the prime **19**, instead of **5**, one of the octaves to which is 20 (see table § 75)—of **19**, instead of **5**—and, if we think proper, of **19** instead of **5**. But, by so doing, we are led to make the long sought for great discovery respecting the origin of the modes denominated major, and minor—and thereby of numerous other particulars which originate in the same cause. As this subject will hereafter constitute an

especial object of attention, it is only necessary here to observe that there are THREE modes—that which has hitherto been called major, I call the *first* mode, that hitherto called minor, I call the *second* mode—to which is to be added, my new discovery of the *third* mode—the nature of which will be explained hereafter.

85. The following is an outline of the GREAT SYSTEM, or system of systems, in which is portrayed the 19 triune systems of which it is composed.

Sub Systems.									Central System		Super Systems.								
1/19	1/17	1/15	1/13	1/11	1/9	1/7	1/5	1/3	1/2	2/1	3/1	5/1	7/1	9/1	11/1	13/1	15/1	17/1	19/1
3	1	11	8	6	2	10	4	7	0		7	4	10	2	6	8	11	1	3
A	♭C	♭D	♭F	♯F	♭B	D	♭A	F	C		G	E	♭B	D	♭G	♯G	B	♯C	♭E
	¶		†	*		§	‡					†		§	*	‡	¶		

Here the fraction, in every separate compartment, stands as the representative of a triune system, and consequently of three simple systems, and the whole as representative of the great system, or system of systems. The unit, in the centre of the system, represents, in its individual capacity, that basis, which, as being the centre to all the other bases, is the absolute centre, or centre of centres, and central root of the great system. But as the representative of an entire triune system, it stands for all the elements appertaining to that triune system. The fraction $\frac{1}{2}$ on the left, and $\frac{2}{1}$ on the right, in the same compartment, is meant to represent not only all the octaves to the central unit itself, but also all the octaves to all the primary ratios of every simple and triune system appertaining to the great system. But since in practice, it is unusual, unnecessary, and inconvenient, except for particular purposes, to fix the attention to any one octave—it follows, that the influence of the prime 2, and all its powers, must also be overlooked. In other words that theory and practice may work together, they must in all cases be rendered commensurate the one with the other, either as being full and complete or as an outline.

86. The numerator units, on the left, represent the central bases to a series of triune systems denominated sub-systems—

**, ††, ‡‡, §§, ¶¶. These sounds are practically, but not theoretically identical.

the denominator units, on the right, represent the central bases to a series of triune systems denominated super-systems—the series of super-systems being in the reverse order of the sub-systems. In both series the fractions express the ratios of the vibrations of the central bases, to the central basis of the central system. For example, the central basis $\frac{5}{1}$, represents a basis, the ratio of whose vibrations is 5 to 1, to the central root of the great system—and the centre basis $\frac{1}{5}$, represents a basis, the ratio of whose vibrations is 1 to 5 to the root of the great system. Moreover, the ratio and interval between the central bases, in regard to any two triune systems, also represents the ratio and interval between all the corresponding particulars appertaining to those two triune systems—that is, the ratio and interval appertaining to **5**, **5**—5, 5—5, 5—**7**, **7**—7, 7—7, 7, &c., in respect to any two triune systems, is, in all cases, the same as the ratio and interval appertaining to their bases.

87. The small figures above the letters, relate to the semitonic scale of the piano, &c.—those of the super-system being counted upwards, and those of the sub-system downwards, from the central point 0. The notes to which these intervals refer, if C be taken as the key-note, or central basis to the central system, is represented by the letters below—and, it is almost needless to add, that by transposition, any other sound may be taken as the key-note, or central basis to the central system. To find the ratio of the central basis of a sub-system, to the central basis of a super-system—multiply the denominator of the former by the numerator of the latter, and the product will give the ratio required. For example, as $\frac{1}{5} : \frac{5}{1} :: 1$ to 25.

88. In the triune systems denominated super-systems, the central root of the great system stands in the relation of a basis to the other central bases—and these in the relation of harmonics to that basis. In the triune systems denominated sub-systems, the central root of the great system stands in the relation of a harmonic to the other central bases, and these in the relation of bases to that harmonic. And hence it is that, to the ear, the effect of the former is such as to convey to the mind the idea of progression, and of the latter the idea of retrogression. The sensation of repose, or state of being at rest, being, as it were, reserved for the suitable elements of the central system. *

* In this work the transition from a basis to any of its harmonics is called *direct*, and from a harmonic to its basis *retrograde*.

89. The reason why in the series of sub and super-systems appertaining to the great system, the bases to the triune systems are not extended beyond the prime 19, is, not because *nature* has placed a barrier at this particular point, but because *art* has done so. In other words, because our keyed instruments are incapable of giving effect to the ratios arising from any of the higher primes, and our notation incapable of representing their effects. And though the voice, and instruments of the violin kind, are not restricted by the nature of their construction, like many others, nevertheless since in such cases the performers would find themselves, as it were, at sea, out of sight of land, and without a compass to direct them, it is questionable whether experiments of this kind could be tried by such means, with any chance of success. At all events the gross ignorance, and false notions, which universally prevail in respect to all that truly relates to music as a science, has hitherto prevented even the contemplation of experimenting in this way, and with such an object in view, either by means of the instruments now existing, or by others especially adapted to the purpose. In short, we never have had, nor, perhaps, for a long time are likely to have, any practical experience in regard to such matters.

90 To show how far the laws relating to truth and order, in nature, transcend the narrow boundaries of art, let us now compare the triune systems appertaining to the great system with the system of 12 mean semitones in the octave, and the inherent defects of that artificial system will become manifest. But first, as the great system demands at least 19 sounds in every octave, merely for the central bases of the triune systems of which it is composed—and since in the artificial system there are but 12 to answer this demand—it is evident that this can only be done by making some of these artificial sounds stand in the place of what ought to be two distinct sounds.

91. The particular cases in which the fact, as above stated, comes into operation, relate to the central bases $\frac{1}{5}$, and $\frac{1}{1}^{3}$, or $\frac{1}{13}$, and $\frac{5}{1} = 1$ to 65,—$\frac{1}{7}$, and $\frac{9}{1}$, or $\frac{1}{9}$, and $\frac{7}{1}$, $= 1$ to 63,—$\frac{1}{17}$, and $\frac{1}{1}^{5}$, or $\frac{1}{15}$, and $\frac{1}{1}^{7} = 1$ to 255,—and, lastly, $\frac{1}{11}$, and $\frac{1}{1}^{1}$, $= 1$ to 121. Now in each of these seven cases the practical interval answering to the product 65 and 63, is $2^6 = 64$,— that answering to the product 255, is $2^8 = 256$,—and that answering to the product 121, is $2^7 = 128$. That is, in practice, the interval represented by the ratio of the central basis of the sub-system,

united to the interval represented by the ratio of the central basis of the super-system with which it is conjoined, will, in every instance, produce the octave (that is some particular octave), § 10. But the true ratio of the octave is 2, 4, 8, 16, 32, 64, 128, 256, &c.—and hence the ratio 65, differs from that octave as 65 to 64—the ratio 63, as 63 to 64—the ratio 255, as 255 to 256—and the ratio 121, as 121 to 128. Now, if these 7 cases be deducted from 19 (the number of central bases really appertaining to the great system), there remain 12, which answers to the number when reduced to practice—that is to the number of semitones in the octave upon keyed instruments.

92. From what has been said it must be evident that, in a practical, or merely key-board, point of view, the study of the great system can offer but little assistance to the student of musical harmony—its domain being too vast and too high, to submit to the shackles of our prim artificial systems—or to square with our stiff and false notions in regard to such matters. Nevertheless, one difficulty, and one which has hitherto been looked upon as insurmountable, it does remove, and that most effectually. The point here alluded to is this—it has been contended that the necessity for false, that is, imperfect ratios, is *an evil for which nature herself is answerable*—that it is a defect *lying at the root of the very constitution of things, and not a mere bungling in regard to it, in matters relating to our own contrivances.* It is maintained, for instance, that let us fix upon what ratios we please, for musical intervals, and then proceed freely to modulate from one key (*i. e.*, triune system) to another, and finally return to the original key—the original pitch of that key can seldom be preserved, except by the admission of falsified ratios. And true it is that, unless we accede to the doctrine here laid down concerning the existence of a great system, it is impossible to rebut this charge.

Now, the reason of this is that, while the existence of the great system is unknown or denied, then in passing from one triune system to another, the ratio of each, in succession, must be calculated from that which precedes it—but not so where the influence of the great system is admitted—since, in that event, the ratio of the several key-notes, is in all cases estimated according to the nature of its connection with the central basis of the central system, and with this only. By this means the unity of a composition is preserved, however long or varied it may be.

It is moreover important to observe that this scale is not an invention, but a *discovery*, the result of analytical investigation and experiment.

In all our reflections and observations upon music, it is important to bear in mind the difference between effects actually present to the senses, and those which are only ideally so by the intervention of the memory—and by the aid of which the former are collected into groups and series, and thereby united so as to form a whole. The effect of the sounds of the triune system, for instance, if heard simultaneously would, to our perceptions, be horrible, notwithstanding the unity actually existing between its several members. But though the particulars relating to this unity are far too numerous for us to perceive, and still less to enjoy in the manner here described, it is not so when small portions only of the entire system are heard at a time, and when the order of the portions selected and presented, is well arranged, and well contrasted one with another. For, when this is the case, besides the gratification arising from the mere sense of the hearing, not only the more exalted affections, but the understanding also becomes awakened, and the pleasure thereby intensified. Now, it has already been shown that in the chord 1, 5, 3, 7, the recollection of the basis 1, induces us not to make the minor third 3 to 7 or 6 to 7 the same as 5 to 3 or 5 to 6—and every one must know that the chord 1, 5, 3, 7, and 1, 3, 5, to which the former naturally proceeds, would, if united, produce a very harsh and bad effect, although nothing is of more frequent occurrence than the progression of the one to the other—or more pure, simple, and natural, than their effects when thus heard in succession. But let us take a still broader view of this subject, and we shall find that, however harsh may be the links by which the members of the great system are sometimes attached the one to the other, *if heard simultaneously*, this can afford no legitimate argument against the truth of the ratios of the intervals assigned them, since such combinations are incompatible with any theory. But supposing these effects to be produced, should we not be distracted in the endeavour to ascertain to which of the two systems the sounds were severally referable—the central basis of that system—and the relation existing between these two systems and two central bases? By rejecting the idea of the simultaneous union of sounds pertaining to two triune systems, the mind becomes entirely disentangled from such perplexities. For, in this case, if all is made to go on correctly and smoothly till the moment of change arrives, and the new key be entered upon at once, as it should be, the moment of transition is reduced to a mere point—that is, the sounds introduced being all referable to the same triune system, the transition is effected instantaneously. And, where the harmony is pure, and the intonation perfect, or very nearly so, the effects of such tonic transitions are intuitively and instantaneously perceived. But though the preceding remarks may serve to remove any doubt as to the real existence of a great system, or sytem of systems—they offer no warrant for the capricious regulation of the pitch of the central bases by resorting to temperament, under the revolting idea that herein nature breaks down. That these are the doctrines and notions hitherto entertained by the comparatively few who have thought at all upon the matter, and which must still prevail where the existence of the Great System is denied, is proved by the following:—Mr. Huyghens, two centuries ago, observed that, "no voice or perfect instrument can always proceed by perfect intervals, without erring from the pitch at first assumed. But as this would offend the ear of the musician, he naturally avoids it by his memory of the pitch, and by tempering the intervals of the intermediate sounds, so as to return to it again."

It is also stated that "a certain monk found by subtracting all the ascents of the voice in a chant from all its descents, that the latter exceeded the former by two commas—so that if the ascents and descents were constantly made by perfect intervals, and the chant were repeated but four or five times, the final sound, which should be the same as the initial, would fall about a whole tone below it. But finding that the voices in his choir did not vary from the pitch assumed, he concluded that the musical ratios, whereby he measured those successive ascents and descents, were erroneous." Now, if Mr. Huyghens or the said monk had looked a little deeper into this matter, they would have found that by subjecting *the different parts of the same chant, to the same sort of scrutiny*—namely, deducting the descents from the ascents—the chances are ten to one that each part would be found to pursue a course of its own—so that while some parts would be working upwards, others would be working downwards!

PART 6.

ON THE ANALYSIS OF CHORDS *per se.*

93. A chord is a combination of two or more musical sounds heard at the same time—and hence it follows that two or more melodies heard at the same time necessarily give birth to a series of chords, and that in this sense the number of chords is illimitable. But since the combinations which occupy the more simple divisions of the bar, are also, in general, more simple in themselves—it is to these only that specific appellations have been given, that the attention has to be directed, and in reference to which innumerable others present mere variegations. Moreover, the number of specific names hitherto given to chords is very limited—but by means of the symbols appertaining to the "true science" every possible combination admits of being significantly represented and expressed.

As the knowledge of the elements of the triune system is all that is required to discover the harmonics and bases of chords, it is my intention here to enter into this subject by way of exercise, before proceeding to other matters. It is expedient, however, to warn the reader against perplexing himself with questions relating to their resolution—the ratio of the vibrations of one to another, where the notation is, or is not, the same—the mode, &c. These things will be explained in their proper place—the matter for investigation here, is the chord considered *per se*, and nothing more.

94. In this work chords are called *simple*, the elements (sounds) of which generate only one basis, and *compound* where its elements generate more than one basis. Hence it follows—(1) that though only two of the harmonics of any basis are sufficient to generate that basis, any combination of the primary harmonics of the same basis, however numerous, can generate that basis only—(2) that the addition of only one super-harmonic, being insufficient to generate another basis, such chords are also to be considered simple.

TABLES OF CHORDS

Containing those which *really* are, with others which only *appear* to be, the chords denominated the MAJOR COMMON CHORD, the MINOR COMMON CHORD, the chord of the FLAT

SEVENTH, the ADDED SIXTH, the DIMINISHED SEVENTH, and the EXTREME SHARP SIXTH:

TABLE

Of real and apparent major common chords.

Nos.	Key of C. Notes.	Generated Bases.	Bases and Primary Harmonics.	Intervals of the Triune System.	Notes.	Generated Bases.	Key.
1	C, G, E	C	1, 3, 5	1, 5, 3	C, G, E	C	C
2	F, C, A	F	**1, 3, 5**	4, 1, 6	C, G, E	C	G
3	G, D, B	G	1, 3, 5	5, 2, 7	C, G, E	C	F
4	♭A, ♭E, C	F, ♭a, C	**19**, 19, △	♭6, ♭3, 1	C, G, E	A, c, E	E
5	♭E, ♭B, G	C, ♭e, G	19, 19, ▽	♭3, ♭7, 5	C, G, E	A, c, E	A
6	♭D, ♭A, F	F	**11, 19, 1**	♭2, ♭6, 4	C, G, E	E	B
7	D, A, ♯f	F	3, **5, 17**	2, 6, ♯4	C, G, e	♭E	♭B
8	♯f, ♯c, ♯a	F	**17, 13, 11**	♯4, ♯1, ♯6	c, g, e	♭C	♭G

95. The above table is divided into 7 columns—in the 1st is placed the chords under examination, the key being C—in the 2nd, the bases generated by those chords—in the 3rd, the symbolic bases and harmonics severally answering to those chords—in the 4th, the intervals of the triune system—in the 5th, the notes severally answering to the generated bases of the 6th column—to the key-notes or central bases in the 7th—to the symbolic bases and harmonics in the 3rd—and to the intervals in the 4th column.

The order in which the particulars relating to every individual chord are placed in these columns is the same throughout.

96. From the peculiar arrangement of the particulars set forth in this table, it will be seen that what is called the major common chord, and thought to be one chord only, because it really is so in respect to the artificial system of 12 semitones in the octave, and the notation, is therein shown to be eight chords, all differing in respect to the elements of the triune system of which they are severally composed, and consequently in the symbols by which they are classified and scientifically represented.

97. But though by the peculiar arrangement of the chords, &c., in this table, the major common chord is represented under a specific number of forms, answering in respect to their effects,

and in a scientific point of view, to so many distinct qualities—it must not from hence be supposed, either in respect to this table, or those which follow, that other varieties do not exist. The varieties actually represented in the above and following tables, are those which stand forth most perspicuously to view, and which most frequently occur.

98. That the above eight chords are all confounded so as to become one in practice, and their distinct individual qualities thereby lost sight of, is rendered apparent from the circumstance that while the key remains the same, the chord varies in its notation, as in column 1—and when the notation of the chord is fixed, as in column 5—then the key, of necessity, changes as in column 7.

Though the reader is here expected to know enough of practical music to perceive that the chords in column 5 are severally the same as those in column 1 transposed, it is not desirable, at present, that he should trouble himself concerning how they can be incorporated into a musical phrase, consistently with nature's laws of harmony, and therefore with good effect. He must wait with patience for this, till other matters have been attended to.

99. From the definition given of a chord in § 90, it does not follow that a chord, considered *per se*, is meant to imply an agreeable, any more than a disagreeable, combination of sounds. That the contrary supposition, in favour of the agreeable, should generally prevail, arises from want of experience and want of observation.

100. A chord which *per se* is agreeable and fascinating to the ear, is representative of a unity of particulars upon a small scale, and having reference to simultaneous order only. In a musical phrase, simultaneous order is united to successive order, and the perception of both gives birth to higher and more varied pleasures, in proportion to the increased magnitude, variety, and beauty of the wider field of operations thus introduced.

101. But it so happens that, in a musical phrase, disagreeable, as well as agreeable chords, *per se*, are continually introduced, in listening to which (the phrase) we lose all consciousness of what, *per se*, is disagreeable, and experience a more refined, and more exalted, pleasure than what any mere chord can possibly produce. Now, if we search into the cause of this, it will be found that the order relating to unity upon the small scale (the mere chord), gives place, as it were, to unity upon the larger scale (the phrase), and this, be it observed, is the law of nature throughout all her operations—and hence it is that though the simple geometrical figures of the square, triangle, and circle, &c.,

may make very pretty playthings for children, nature disowns them in almost all her productions.

102. When harmony is looked upon as a combination of melodies, having a certain relation the one to the other, and not of necessity, as a series of chords—then, and not till then, it will be perceived that the natural flow of the melodies of which it is composed, is that which demands our most careful consideration—and consequently, that dogmas concerning petrified chords, cannot lead to a knowledge of music, since a knowledge of music, particularly of the triune system, is required in order to explain any chord. Harmony is therefore the necessary result of a combination of two or more melodies, and only occasionally the result of a series of *per se* agreeable chords.

103. To show how the bases and harmonics in the above table of chords has been obtained, we must turn to § 72, wherein it will be seen that, the key being C, the sounds of the 1st chord C, G, E, are all referable to C, as its basis—to which basis G is the harmonic 3, and E the harmonic 5. It has already been shown that C 3, is the same as C 1, and G 3, the same as G 1, —and it will be seen that the chords throughout are obtained by *spelling* them, note by note, by reference to the triune system, and not by reference to their practical intervals. It happens, however, in some instances, that a note of the same denomination is found to occupy two positions, in which case the rule to be observed is, not unnecessarily to involve *two* bases instead of one, or *three* instead of two.

104. That the chords in a line with the numbers 1, 2, 3, are all alike, with the exception of their position in the triune system, is evinced by the circumstance that the symbols by which they are represented in column 3, though differing in character, are numerically the same—as are also their intervals, in respect to their primary bases, though not in respect to the central bases or key-note.

105. As the ratio and interval between corresponding harmonics of the triune system must necessarily be the same as their bases, it follows that when the harmonic 19 is found in combination with 19—19 with 19—or 19 with 19—the former stands in the relation of a basis to the latter, and the latter in the relation of a harmonic to the former. To this kind of basis and harmonic, I affix the term *medial*, for reasons to be explained hereafter. These medial bases and harmonics are represented in

the table by a brace passing from the one to the other, and also from one note to the other in the chords to which they relate.

106. The symbol △, in the 3rd column, represents the key-note under the twofold quality of central basis, and harmonic 3 to the sub-basis—and ▽, in the twofold quality of super-basis, and harmonic 3 to the central basis. For example, △ is the central basis to 𝟙𝟡, but as the representative of the harmonic **3**, in conjunction with **19**, it will generate **1**. In like manner ▽, is the central basis to 1𝟡—but as the representative of the harmonic 𝟛, in conjunction with 𝟙𝟡, it will generate 𝟙.

107. The chords, &c.. in a line with the numbers 1, 2, 3, on the left, and united by a brace on the right and left, are *real* major common chords, because they are what they appear to be, all below being only *apparent* major common chords, because they are *not* what they appear to be.

The three chords in connexion with the numbers 4, 5, 6, and united by a brace, are *diatonic* chords—and the two in connexion with the numbers 7, 8, and also united by a brace, are *chromatic* chords. The chromatic notes are distinguished by small letters.

TABLE

Of real and apparent minor common chords.

Nos.	Key of C. Notes.	Generated Bases.	Bases and Primary Harmonics.	Intervals of the Triune System.	Notes.	Generated Bases.	Key.
1	C, G, ♭E	C	𝟙, 𝟛, 𝟙𝟡	1, 5, ♭3	C, G, ♭E	C	C
2	F, C, ♭A	F	**1, 3, 19**	4, 1, ♭6	C, G, ♭E	C	G
3	G, D, ♭B	G	1, 3, 19	5, 2, ♭7	C, G, ♭E	C	F
4	A, E, C	F, a, C	**5, 5**, △	6, 3, 1	C, G, ♭E	♭A, c, ♭E	♭E
5	E, B, G	C, e, G	5, 5, ▽	3, 7, 5	C, G, ♭E	♭A, c, ♭E	♭A
6	D, A, F	F	3, **5, 1**	2, 6, 4	C, G, ♭E	♭E	♭B
7	B, ♯f, D	F . . G	5, **17**, 3	7, ♯4, 2	C, g, ♭E	♭G . . ♭A	♭D

108. In the above and following tables, the bases and harmonics are obtained from the triune system, §. 72, the same as for that in § 94. Each of these tables are also divided into 7 columns —and, in several other particulars, arranged in the same manner,

and for similar purposes, which it is unnecessary here to recapitulate.

109. The only circumstance to which particular attention is required in regard to this table, is, that the 4th and 5th chords are here not real, but apparent minor common chords. Thus in the table of real and apparent major common chords, there are two (the 4th and 5th), which appertain to the minor mode, and which are therefore only apparent major common chords—while on the contrary, in the table of real and apparent minor common chords, there are also two (the 4th and the 5th), which appertain to the major mode, and which are therefore only apparent minor common chords.

As these facts are pregnant with important consequences, the reader will do well to examine and reflect upon them carefully. Moreover, as the habit of *spelling* chords, by reference to the key, and triune system, is a most useful exercise, it is earnestly recommended to do so—first, by reference to examples in the key C, and after this, to other keys. For, be it observed that, as in language, so in music, spelling is the proper introduction to reading, and not *vice versa*.

It is remarkable that the two apparent minor common chords pertaining to the major mode, generate in both instances, a real major common chord—and that the two apparent major common chords, pertaining to the minor mode, generate, in both instances, a real minor common chord. See the bases to the 4th and 5th chords in the above table.

TABLE

Of real and apparent chords of the ♭7th with the major and minor 3rd.

Nos.	Key of C. Notes.	Generated Bases.	Bases and Primary Harmonics.	Intervals of the Triune System.	Notes.	Generated Bases.	Key.
1	C, G, E, ♭b	C	1, 3, 5, 7	1, 5, 3, ♭7	C, G, E, ♭b	C	C
2	F, C, A, ♭e	F	1, 3, 5, 7	4, 1, 6, ♭3	C, G, E, ♭b	C	G
3	G, D, B, f	G	1, 3, 5, 7	5, 2, 7, 4	C, G, E, ♭b	C	F
4	D, A, ♯f, C	F	3, 5, 17, 3	2, 6, ♯4, 1	C, G, e, ♭B	♭E	♭B
5	A, E, ♯c, G	F, a, C	5, 5, 17, 3	6, 3, ♯1, 5	C, G, e, ♭B	♭A, c, ♭E	♭E
6	F, C, ♭A, ♭e	F	1, 3, 19, 7	4, 1, ♭6, ♭3	C, G, ♭E, ♭b	C	G

110. By this table it will be seen—(1) that the 1st, 2nd, and 3rd are real chords of the ♭7th with the major 3rd—(2) that the 4th and 5th are chromatic chords of the added 6th, and, consequently, only apparent chords of the ♭7th—and (3) that the harmonic 7 being chromatic, the chord is therefore chromatic.

The reason why the chord of the ♭7th with the minor 3rd (No. 6) can only take place upon the sub-basis, is, because it is in this position only that a natural resolution can be effected. That the harmonic 7 is really chromatic will be seen hereafter.

TABLE

Of real chords of the added sixth.

Nos.	Key of C. Notes.	Generated Basis.	Basis and Primary Harmonics.	Intervals of the Triune System.	Notes.	Generated Basis.	Key.
1	F, C, A, D	F	1, 3, 5, 3	4, 1, 6, 2	C, G, E, A	C	G
2	F, C, ♭A, D	F	1, 3, 19, 3	4, 1, ♭6, 2	C, G ♭E, A	C	G
3	F, C, ♭A, ♭D	F	1, 3, 19, 11	4, 1, ♭6, ♭2	C, G, ♭E, ♭A	C	G

111. It is here important to observe—(1) that the chord of the ♭7th cannot exist in the key of C with the notes as above, simply because D and ♭D are not bases in that key—(2) that the chord of the added 6th cannot exist in the key of C, with the notes F, ♭A, C, ♭E, simply because ♭A is not a basis in that key. That there are many chromatic modifications of the above three diatonic chords of the added 6th, and also that the last of these is diatonic, as well as the 1st and 2nd, will be seen hereafter.

112. The reason why the chord of the added 6th can only rest upon the sub-basis, is, because it is only in this position that its natural progression can be anticipated and take place. It is, moreover, to be remarked, that the added 6th, as a simple chord, cannot possibly exist, either upon the central or super-basis.

By treating the harmonic 3 as the basis to the chord of the added 6th (No. 1,) it is, in appearance, converted into the chord of the flat 7th—and by the omission of the harmonic 3, it is, in appearance, converted into the minor common chord,—and by the same means No. 3, is, in appearance, converted into the major common chord. That these facts have been the fertile source of a vast amount of altercation, and learned discussion, is known to every one conversant with such matters.

Upon examination it will also now be seen that, by treating the major or minor 6th to the central or super basis, as the basis to the chord of the added 6th, to either of those intervals of the scale, they also become in appearance converted into flat 7ths—and that in either of these cases, by the omission of the 5th, such chords of the added 6th are converted into apparent major or minor common chords.

113. The reason why the chord of the added 6th is classed among simple chords, is, because by it, one basis only is generated—although in respect to the triune system it is a compound

chord, because in reference to its primary harmonics two bases are involved, § 94. But though three of its sounds are primary harmonics to the sub-basis, and only one to the super-basis—nevertheless, as the primary harmonics to the central basis are secondary to the sub-basis—and as the primary harmonics to the super-basis are secondary to the central, and tertiary to the sub-basis—the sub-basis is consequently the primary basis to the three and a tertiary basis to the fourth sound, in respect to each of these three chords. It is moreover a remarkable circumstance that whenever it happens, by reference to the triune system, one of two primary bases would be discordant with the other parts of the harmony, such *referential* bases are never generated.

TABLE

Of chords of the diminished or extreme ♭7th.

Nos.	Key of C. Notes.	Generated Bases.	Bases and Primary Harmonics.	Intervals of the Triune System.	Notes.	Generated Bases.	Key.
1	♯c, G, E, ♭b	C	17, 3, 5, 7	♯1, 5, 3, ♭7	♯c, G, E, ♭b	C	C
2	♯f, C, A, ♭e	F	**17, 3, 5, 7**	♯4, 1, 6, ♭3	♯c, G, E, ♭b	C	G
3	♯g, D, B, f	G	17, 3, 5, 7	♯5, 2, 7, 4	♯c, G, E, ♭b	C	F
4	B, F, D, ♭A	F . . G	5, **1**, 3, **19**	7, 4, 2, ♭6	♯C, G, E, ♭B	G . A	D
5	♯d, A, ♯f, C	F . . G	13, **5, 17, 3**	♯2, 6, ♯4, 1	♯c, G, e, ♭B	♭E . F	♭B
6	E, ♭b, G, ♭d	C . . G	5, 7, ▽, 11	3, ♭7, 5, ♭2	♯C, g, E, ♭b	A . E	A
7	♯a, E, ♯c, G	F . . C	**11, 5, 13, 3**	♯6, 3, ♯1, 5	♯c, G, e, ♭B	♭A . ♭B	♭E
8	A, ♭E, C, ♭g	F . . C	**5, 19, 3,** 11	6, ♭3, 1, ♭5	♯C, G, E, ♭b	A . E	E

114. In this list the first three chords are simple, and all the others compound. As it would be premature here to enter into a scientific investigation of the great variety of facts appertaining to these chords, but more especially to the three first—the only points to which I shall advert, at present, are those relating to actual practice in respect to keyed instruments, the central basis or key, and the notation.

On the piano key-board, the notes of this chord being transposable into *three* positions only, for all the twelve keys, it is evident that each position must, as it were, do duty for four different keys. For example the four chords:—

Nos.	17	3	5	7	
1	♯c	G	E	♭b	in the key of C
2	g	♭D	♭B	♭f	. . . . . ♭G
3	e	♭B	G	♭d	. . . . . ♭E
4	♯a	E	♯C	g	. . . . . A

are all the same on the piano, in regard to which it will be observed—(1) that the harmonic symbols above are severally applicable to all the notes below—(2) that, disregarding their denominations, the sound in each of the series, both vertically and horizontally, are the same, and produce, practically, a series of minor 3rds—(3) that the bases resulting therefrom give rise to another series of sounds and minor 3rds.

115. The following are the only three sounds appertaining to this chord, which differ one from the other on the piano,

♭E	♯F	A	C
D	F	♭A	B
♯C	E	G	♭B

and since, as before observed, each of these have to do duty for four keys—hence it is, that the above three practical varieties, are adapted to become substitutes for $3 \times 4 = 12$,—that is, for the 12 keys appertaining to the semitonic scale of the piano, &c.

116. It is worthy of notice, that the chord of the diminished or extreme ♭7th is also denominated the *equivocal chord*—a name singularly significative, not only of the truths above stated in regard to it, but also of the dubious application of all that can possibly be said of it, and the rules relating to it, by those whose ideas of the science of music are based upon the shaky foundation of conventional names, the notation, and the piano key-board—and not upon things or facts for a foundation, and thence to names.

117. In short, transposition in reference to the notation, may be said to embrace 17, and in respect to the pianoforte-key-board 12, as its general, and apparently universal multiplying power—while on the contrary, in other instances, it reduces a variety of really dissimilar chords, sometimes appertaining to the same and sometimes to different keys, to a comparatively few in appearance, § 95—118.

TABLE

Of chords of the extreme sharp 6th.

Nos.	Key of C. Notes.	Key of C. Generated Bases.	Bases and Primary Harmonics.	Intervals of the Triune System.	Notes.	Generated Bases.	Key.
1	♭E,G,♭b,♯c	C	19,3,7,17	♭3,5,♭7,♯1	♭E, G,♭b,♯c	C	C
2	♭a, C, ♭e,♯f	F	**19,3,7,17**	♭6,1,♭3,♯4	♭E, G,♭b,♯c	C	G
3	♭b, D, f,♯g	G	19,3,7,17	♭7, 2, 4,♯5	♭e, G,♭b, ♯c	C	F
4	C, E, G, ♯a	F . . C	△,5,3,11	1, 3, 5, ♯6	♭E,G,♭B,♯c	♭A–♭E	♭E
5	♭d, F, ♭a, B	F . . G	11,1,19,5	♭2, 4, ♭6, 7	♭e, G,♭b,♯C	G——A	D

118. The chord of the extreme sharp 6th, being the same on keyed instruments as the chord of the ♭7th, the value and importance of the adoption of that natural means of arriving at the truth in regard to it, by spelling, note by note, in reference to the key and triune system must be evident to every one the least acquainted with such matters. Thus, if we propose to refer to the notes, ♭A, C, ♭E, ♯F, to one basis only, the key being C, it will be found that that basis is F—and hereafter abundant, practical, scientific, and experimental evidence, will be given that this interpretation, and this only, is correct.

In the keys ♭D, ♭A, ♭E the same practical chord has ♭A for its basis—the note ♯F being changed to ♭G.

How the basis becomes changed from D to ♭A (a flat 5th) by changing ♯F to ♭G, in agreement with the assumed ratio 125, 128, I leave the theorists of this incoherent system to explain. In justice, however, to their real merits, the attempt to do this, would evince a degree of folly of which I am sure not one of them is at all likely to be guilty. That they have been caught in a snare, in regard to music, they, more than others, must now clearly perceive. This being the case, the best thing they can do, is to wipe off the blot, which their inconsiderate interference with this subject has occasioned to their reputation—by casting aside, at once, the errors which originated, not with themselves, but their forefathers. It is, indeed, a matter of surprise that they should, at any time, have been so far deluded, as to attempt to rear a scientific fabric upon so slender a foundation as the three isolated facts relating to the primes 2, 3, 5—and then, pertinaciously continue so long to uphold this visionary system at all risks, and in defiance of the pressure of evidence, flowing from all quarters in opposition to it, and denouncing it, as being false, and rotten to the core.

119. The chord No. 2, is the only position of the scale in which the chord of the extreme sharp 6th can properly occur, because this only admits of a natural resolution.

120. The chord of the extreme sharp 6th has confessedly been a source of great perplexity to theorists. That its effect is most beautiful is acknowledged, but its true basis has hitherto been sought in vain. Like other chords, it has always been examined as an isolated fragment, thus not as being connected with any key and scale. It has not therefore been known to consist of three chromatic, and only one diatonic harmonic—and this has saved it from being looked upon as a mere cluster of passing notes.

It is requisite to remind those who have read other works upon the theory of music that, in this, there are no notes that come under the denomination of *passing notes*—EVERY NOTE BEING EQUALLY SUSCEPTIBLE OF EXAMINATION AND EXPLANATION BY REFERENCE TO ITS BASIS. Of course, those who do not like the trouble and minute attention, which this requires, are at liberty to *pass* over as many notes as they please—and this may be done at any time, with common-place passages, which have already been examined, and with those which have nothing in them worth attending to. But it would be absurd to deny the existence of a basis to each note upon this ground. In fact the term *passing notes* affords a very broad and convenient opening for evasion and retreat, whenever matters are found absolutely inexplicable, or at variance with received principles and rules.

121. The least attention to the preceding seven tables of chords, is sufficient to show the importance of referring all harmony, and consequently all chords, to their bases—whereby, alone, all real differences and distinctions are perspicuously brought forth to view.

Besides, what can be more natural in the study of music, than to make the sounds themselves the primary objects of attention—or more unnatural than to overlook the sounds themselves, and in utter ignorance of what, in respect to their true pitch they really are, to set up *intervals*, that is, the *differences* between the *thereabouts* of the sounds, as the foundation of the science of music.

122. What the difference is between the chord of the extreme ♯6th and that of the ♭7th, whose sounds are the same on the piano and what the practical *uses* to which the knowledge of such matters can be applied will be shown hereafter But, before I commence with a new subject, I request the reader's attention to the following important points, all of which are more or less closely connected with what has already been advanced.

123. AS THE TRIUNE SYSTEM IS DERIVED FROM THREE BASES ONLY, IT IS INCORRECT IN THE EXPLICATION OF HARMONY TO REFER TO BASES EXTRANEOUS TO THAT SYSTEM—that is, to the bases of more than one triune system at the same time.

That medial bases make no exception to this rule is evident, since in no instance do these bases constitute the foundation of a distinct series of harmonics, but merely point to the fact that the interval between corresponding harmonics must of necessity be the same as that between their respective bases.

124. IT IS INCORRECT TO ASSIGN A DIFFERENT BASIS TO ANY CHORD ON ACCOUNT OF THE INTRODUCTION OF AN ADDITIONAL SOUND. For example, if, in the key of C, the primary basis to the notes F, A, C, be F, it cannot be altered by the addition of the note D. And, upon the same principle, since the introduction of the note D cannot alter the primary basis to the notes F, A, C, neither can the basis to these sounds alter that to which D is referable. If, therefore, the primary basis to the notes F, A, C, be F, and the primary basis to the note D be G —then must the primary bases to the chord F, A, C, D, be both F and G. It has already been shown, § 111, that F is the primary basis to three out of four of the notes of this chord, and a tertiary basis even to the fourth.

The theorists who consider these notes as forming a chord of the 7th in the key of C are, of course, driven to make D the basis; but:—

125. Let it constantly be borne in mind, that the foundation of the science of music, rests upon the fact, THAT THE BASES OF CHORDS AND CONSEQUENTLY OF HARMONY ARE ENTIRELY DEPENDANT UPON THE ELEMENTS OF THE TRIUNE SYSTEM OF WHICH THEY ARE COMPOSED.

PART 7.

OF THE TRIUNE SYSTEM IN RESPECT TO CHORDS—THAT IS, HARMONIC COMBINATIONS, WHENCE PROCEED THE FIRST, SECOND, AND THIRD MODE.

126. Were it true that nature, in respect to the triune system, did nothing but produce sounds *ad infinitum*, in one dull routine upwards, in the elements of which there were no latent powers of any kind, the fact would be without a parallel in any of her operations. But, as this is not the case, it becomes my task to develop these secret, and in respect to their causes, deep and mysterious influences, to their source.

Had the Creator not bestowed on man perceptive as well as intellectual faculties, he never could have discovered the reality of even the existence of the latent powers above alluded to, and consequently nothing whatever concerning them. For, who by investigating the mere elements of the triune system, (§ 72) can discern the peculiar consociations, discriminations, attractions, and affinities, which the perceptions of mankind attest, to belong to it—and which will hereafter be seen to constitute the source and foundation of all our knowledge in respect to it? But, these interior facts being perceived, and acknowledged—the scrutinizing powers of the intellect become available, useful, and important, because their operations are then restricted to the sphere which properly belongs to them. Uncontrolled by the beacons, checks and safeguards, afforded by the more interior, and consequently superior, faculties appertaining to humanity, the solitary self-governed power of the intellect, is prone continually to go astray, and exhaust all its energies in building castles in the air.

These considerations will account for the utter failure of mathematicians, in all their efforts to investigate the laws which relate to, and govern our feelings in respect to music, notwithstanding the numbers who have labored in this deparment of science, during a space of not much less than 3000 years. But since the idea of placing the perceptive, above the intellectual faculties, is not particularly flattering to persons engaged in intellectual pursuits, and opposed to the notion commonly entertained upon the subject, of course I do not expect every one to assent to it.

127. In popular phraseology a scale is—(1) a series of sounds, progressively arranged, in an orderly manner, from the lowest upwards, or *vice versa*—(2) an ascending or descending series of notes, intended as a practical exercise for the fingers or voice

—(3) any arbitrary, or fanciful, series of notes, set forth, and displayed, with the view to some especial object. But, in this work, by the term system, instead of scale, it is to be understood the tabular exhibition, or formula, representative of that series of sounds, the elements of which, with respect to their vibrations, being derived entirely from primary numbers, are united, the one with the other, by the strongest possible affinity. And as the primes are the germes, or elements, whence all other numbers are derived, it follows from hence that a system of sounds thus constituted, is a system composed entirely of germes or elementary essences.

128. The following is an altitudinal display of the three diatonic formations appertaining to the first period of the development of the triune system, commencing from its root, the sub-basis; and proceeding thence to its apex, the super-terminant.

Bases and diatonic harmonics of the triune system.

Number of Qualities.	Sub-basis, with its primary, secondary, and tertiary diatonic harmonics.	Central basis, with its primary and secondary diatonic hrms.	Super-basis with its primary diatonic harmonics.	APPELLATIONS.					Practical example, the key being C.
1 . . .	99 . . 27	33 . . 9	11 . . 3	. . .	. . .	Super tmnts.	. . .	. . . . .	♭ D ♯
2 . . .	171 . . 45	57 . . 15	19 . . 5	. . .	. . .	Super mnts.	. . .	Central mdl. terminants.	♭ B ♯
2 Central or	33 . . 9	11 . . 3	. . 1 ▽	. . .	Central termnts.	Super basis.	. . . .	. . . . .	♭ G G ♯
3 Quiescent	57 . . 15	19 . . 5	. . . .	. . .	Central mednts.	. . .	Sub mdl. termnts.	Central mdl. bases.	♭ E ♯
2 Position	11 . . 3	. . 1 .	. . △	Sub tmnts.	Central basis.	. . . .	. . . .	. . . . .	♭ C C ♯
2 . . .	19 . . 5	. . . . .	. . . .	Sub mdnts.	. . . .	. . . .	Sub mdl. bases.	. . . . .	♭ A ♯
1 . . .	. . . 1 . . .	. . . . .	. . . .	Sub basis.	. . .	. . . .	. . . .	. . . . .	. F . .

129. In order to proceed, step by step, in the examination of this system of sounds, let us first imagine ourselves limited to the use of the three bases **1**, 𝟙, 1, which in the key of C represent the notes F, C, G. With such a paucity of materials, even with the addition of their octaves, it is obviously impossible to do much, either in the way of melody or harmony, but:—

If to these elements we add **3**, 𝟛, 3—as **3** is identical with 𝟙, and 𝟛 with **1**, the only additional sound gained is 3, which represents the note D and its octaves. But no one ever has written, or ever can write, anything worth hearing, if confined to the use of the notes F, C, G, D. Wherefore;—

130. Let us proceed to enrich the system by the introduction

of the additional symbols **5**, 𝟝, 5, which, in the key of C, represent the notes A, E, B. So important is the introduction of the prime 5, that with this, and the lower primes we are enabled to develop all the elements of the 1st mode, and thereby to lay the foundation of melody and harmony.

131. It will be seen that, the key being C, the symbols **1**, **5**, **3**, represent the notes **F**, **A**, **C**—the symbols 𝟙, 𝟝. 𝟛, the notes 𝔽, 𝔸, ℂ—and the symbols 1, 5, 3, the notes G, B, D—and that from hence is obtained the diatonic series 𝟙, 3, 𝟝, **1**, 1, **5**, 5, 𝟙, answering to the notes ℂ, D, 𝔼, **F**, G, **A**, B, ℂ. This is commonly called the scale of nature, and so it really is—and, as being the most simple, it is also the *first* development of a system capable of being applied to practice. But, though the first, it does not not follow that it is the only system, for the elements of which we are indebted to nature—or, that the more complete developments of that which is merely initial, are less natural, simply because higher, more refined, and more remote elements are successively introduced.

132. The small numerals, under the head sub-basis, with its primary, secondary, and tertiary, diatonic harmonics, represent the products arising from the multiplication of the primary and secondary harmonics, of the central basis by 3, which are thereby referred, as harmonic products, to the sub-basis. In like manner the small numerals under the head central basis, with its primary, and secondary, diatonic harmonics, represent the products arising from the multiplication of the primary harmonics of the super-basis by 3—which are thereby referred, as harmonic products, to the central basis.

133. It is here to be remarked, that the prime 5 constitutes what is called the *arithmetic* mean, between 4, the double octave to the unit, and 6 the octave to the prime 3. That is, in the series, 4, 5, 6, the number 5 is the arithmetic mean. The rule for finding the arithmetic mean, is—Divide the sum of the extreme terms by 2, and the quotient is the mean term. For example, $4 + 9 = 13 \div 2 = 6\frac{1}{2}$ the arithmetic mean. There is also what is called the *geometric* mean, the rule for finding which, is—Extract the square root of the product of the two extreme terms, and the result is the geometric mean. For example, $4 \times 9 = 36$, and the square root of 36 is 6, which is therefore the geometric mean.

134. As the elements of the simple system, and conseqently of the simple systems of which the triune and great systems are

formed, are entirely composed of primary ratios it is evident the geometric mean is inapplicable thereto. The geometric mean is therefore entirely confined to the tempered intervals of artificial systems, of which that of 12 mean semitones in the octave constitutes the chief.

Certain authors have also conjured up what they call the *harmonic* mean—but as this consists in the mere invertion of the terms of the arithmetic mean, nothing more need be said about it.

135. But, besides the prime 5, which nature adopts, as a mean term, between the two extreme terms 4, 6—it will be found that nature also adopts the prime 19 as another mean term, between the very same two extreme terms—that is between the two extreme terms 16, 24, the octaves 4, 6—and in so doing ignores the fact that 19 is neither the geometric nor harmonic mean. From this new prime we obtain the symbols **19**, 𝟙𝟡, 19, and thereby the three additional sounds ♭**A**, ♭𝟙𝟡, ♭**B**, which constitute the elements peculiar to the 2nd mode. It is important here to observe that the effect of the ratio 16 to 19, when used as a basal minor third, is most beautiful, and highly characteristic of the mode to which it belongs. Hitherto, the minor mode has been represented either as a purely artificial production, or else, as being obtainsd from the elements of the major mode, by resorting to an artifice similar to that of describing the configuration of a man, after the similitude of his appearance, when standing upon his head.

136. Great has been the consternation and perplexity into which theorists have been thrown, respecting the difference between, what they call, the ascending and descending scale of the minor mode. The symbolic representation of the ascending series is 𝟙, 3, 𝟙𝟡, **1**, 1, **5**, 5, 𝟙, and of the descending series 𝟙, 19, **19**, 1, **1**, 𝟙𝟡, 3, 𝟙. But in respect to this it is to be observed, (1) that an ascending or descending series of notes does not represent a scale or system, but merely a particular melody—and since there are laws, of universal application, relating to melody—this melody is, of course, subject to the dominion of those laws. For example, the descending series represented above, is not so good, that is, less strictly in agreement with the laws of nature than the following, 𝟙, 7, **19**, 1, 7, 𝟙𝟡, 3, 𝟙 for reasons which will be shown hereafter. (2) The notion that the sounds of the so-called ascending and descending major scales are the same, is not true—whence it follows that this

circumstance, in respect to the minor mode, is not an exceptional case, peculiar to this mode. For example, the ascending major scale is 1, 3, 5, **1**, 1, **5**, 5, 1, and the descending 1, 5, **5**, 1, 7, (not **1**) 5, 3, 1.

137. It is to be noticed that, except where the harmonic 7 occurs, the character of the symbols change with every note—thereby significantly pointing to the fact, that in ascending diatonically to the next note above, or descending to the next note below, the introduction of a new basis is the necessary consequence. The reason the harmonic 7 forms an apparent exception to this rule, is, because this harmonic is not diatonic, but chromatic, as will be seen hereafter.

138. With the view to shuffle over this self-imposed difficulty about the so-called minor scale, several theorists have recently entered into a league to represent the descending series the same as the ascending. Thus, for the ascending 1, 3, 10, **1**, 1, **10**, 5, 1, and for the descending, 1, 5 ,**10**, 1, **1** (?), 10, 3, 1. But, here it is to be observed, (1) that since, consistently with its true analysis, the 4th of the scale in ascent is **1**, and in descent is **7**, the sound represented by that interval is not the same in both cases, notwithstanding its being so in respect to the notation, and upon keyed instruments. (2) If these harmonic ratios represent the only correct progressions of the so-called minor scale, all others must be incorrect—and if so, the compositions of all our most esteemed composers are replete with errors. (3) If, on the contrary, other progressions of this kind are also good, then are there more minor scales than one—or if not, the minor mode, is, in its own nature obscure and indefinite, and consequently susceptible of being variously interpreted.

139. Having now pointed out the variable nature of the sounds here called mediants, and the origin of the 1st and 2nd modes, it now remains to be shown that there is also a 3rd mode—which though it has hitherto been entirely overlooked by theorists, exists more or less in the works of every great master. That it exists in the works of Handel, J. S. Bach, Corelli, and still more abundantly, in the works of Mozart, and Beethoven, will be shown hereafter.

140. As the existence of a 3rd mode is unknown, of course it has no name—and as the terms major and minor have not the remotest connexion with anything appertaining to nature, but

only with the pianoforte key-board, and the semitonic scale thence derived, the sooner they are laid aside the better. The denomination 1st mode, 2nd mode, and 3rd mode, agrees with the order of their natural development—for though all are from nature—the 1st is the most simple, and that to the perception of which all mankind are first awakened—the 2nd less simple and more congenial to a state receding from infancy—and the 3rd, though equally natural, still farther removed from simplicity, and consequently more congenial to adults—or rather, if I may be allowed the expression, to adult perception.

141. Though the diatonic elements peculiar to the 3rd mode differ from those of the 1st and 2nd mode, by the substitution of the harmonic **11** instead of **3**,—𝟙𝟙 instead of 𝟛,—and 11 instead of 3, it must not from hence be supposed that the bases 𝟙, 1, are therefore displaced or changed—for though **3**, 𝟙—and 𝟛, 1, are identical as to pitch, it is the quality **3**, and not 𝟙, that is changed to **11**,—and the quality 𝟛, and not 1, that is changed to 𝟙𝟙.

142. It is also to be noted that, as the difference between the 1st and 2nd mode depends entirely upon the adoption of the prime 5, or 19, as the mediant to the 5th, between the two extreme terms 2, 3—thus 2, 5, 3, or, what amounts to the same thing, 4, 5, 6, and 2, 19, 3, or 16, 19, 24—so, in like manner, the prime 11 is diatonically introduced in the constitution of the 3rd mode, as the mediant to the 8th or octave, between the two extreme terms 1, 2,—thus 1, 11, 2, or 8, 11, 16.

143. It is most desirable that the reader should himself investigate this altitudinal display of the triune system, § 128, since his ready apprehension of what will be said hereafter greatly depends upon a perfect knowledge of all that relates to this subject. If this be understood, every thing relating to fixed and serial chromatic harmonics will also be understood—but if not, the whole will probably appear abstruse and obscure. The reader should, in fact, reflect upon all the circumstances relating to it—as for instance—(1) that, 5 being elevated to the octave 20, and 3, to the octave 12, the relation between the two mediants 19, 5, is as 19 to 20, and between the two terminants 11, 3, as 11 to 12,—(2) that the terminant of the sub-system constitutes the basis to the central system, and the terminant to the central system the basis to the super-system—(3) that the three bases, and the three mediants, appertaining to the 1st and 2nd modes, supply six out of the seven notes of the system—

(4) that the reason why, in these two modes, the three terminants add but one additional elementary sound, is, because two are identical with the bases—(5) that this system contains the elements of all the simple harmonics, that is, of all the harmonics, or chords, to be obtained from the primes 1, 2, 3, 5, 19, 11—(6) that these elements, which constitute the diatonic or rudimental system, are the initials of the perfect system, and which, with an occasional exception, symbolically represent all the sounds appertaining to the works of the elder masters—(7) that without a due intermixture of diatonic harmonics, the chromatic can by no means be brought into practical use—although the diatonic can be used independently of the chromatic.

144. The two braces by which the two sub-medial bases are respectively connected with the sub-medial terminants—and these again, as central-medial bases, with the two central-medial terminants—point to the relation existing between these sounds, as corresponding with that of a basis and its primary harmonic 3—and also to the relation existing between the two sub-medial-bases, and the two central-medial terminants, as corresponding with that of a basis and its secondary harmonic $3^2 = 9$.

145. Lastly, if we enumerate the particulars set forth in regard to the bases and diatonic harmonics of the triune system, in connexion with their appellations, and the practical example in the key of C, it will be seen that the number of their qualities, are respectively, 1, 2, 2, 3, 2, 2, 1, as stated in the first column, whether we proceed by ascending, or descending. Furthermore, that the key-note constitutes the central basis of the system—and the common chord of that basis, the central chord of the system. That the central position of the basis of this chord, and of the chord itself, is the cause of their peculiar adaptation to constitute the elements of the closing note, and chord, in every composition, must be evident. The central basis is therefore the quiescent position in respect to the other bases, and the common chord to this basis, the quiescent position, in respect to all other chords.

PART 8.

OF THE METHOD OF FINDING THE RATIO OF SINGLE NOTES, AND CHORDS, WHICH APPERTAIN TO DIFFERENT TRIUNE SYSTEMS—AND WHICH, NEVERTHELESS, ARE THE SAME UPON KEYED INSTRUMENTS, AND FREQUENTLY, THOUGH NOT ALWAYS, THE SAME IN RESPECT TO THEIR NOTATION.

146. Before I proceed to the explication of the second period in the development of the triune system, wherein fixed chromatic sounds are introduced, I wish again to engage the reader's attention to the altitudinal display, in respect to the great strength of all its parts, arising from the union of each part with the whole, and of the whole from the many wonderful circumstances relating to its parts, as members of a truly social system. For, what can be more wonderful than the various ties of affinity, which bind its parts so closely into one? What more wonderful than the facts relating to its bases and harmonics—as, for instance, that there should be not only *numerically* three of each, but three of each in respect to *quality?* The denominations which relate to position, and the triple conjunction of the three similar elements which constitute the numerical triads, are *sub*, *central*, *super*—and those which relate to the triads of quality, are *primary*, *secondary*, *tertiary* (§ 10). To these must be added the term *medial*, to designate collateral affinities. Of medial bases and medial harmonics, four are primary, and stand in the relation 1 to 3—that is of a basis to its primary harmonic 3—and two are secondary, and stand in the relation 1 to $3^2 = 9$ —that is of a basis to its secondary harmonic 9. And again, what more wonderful than the facts relating to the orderly increase and decrease of these numbers and qualities from 1 to 3, and back again from 3 to 1, whether counted from below upwards, or from the top downwards?

147. In proceeding to find the ratio of any one or more notes of the same denomination, or which are the same upon keyed instruments, and in respect to the semitonic system—but which appertain to two different triune systems—first draw a line, in connexion with a brace thus, {————, then ascertain, (1)

Which of the two key-notes stands in the relation of a basis to the other—and, consequently, which stands in the relation of a harmonic to that basis. For convenience, let us call the first, the basal, and the last, the harmonic key-note. (2) Put the numeral representing this relation in front of the brace thus, $3\{$———. (3) Mark, symbolically, the harmonics of the basal key-note, below the line—and those of the harmonic key-note severally answering to the corresponding note, above the line. (4) Separate the harmonics of different notes by short perpendicular lines thus:— .

$$3\left\{\begin{array}{c|c|c|c} 1 & 5 & 3 & \cdot 7 \\ \hline 3 & 17 & 5 & 11 \end{array}\right.$$

Being thus duly prepared, let the following rules be attended to, and the ratios required will, in every case be obtained.

Rule 1.

148. MULTIPLY THE SYMBOLS IN THE UPPER ROW BY THE COMMON MULTIPLIER IN FRONT OF THE BRACE. For instance, in obedience to this rule we are required to multiply, severally, the symbols 1, 5, 3, 7, in the preceding example, by 3.

Rule 2.

149. IF THE CHARACTER OF THE SYMBOL BELOW, BE THE SAME AS THE ONE ABOVE, THE OPERATION IN RESPECT TO ANY NOTE, IS AT AN END—EXCEPT WHERE IT BECOMES NECESSARY TO ELEVATE EITHER OF THEM TO THE CORRESPONDING OCTAVE.

Here, however, after obtaining 3, as the product of 1×3, from the upper row, we find the same number in the lower, in which case the sound represented is also the same—and consequently, the ratio of their vibrations $= \frac{1}{1}$.

Rule 3.

150. IF THE CHARACTER OF ONE SYMBOL REPRESENT THE PREFIX *sub*, AND THE OTHER *central*, MULTIPLY THE *central* BY 3, THAT BOTH MAY REFER TO THE SAME BASIS—FOR THE SAME REASON ALSO, IF ONE SYMBOL REPRESENT THE PREFIX *super*, AND THE OTHER *central*, MULTIPLY THE *super* BY 3—AND IF ONE SYMBOL REPRESENT THE PREFIX *super*, AND THE OTHER *sub*, MULTIPLY THE *super* BY $3^2 = 9$

151. If we now multiply 5, the next symbol in the upper row, by 3 (rule 1), we obtain 15—but as this symbol represents the prefix *super*, and the lower *sub*, we multiply 15 by 9 = 135 (rule 3), and then elevate **17**, the symbol of the lower row, to the corresponding octave—that is to the octave $\mathbf{17} \times 2^3 = 136$. From hence it follows that the sound represented by 5 in the upper row, is flatter than that represented by **17** in the lower row, in the ratio 135, 136.

152. The next symbol 3×3 (rule 1), $= 9$, and 9×9 (rule 3), $= 81$—which when compared with the lower symbol $\mathbf{5} \times 2^4 = 80$, shows that the sound represented by the symbol 3, in the upper row, is sharper than that represented by **5** in the lower row, in the ratio 81, 80.

153. The last symbol in this example is 7. Now $7 \times 3 = 21$, and $21 \times 3 = 63$, which on being compared with the lower symbol $\mathbf{1\!\!1} \times 2^6 = 64$, shows that the sound represented by 7, in the upper row, is flatter than that represented by **1\!\!1**, in the lower row, in the ratio 63, 64.

154. In ex. 21 the key is C, and the bases f, C, g—in ex. 22 the key is G, and the bases c, G, d. Here the chord ♯F, A, C, D, is the same in both examples, so far as keyed instruments and the notation is concerned. But it has just been shown that ♯F, **17**, in the key of C, is sharper than ♯F, 5, in the key of G, in the ratio 136, 135—that the note A **5**, in the key of C, is flatter than A 3, in the key of G, in the ratio 80, 81—and that the note C **1\!\!1**, in the key of C, is sharper than C 7, in the key of G, in the ratio 64, 63. Three notes out of four are therefore not the same—two, in the key of C, being sharper, and one flatter—the discrepancies between them, as it were, pulling in opposite directions. It will moreover be found that, in the example in the key of C, the note ♯F may be very well substituted for ♯F, but not so with the example in the key of G. There are other points worthy of notice in respect to these examples, for the investigation of which the reader is not yet duly prepared—but let him well consider what has been said, and he will no longer submit to suffer conventional signs to occupy his mind, to the exclusion of the things signified. Let him, upon all occasions, restrict conventional signs to *use*, and not suffer them to grow into an *abuse*. In the former instance he is their master, in the latter their slave.

155. In ex. 23, 24, the first chord in the key of C is ♯G **17**,

B 5, E 5—in the key of E it is ♯G 5, B 3, E 1—which, when properly set out for analyzation will appear as follows:—

$$5\left\{\begin{array}{c|c|c} 5 & 3 & 1 \\ \hline 17 & 5 & 5 \end{array}\right.$$

Then, proceeding according to the rules § 148—150, $5 \times 5 = 25$ and $17 \times 3 = 51$. Elevate 25 to its octave 50, and we have the ratio 50, 51—whence it follows that ♯G 5 is flatter than ♯G 17 in the ratio 50, 51. The two next symbols are $\frac{3}{5}$—but, instead of 3, we may substitute 1, the sound represented thereby being the same. Then $1 \times 5 = 5$, and as this corresponds with the number in the lower row, it follows that the sound represented thereby is the same—and the ratio $\frac{1}{1}$. The sound represented by the two next symbols $\frac{1}{5}$ is also the same, the numerals being the same and the process the same.

156. I now proceed to the next chord, in which the notes are again the same, in both keys. Here, in the key of C, the symbols representing the chord, are **5**, **3** (or 1), 5—in the key of E, **1**, **19**, **3** (or 1)—which arranged in form will stand thus:—

$$*5\left\{\begin{array}{c|c|c} 1 & 19 & 1 \\ \hline 5 & 3 & 5 \end{array}\right.$$

The analysis for the symbols $\frac{1}{5}$, is $1 \times 5 = 5$ which being the same as for the note below, the sound represented is also the same—for the symbols $\frac{19}{3}$, $19 \times 5 = 95$, and $3 \times 2^5 = 96$. The note C **19** is therefore flatter than C **3** in the ratio 95 to 96. The circumstances relating to the symbols $\frac{1}{5}$, are the same as for $\frac{1}{5}$.

157. The student will do well to symbolize all the other chords in the preceding and following examples, by the adoption of the *spelling* principle pointed out § 103. And, in addition to this, he may also ascertain the ratio between other

notes, by attending to the three rules laid down for that purpose, § 148—150.

158. The use and importance of symbolizing notes and chords will now, I trust, be clearly perceived—and more especially when we reflect upon the fact that, as in algebra, every formula is of universal application, not being confined to any one key, or to the conjunction of any two keys in particular.

159. In ex. 25, 26 the second chord in the key of C, is the same as the third chord in the key of ♭B. These two examples being detached, the ratio between the basal key-note C, and the harmonic key-note ♭B, is taken as 1 to 7—not because it is so, or must be so, in this particular instance, but because examples exist, in which this ratio is really involved. The symbolic representation of the chord F, A, C, in both keys, is as follows:—

$$7\left\{\begin{array}{c|c|c} 1 & 5 & 3 \\ \hline \mathbf{1} & \mathbf{5} & \mathbf{3} \end{array}\right.$$

The analysis is $\mathbf{1} \times 7 = 7$—and $7 \times 9 = 63$,—and $\mathbf{1} \times 2^6 = 64$. Hence the ratio of **1** to **1** is 63, 64. Again $\mathbf{5} \times 7 \times 9 = 315$, and $\mathbf{5} \times 2^6 = 320$. But the ratio 315, 320, when reduced to its lowest terms, is also 63 to 64. Lastly $3 \times 7 \times 9 = 189$,—and $\mathbf{3} \times 2^6 = 192$. But the ratio 189, 192, reduced to its lowest terms, is again 63 to 64.

160. I have gone through the details of this example merely to show that, whenever the symbols are numerically the same, but differ as to quality (*i. e.* as *sub*, *central*, and *super*), it is only requisite, in order to find the ratio of the one to the other, to multiply the number of its respective prefix (3 or 9), § 150, by the common multiplier in front of the brace (the ratio of the two key-notes) for the ratio of the upper symbol—and, after this, to find the corresponding power of 2 for the lower. For example, $9 \times 7 = 63$ and $2^6 = 64$—and, in every instance, 63 to 64 is the ratio of the sounds answering to the same note, in the above examples.

161. Casual instances will also occur, in which the common multiplier, in front of the brace, being the same as one of the numerals in the lower, the usual operation may be shortened by treating them both as units—that is, by taking no notice of either.

162. Any arithmetician will at once perceive that, in all that is here said and done, the object in view is merely to find a

common denominator, expressed in its lowest terms, for the sound or sounds represented by the two symbols answering to the same note on keyed instruments—and which may or may not, be also the same in respect to its notation—and, consequently, that when the sound thus represented is the same, the terms of the ratio will also be the same, the representation of which is $\frac{1}{1}$.

163. If the ratio of the two key-notes C, ♭B, be taken as 9 to 8, in this case, ♭B becomes the basal and C the harmonic key-note—and then the formula will stand thus :—

$$9 \left\{ \begin{array}{c|c|c} \mathbf{1} & \mathbf{5} & \mathbf{3} \\ \hline 1 & 5 & 3 \end{array} \right.$$

But here the numerals being the same, and the common multiplier (9) the same as that relating to quality *(super, sub)*, it is evident that the sound represented in each case is also the same. Wherefore, to multiply all the symbols in the upper row by 9, for one purpose, and those in the lower also by 9 for another, is evidently useless.

$$\frac{9}{5 \text{ or } 10} \left\{ \begin{array}{c|c|c} \mathbf{1} & \mathbf{5} & \mathbf{3} \\ \hline \mathbf{1} & \mathbf{5} & \mathbf{3} \end{array} \right.$$

164. If the ratios of the two key-notes be assumed as $\frac{9}{5}$ or $\frac{9}{10}$ as above—since these are both harmonic key-notes, which refer to ♭A as the basal key-note—that is, to ♭A, as the central basis, or root of the great system—the process required in the analysis, is, to multiply the symbols answering to the key of C (the harmonic 5 to ♭A) by 5—and the symbols answering to the key of ♭B (the harmonic $3^2 = 9$ to ♭A) by 9. For example $\mathbf{1} \times 9 \times 9 = 81$—and $\mathbf{1} \times 5 \times 2^4 = 80$—whence it follows, that in this case, the ratio **1** to **1** is 81, 80. $5 \times 9 \times 9 = 405$—and $\mathbf{5} \times 5 \times 2^4 = 400$—in its lowest terms 81, 80. Lastly $3 \times 9 \times 9 = 243$—and $\mathbf{3} \times 5 \times 2^4 = 240$—also in its lowest terms 81, 80. Wherefore, in this case, the notes F, A, C, are all higher in the key of C than in the key of ♭B, in the ratio 81 to 80.

165. Before we pass on to the next common multiplier, 11, let us ascertain the ratio of the vibrations of the note ♯G **17** and ♯G **11**—B 5 and B 17—the ratio of the two keys being as 1 to 7, and the keys chosen for the example C and ♭B.

$$7\left\{\begin{array}{c|c}\frac{11}{17} & \frac{17}{5}\end{array}\right.$$

The analysis is $11 \times 7 = 77$, and $17 \times 9 = 153$. Hence the ratio of 11 to 17 is as $77 \times 2 = 154$ to 153. $17 \times 7 = 119$, and $5 \times 3 \times 2^3 = 120$. The ratio 17 to 5 is therefore 119, 120.

166. In ex. 27, 28, we find the chord F, ♭A, ♭D, the symbolic representation of which is:—

$$11\left\{\begin{array}{c|c|c}\frac{5}{1} & \frac{3}{19} & \frac{1}{11}\end{array}\right.$$

the analysis is $5 \times 9 \times 11 = 495$—and $1 \times 2^9 = 512$. The ratio 5 to 1 is, therefore as 495 to 512. $3 \times 9 \times 11 = 297$—and $19 \times 2^4 = 304$. The ratio 3 to 19 is, therefore, 297, 304. Lastly $1 \times 11 = 11$, and as this coincides with the 11 below, the sound, in this case, is therefore identical.

167. In the preceding example, the basal is the lower, and the harmonic, the upper key-note—but since upon keyed instruments, though not as to the ratio of its vibrations, or the notation, the ♭5th (C, ♭G) is the same as its invertion the ♯4th (♭G, C). I again proceed to analyze the last example upon the supposition that ♭B is the basal, and C the harmonic key-note—in which case the symbolic representation of the chord F, ♭A, ♭D will stand thus:—

$$11\left\{\begin{array}{c|c|c}\frac{1}{5} & \frac{19}{3} & \frac{11}{1}\end{array}\right.$$

168. But as the symbols now represent inverted intervals, the mode of operation must be changed; the numerals below being treated according to the rules for those above, and *vice versa*. Or, instead of this, the process may be carried on as usual, without inversion, provided the ratio or ratios relating to each note, be inverted, when the operation is completed. For example, it has been shown that the ratios of the sounds of the chord F, ♭A, ♭D, § 166 are:—

Symbolic Representation.	Ratios.	Notes.	Basal Key-note.	Harmonic Key-note.
$11\left\{\begin{array}{c\|c\|c}\frac{5}{1} & \frac{3}{19} & \frac{1}{11}\end{array}\right.$ =	$\begin{array}{c\|c\|c}\frac{495}{512} & \frac{297}{304} & \frac{1}{1}\end{array}$	F, ♭A, ♭D.	C.	♭B

and hence it follows that § 161 must represent the same ratios inverted:—

Symbolic Representation.	Ratios.	Notes.	Basal Key-note.	Harmonic Key-note.
$11\left\{\frac{1\,\|\,19\,\|\,11}{5\,\|\,3\,\|\,1}\right.$	$\frac{512\,\|\,304\,\|\,1}{495\,\|\,297\,\|\,1}$	F, ♭A, ♭D.	♭B	C.

169. The first chord in ex. 29, is the same in respect to keyed instruments, and the notation, as the second chord in ex. 30. The third chord in ex. 29, is the same as the first and last in ex. 30. The formula relating to these two chords are:—

$$13\left\{\frac{5\,\|\,3\,\|\,1}{1\,\|\,19\,\|\,11}\right. \qquad 13\left\{\frac{1\,\|\,3\,\|\,5}{19\,\|\,19\,\|\,1}\right.$$

Proceeding in obedience to the rules, the ratios and notes are:—

$$\text{Keys.}\left\{\frac{\flat\text{A}.\,65\,\|\,39\,\|\,104}{\text{C}.\,64\,\|\,38\,\|\,99}\right. \qquad \text{Keys.}\left\{\frac{\flat\text{A}.\,39\,\|\,39\,\|\,65}{\text{C}.\,38\,\|\,38\,\|\,64}\right.$$

F, ♭A, ♭D. ♭A, ♭E, C.

170. It is stated, § 19, that the harmonic ratio 13 represents the ♯5th—but however desirable it may be to adhere to rule, and consistency, in regard to the notation of the sounds appertaining to the *same* triune system—it would evince sheer folly, and pedantry, to extend the application of these rules to the denomination of different central bases, whatever may be the ratio of the one to the other, in defiance of the inconveniences that might result therefrom. I have therefore thought proper to write the last example in the key of ♭A, with 4 flats, rather than ♯ G, with 8 sharps.

171. As the direct ratio, 1 to 13, cannot be distinguished, upon keyed instruments, from the inverted ratio, 5 to 1, and *vice versa*—in the present examples, ♭A may be understood as the basal, and C as the harmonic, key-note. And since this last interpretation is the simplest, it ought to be preferred—and the more so, as we are entirely destitute of the means of making any practical use as bases of the ratio 1 to 13, or its invertion 13 to 1—and consequently know nothing concerning its effects.

172. The formula relating to the two preceding examples, upon the supposition that ♭A, is the basal, and C, the harmonic, key-note, are:—

$$5\left\{\frac{1}{5}\,\middle|\,\frac{19}{3}\,\middle|\,\frac{11}{1}\right| \qquad 5\left\{\frac{19}{1}\,\middle|\,\frac{19}{3}\,\middle|\,\frac{1}{5}\right|$$

Ratios and notes answering to the above:—

$$\text{Keys.}\left\{\frac{\text{C}.\ 1}{\flat\text{A}.\ 1}\,\middle|\,\frac{95}{96}\,\middle|\,\frac{495}{512}\right. \qquad \text{Keys.}\left\{\frac{\text{C}.\ 95}{\flat\text{A}.\ 96}\,\middle|\,\frac{95}{96}\,\middle|\,\frac{1}{1}\right.$$

F ♭A ♭D ♭A ♭E C

173. In ex. 31, the third chord is the same upon keyed instruments, and, with the exception of one note, the same as to the notation, as the first chord in ex. 32. The formula relating to this chord is:—

$$17\left\{\frac{1}{19}\,\middle|\,\frac{5}{3}\,\middle|\,\frac{3}{7}\,\middle|\,\frac{7}{17}\right.$$

Ratios and notes answering to the above:—

$$\text{Keys.}\left\{\begin{array}{l|l|l|l|l}\flat\text{D}\ 153 & 255 & 459 & 63 \\ \hline \text{C}\ 152 & 256 & 448 & 64\end{array}\right.$$

♭A C ♭E ♭G ♯F

174. The symbols **19, 3, 7, 17** represent what is called the chord of the extreme ♯6th, and 1, 5, 3, 7, the chord of the ♭7th. These two chords frequently pass from one to the other, and the symbolic numerals and ratios, represent the transition, through the medium of the same notes, from any key to the semitone above—and *vice versa*.

Let the reader now turn back to § 117—119, and again read what is there said concerning the chord of the extreme ♯6th, and the chord of the ♭7th—and he will perceive the importance of attending to every circumstance relating to them—and should he be a performer upon any instrument, let him attend to their effects, and compare the one with the other. Even on the piano, if he has any taste for music, and any ear for the truth of its intonation, his feelings in respect to what is required will at once determine who is right in regard to its theory.

Consistently with my views of the science—(1) not one sound in the above two chords is the same—(2) the relation between the two key-notes is specifically and unequivocally stated—(3) in the chord of the extreme ♯6th the note ♭A being lower, and ♯F higher, than the

greater, while in both instances these notes are made to approximate, or, in other words, to impinge upon those to which they naturally tend, and upon which they must finally resolve—a fact peculiar to my theory, and in agreement with practice. (4) The note ♯F being an ascending chromatic, must ascend—and if accompanied by a descending progression, the note in that part must be ♮F, notwithstanding its being made sharp in any other part—a law confirmed universally in practice.

In regard to other theories the matter stands thus—(1) for many years D was the accredited basis, the note ♭A being taken "by licence" instead of ♮A—the note ♭E as "a borrowed note," instead of D, and the key *ad libitum*—(2) the fashion having at length changed, the note ♯F is the arcredited basis, the key being still *ad libitum*—(3) the notes ♭A, C, ♭E being declared the same in both keys, and ♯F in submission to a mere abstract calculation, made lower than ♭G, the difference between ♯F and ♭G becomes the sole cause of the difference between their bases—which difference is, nevertheless, most unaccountably made to involve the interval of the ♯4th—(4) the circumstances relating to the union of ♯F in ascent with ♮F in descent, being very prudently overlooked.

That these inconsistencies in regard to the chord of the extreme ♯6th, is indicative of rottenness within, and not a mere scratch upon the surface, no one can doubt, who has any penetration in regard to such matters. The only peculiar circumstance relating to it, is, that here the eruption makes its appearance, so as to render it difficult to find a plaster, to cover it over. Moreover, the metaphysics relating to licences, borrowed notes, &c., may certainly be said to present as good a specimen of pure nonsense as can anywhere be found. To show that false theories are not altogether harmless, I observe that there exists an ever increasing tendency in modern compositions to substitute the accredited, but false basis instead of the harmonic 7—that is, the key being C, to substitute ♭A, C, D, ♯F, for ♭A, C, ♭E, ♯F—or for the more simple, and elegant combination ♭A, C, ♯F.

175. Upon the supposition that the ratio of the key-notes, C, B (ex. 33, 34), is 16 to 15—the analysis of the chords E, G, B, and G, D, B, which will be found in both examples, is:—

$$15\left\{\begin{array}{c|c|c} 1 & 19 & 1 \\ \hline 5 & 3 & 5 \end{array}\right. \qquad 15\left\{\begin{array}{c|c|c} 19 & 19 & 1 \\ \hline 1 & 3 & 5 \end{array}\right.$$

Ratios and notes answering to the above:—

$$\text{Keys.}\left\{\begin{array}{c|c|c} B\ .\ 1 & 95 & 1 \\ \hline C\ .\ 1 & 96 & 1 \\ E & G & B \end{array}\right. \qquad \text{Keys.}\left\{\begin{array}{c|c|c} B\ 95 & 95 & 1 \\ \hline C\ 96 & 96 & 1 \\ G & D & B \end{array}\right.$$

176. The third chord, in ex. 35, in the key of C minor, is the same in respect to keyed instruments, and the notation, as the second chord in ex. 36, in the key of ♭E major—the third chord in ex. 37, in the key of C major, the same as the second in ex. 38, in the key of E minor.

The formula relating to ex. 35, 36, is:—

$$19\left\{\begin{array}{c|c|c} 1 & 5 & 1 \\ \hline 19 & 3 & 19 \end{array}\right.$$

The ratios and notes answering to the above are:—

Keys.			
	♭E . 1	·95	1
	C . 1	96	1
	♭A	C	♭E

The formula relating to ex. 37, 38, is:—

5			
	1	19	𝟙
	5	3	5

The ratios and notes answering to the above are:—

Keys.			
	E . 1	95	1
	C . 1	96	1
	A	C	E

Having now given specimens of the method of ascertaining the ratio between notes and chords which are the same upon keyed instruments, and, in most instances, the same as to the notation also—any one, who duly reflects upon the subject, must be driven to the conclusion that, either the theories which ignore these particulars must be rotten to the core, or else, that the materials of the fabric I am endeavouring to establish, are interwoven with unparallelled ingenuity. Wherefore, let every one settle this point in agreement with his own notions. I know the final result, and that is my chief concern. With the intermediate process of fermentation, ebullition, and precipitation of dross, I have no wish to be concerned.

177. It will be seen that the specimens above alluded to, afford one or more examples, in which the primes 3, 5, 7, 9, 11, 13, 17, 19 are successively introduced, as the common multiplier.

The reason why I stop at 19, is, because none of the higher primes can be practically distinguished from these, § 71.

178. In order to show how many things are interiorly involved, where in externals there is little or no difference to be seen—I here introduce a representation of the vibrations of the chords described in § 154—176. See figs. 19, 20.

179. In regard to the figs. 19, 20, let it be observed that the pitch of the note D being the same, the dots representing its vibrations, are set exactly at the same distance apart. But, as the ratio of the notes ♯f, a, c, are not exactly the same in fig. 20 as in fig. 19, the spaces between the dots representing the vibrations of these notes, are varied accordingly. That the ratio 24, 27, is as 8 to 9,—that 20, 24, as 5 to 6—6, 8, as 3 to 4,—is

shown by the short lines which connect these ratios together—and that the ratios between these notes, are, as stated in fig. 22, has been proved, § 154.

180. By examining figs. 21—24 the truth of what has been said concerning real and apparent major and minor common chords,—that is, that the apparent major appertains to the 2nd or 3rd mode—and the apparent minor to the 1st mode—becomes a matter of occular demonstration—also that, in both instances, these apparent chords involve *three bases*, of which the medial basis is the only one that changes with the mode.

181. In these very peculiar instances, as well as in all others, cause and effect are seen, as it were, face to face, so as to afford the means of perceiving in the latter an effigy of the former—for, let me ask, who can be insensible to the mental commotion occasioned by the natural running accompaniment, fig. 24—or to the contrast, between it, and the placid effects, arising from either of the two common chords when real? Where can we find the artist who can imitate this accompaniment? And who, after this, will maintain that music is an art only, and not also a science? That it is so, in reference to our productions, is in a great measure true. But science, properly so called, has respect to the Creator, of whose perfect and immutable laws, it is the exponent. That the three bases appertaining to the two apparent common chords should produce a natural running accompaniment—and that this, to an attentive listener, should create in the mind the various effects of deep pathos, accompanied with a sort of impatient enquiry into the cause of the unlooked for *interruption**—may well excite surprise and admiration.

182. As only one basis has heretofore been assigned to any chord, while, nevertheless, it has in this work been shown, that a chord consisting of only three notes, *may* generate three bases—it is here necessary to inform the reader, that the *maximum* of generated bases, appertaining to any given number of sounds, corresponds, in every instance, with what are called TRIANGULAR NUMBERS. For example, the numbers 1, 3, 6, 10, 15, 21, 28,

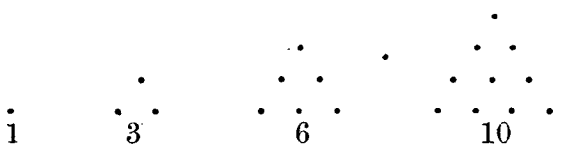
---

* N.B.—These are the two chords that constitute the elements of the so-called *interrupted* cadence. Thus, from feeling, we obtain truth, expressed in the language relating to feeling—but in respect to the science of music, instead of truth expressed in the language of science, we have hitherto been presented with nothing but a tissue of falsehoods, ostentatiously decked out in the garb of science.

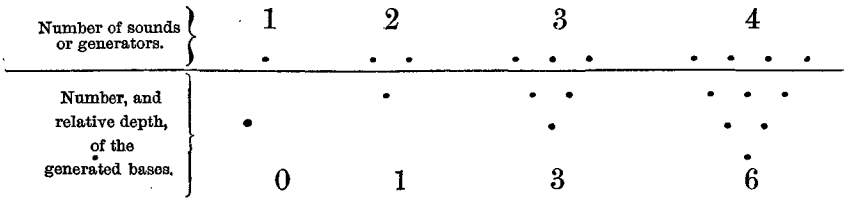
&c., are formed by adding the successive terms of the series of natural numbers thus:—

$$1 = 1$$
$$1 + 2 = 3$$
$$1 + 2 + 3 = 6$$
$$1 + 2 + 3 + 4 = 10, \text{ \&c.}$$

They are called triangular numbers, because the number of points, corresponding to any term, can always be placed in the form of a triangle—for instance:—



By inverting the usual position of the triangular forms, the dots become representative, not only of the maximum of the generated bases, but also of their number, and relative depth, according to degrees.



By this it will be seen, that 1 generator only, produces no basis—that 2 generators produce 1 basis only—that 3 generators produce 2 bases of a higher, and 1 of a lower degree—that 4 generators produce 3 bases of a higher, 2 of a lower, and 1 of a still lower degree. It may here be observed that the generated bases of the first or highest degree result from simple ratios only—those of the second degree from square numbers—and those of the third degree from cube numbers, &c.

183. This law, however, though true in itself, can never practically come into operation, so as to produce more than *three* bases at once—because the only combinations the ear will admit, in practical music, are those derived from the triune system. For, let it be observed that, even upon the supposition

that all the primary harmonics, *ad infinitum*, of any primary basis could be simultaneously produced—the primary basis is the only sound that would be generated thereby. But, the facts relating to these things, and their application to practice, will be seen hereafter. An attentive observer of nature, will, however, even now see cause for admiration—for, where this not the case, the generated bases might give birth to all sorts of discordant sounds, which consistently with the peculiar arrangements, and the laws of order, appertaining to the triune system, is impossible.

184. In ex. 39, 40, G is the primary basis to the chord G, B, D, F, and C a secondary basis. Fig. 25 represents the vibrations of this chord, wherein the ratios 2, 3, 5, 7, and the short cycles, refer to the primary basis G—and the ratios 6, 9, 15, 21, and the long cycles, to the secondary or sub-basis C.

As this chord consists of 5 parts, and these parts of the conjunction of 3 and 2 consecutive letters (B, C, D, and F, G, in the key of C), theorists have been sadly perplexed to know what to make of it, and what to say about it. Wherefore, in this dilemma they have been induced to cut the matter short, by affixing to the sound G the epithet *pedal harmony, or pedal note.* But though, in music for the organ, this sound, as being the sub-basis, frequently occupies the lowest part, and thereby becomes a note for the foot and pedal, it nevertheless sometimes occupies a higher, and especially the highest part. But, be this as it may, what the mere dubbing of a sound with a name having reference to the foot and a piece of wood, has to do with the explication of harmony, I am utterly at a loss to conceive.

185 Having no wish to spin out this work to an unnecessary length, by dwelling upon trifles, relating to the fact that the position of the notes $\begin{matrix} G \\ E, \\ C \end{matrix}$ may be changed to $\begin{matrix} C \\ G, \\ E \end{matrix}$ and $\begin{matrix} E \\ C \\ G \end{matrix}$—I have as yet said nothing about the inversion of chords, because a few remarks in reference to this very simple, and not very important subject, is all that can be required—and, the best time to do this, will be when we enter upon the analysis of entire phrases, and movements, taken from the works of great masters —that is, the masters whose works can abide the ordeal to which this new method of analysis will render them subject.

PART 9.

OF THE SECOND PERIOD OF THE DEVELOPMENT OF THE TRIUNE SYSTEM, WHENCE PROCEED THE HARMONICS DENOMINATED FIXED CENTRAL CHROMATICS, AND THE COMBINATIONS THENCE DERIVED.

186. It has been shown, § 78, that if the diatonic sounds appertaining to the triune system be alphabetically disposed, the bases of no two adjacent sounds are the same. That this series of sounds should be more consonant with the laws relating to melody and harmony in ascending than descending, has nothing whatever to do with the fact herein recorded, relating simply to their relative position—and which is also true whether the diatonic sounds be those of the first, second, or third mode. As the necessary consequence it follows—(1) that the character of the symbol of every two adjacent diatonic harmonics, must in every instance differ—(2) that all melodies strictly diatonic, and which proceed by ascending to the next note above, or by descending to the next note below, must necessarily announce a new basis at the introduction of every new note—(3) that thus incessantly, and unceremoniously, to vary the bases, conveys to the mind the idea of something rude, abrupt, and inelegant.

187. It will now be shown—(1) that by the introduction of fixed central chromatics, appertaining to the second period, in the development of the triune system, the position of those relating to the first period, undergo no alteration, whatever may be the mode—(2) that the elements of the first period are indispensable to the second, but not so the second to the first—(3) that those exclusively appertaining to the second, are adapted, as it were, to round off the corners, and rough angular forms, of the first, by the insertion of intermediate sounds, between those of the first period—that in so doing, the simplicity and grandeur of the outline, and general contour of the entire figure, is neither destroyed, nor, in any way, effaced.

188. Upon the understanding that the reader is familiarly acquainted with the construction of the altitudinal display, § 128, and all that has been said concerning it—I here ask what would be his natural course, if desirous to proceed farther—and, if left to himself to find out the true ratios of the inter-

mediate sounds of the second period, with which modern music abounds? Would he not, in such a case, appeal to the ear in the first place—mark down carefully the facts thus obtained—and then endeavour to find out the cause, or governing principle relating to them?

189. But let a similar case be put in reference to some other subject, and the preposterous nature, and arrogant conceit, of attempting to build upon a foundation not based upon facts, will appear in its true colors. Let it here be supposed, for instance, that an animal of an unknown species, is reported to have been found in some remote part of our globe. In such a case, would our naturalists, upon hearing this report, proceed to determine the nature, appearance, size, form, &c., of this newly-discovered animal, by sticking together the limbs of others, with which they were acquainted, and thus fabricate a monster as its representative? Or, would they not proceed by collecting all the information they could concerning it—and, if an opportunity occurred, would they not examine it, with all possible care and attention?

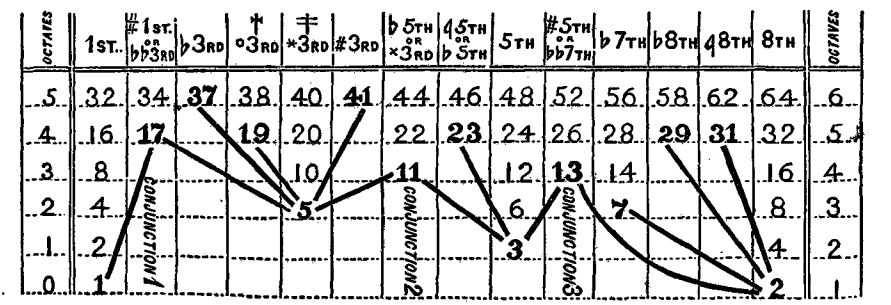
If this be the natural and only common-sense method in this case, why not so in music? If the mathematics is unavailable for the discovery of facts of this kind, it is equally so in regard to music. It is evident that discoveries, such as these, do not lie within the sphere of mathematical operations, which consists in bringing to light the more or less remote consequences resulting from admitted facts or truths, and in pointing out the harmony and consistency which reigns throughout. But, to do this, the truths of nature must be compared one with another, and not a natural truth with one which is artificial, since this can lead to nothing but fallacies. As for instance:—

190. It is true that the ratios of the vibrations of the major 3rd being 5 to 1, that of two major 3rds must be $5 \times 5 = 25$ to 1. It is also true that in respect to the pianoforte key-board, and the notation, two major 3rds (C, E, and E, ♯G, for instance), produce a ♯5th—but to infer from hence that this is the true ratio of the basal ♯5th is illogical and false, since only one of the two facts whence this idea is derived, is obtained from nature, the other being derived from the machinery relating to an instrument of human contrivance. Now, by experiment, it is found that for the interval of the basal ♯5th, the ear demands the primary ratio 13, or $13 \times 2 = 26$, and not 25—and since there is no disputing the fact, as such—or pushing it aside,

except in reference to ourselves, by shutting our eyes and ears —we are compelled to let it remain, however irreconcilable it may be to our preconceived notions. But if it be afterwards discovered that, what at first appeared as an isolated fact, is not so, and that so far is this from being the case, it is, in truth but one of the items of the universal law of order, relating to such things—that is, to the existence of a stupendous system of orderly arrangements, respecting the vibrations of musical sounds—such as no artificial system can possibly rival, either for simplicity, unity or variety—this apparently unimportant, and isolated fact, respecting the basal ♯5th, will then be seen in its true light—as one of the links in a chain of innumerable other distinct truths, and the whole as one universal truth.

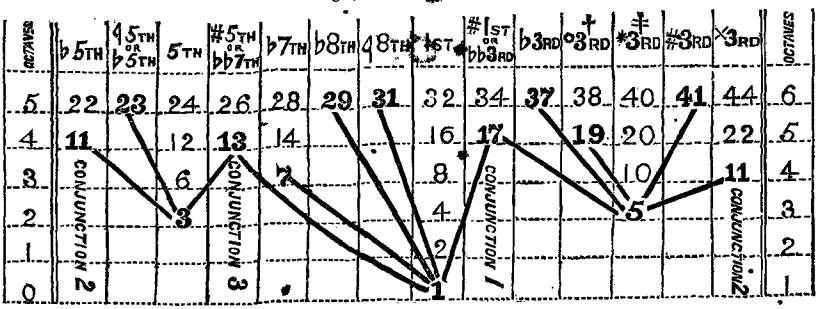
191. Having, by way of example, alluded to the circumstances relating to the basal ♯5th—it is now to be observed that many other similar instances could be pointed out, were it necessary—the general result of which, is, that in actual practice those chromatic sounds, which, in obedience to the ear, proceed to the next diatonic note above, are uniformly sung, and upon the violin, &c., played, sharp—those which, in obedience to the ear, proceed to the next diatonic note below, are uniformly sung, and played, flat—in direct opposition to the dogmas laid down by the acousticians. That singers, violinists, &c., are right in so doing is shown by a variety of particulars in the ratiometer—the truth of which is moreover verified by the following:—

Representation of the affinities or peculiar consociations of numbers, one with another, in respect to the simple and triune system—the unit-sound or basis being placed on the left, and the harmonic 2 on the right.



† °3rd stands for the minor 3rd.
‡ *3rd stands for the major 3rd.

Representation of the affinities or peculiar consociations of numbers, one with another, in respect to the simple and triune system—the unit-sound or basis being placed in the centre.



192. By the above diagrams are represented the peculiar consociations existing among the ratios of the vibrations of sounds generally. This is shown in the first diagram by placing the basis on the left, and its octave on the right—in the second diagram by placing the basis and its octaves in the centre—these two diagrams being in other respects the same. The intervals answering to the harmonics are placed above, and the harmonics themselves at different elevations, in agreement with the octave in which they occur, as marked at the two extremities—and the harmonics of every series lie between the octave marked on the left and right. It will be seen by the thick lines, radiating from the diatonic centres 1, 3, 5, that the consociations, or family connexions, of the basis, are the primes 13, 7, 29, 31, in descent, and 17 in ascent—those of the prime 3 in descent are 11, 23, and in ascent 13—those of the prime 5 in descent are 17, 37, 19, and in ascent 41, 11.

193. What the ratio is, between these numbers, and others, appertaining to the same octave, is also pointed out by those in the same parallel. In every instance, the primes are distinguished from those relating to octaves, by being thicker.

Though all the circumstances relating to these two diagrams, must, if understood, appear very extraordinary, and such as to preclude every idea of their being artificially got up, and contrived—nevertheless, that relating to the coalition of their three distinct members—or, in other words, to the conjunction of the higher extremity of one member, with the lower extremity of another, and *vice versa*, is certainly one of the most remarkable. The extent to which the influence of the mode operates, to vary the position of the connecting links, which

unite the ascending with the descending radiants, will be pointed out hereafter.

194. The general rule, in regard to the notation, adopted in this work—and which is in agreement with the ordinary practice of all composers, is, to write chromatic notes upon the same degree of the scale, as the diatonic, in whose place they may be said to stand. But though 7, and sometimes 13, are really descending chromatics, to which the basis is the diatonic relative—as the operation of this rule would, in these two instances, be attended with extreme inconvenience, it is necessarily set aside. For example, the chord of the ♭7th, G, B, D, F, would, if written in coformity with the above rule, be named, G, B, D, ♭♭G—that is, the ♭7th, as a descending chromatic to the basis, is in reality a ♭♭8th, and the descending chromatic harmonic 13, a ♭♭♭8th. But, however absurd it would be to write in this fashion, it is important, nevertheless, that it be distinctly understood, that these two descending chromatics really do appertain to the basis, as its diatonic relative.

I am fully aware how startling this will appear to many, especially before time and reflection shall have rendered the ideas relating to it clear and familiar. Let it, however, be observed that the sole reason for making an exception to the rule, in the case of these, or other chromatic harmonics, is, not on account of any peculiarity inherent in themselves, but simply on account of the magnitude of the intervals produced thereby, in reference to the basis, or some one of its octaves. It is moreover to be observed, that as the intervals between diatonic and chromatic relatives vary with every particular harmonic, some one must be the largest. To this may be added, the self-evident fact, that, though man has thought proper to link music with the alphabet—and has even acted wisely in so doing—this affords no excuse for either the presumption, or folly, of attempting to subjugate the ordinances of nature to that, which is nevertheless purely arbitrary and conventional.

195. The diatonic, and medial chromatic harmonics, which stand consociated, the one with the other, in the above diagrams are, in this work, denominated RELATIVES,—the diatonic harmonic being called the *relative diatonic*, and the chromatic harmonics *relative ascending*, or *relative descending*. The term relative being adopted, significantly, to draw attention to the fact of their being the offspring of the same parental sound or basis. Furthermore:—

196. The sound to which any chromatic harmonic tends—to which the ear demands that it should proceed—and upon which it is said to resolve—is in this work denominated RESOLVENT. It is to be borne in mind—(1) that a resolvent is always a diatonic harmonic—(2) that at the introduction of every resolvent the basis, and consequently, the character of the symbol, is always changed.

197. Examples 41—74 will serve to illustrate, practically,

as far as can be done at present, the circumstances relating to the above diagrams. The student is requested to attend to the observations which follow, and also to compare them, severally, with the diagrams.

198. The first semibreve in every example is the diatonic relative, and the last the resolvent—between these, in black notes, are placed the relative chromatic harmonics. Besides the harmonic ratios, symbolically expressed, placed over the notes, there are small numerals representing, in its lowest terms, the ratio of the chromatic sounds, one with another, and with their diatonic relatives and resolvents. Also the ratio, in its lowest terms, of the diatonic relative and resolvent.

These particulars are here given in detail, with the view to direct the reader how to proceed himself, in similar cases—and also to draw his attention to the facts thus brought to light—so that while viewing the symbols only, he may get into the habit of perceiving the real ratios involved—and not become confused from the circumstances relating to octaves, or a change in the basis. §160—162.

199. Should some of these examples appear rather strange, it is to be recollected that they are not here presented in connexion with suitable passages—nevertheless, I feel confident, the reader will, in every instance, perceive the consistency and harmony between their effects, and the analysis here given of them. Should, however, any one feel disposed to cavil against these examples, and the exposition given of them—let him show how it is, that in ascending chromatically the basis, or octave to the basis, passes to the semitone, and not to the wholetone, above—while on the contrary, in descending chromatically, the basis, or octave to the basis, passes to the wholetone, and not to the semitone, below, except in conjunction with the wholetone. In other words, can any one explain these peculiarities, by reference to a principle, bearing more directly to the point, or more simple, than that which refers the ascending chromatic to the prime 17, in connexion with the octave $2^4 = 16$, and this descending chromatic to the prime 7, in connexion with the lower octave $2^3 = 8$? It is scarcely necessary to remark that the prime 17 is above, and not below, $2^4 = 16$—and the prime 7 below, and not above, $2^3 = 8$—and that the numbers on the opposite side of these powers of 2, are, in both instances, products.

PART 10.

NATURE—ART.

200. In ex. 75—80 are presented the series of sounds commencing from the basis, and proceeding, gradually, with the harmonics 2——41, as produced by nature, from a vibrating string or pipe—in connexion with the sounds appertaining to the artificial semitonic scale.*

201. In ex. 75, the vibration 1 represents the lowest sound, and 2 its octave—and since there is no intermediate number between 1 and 2, there is, consequently, no intermediate harmonic sound between the basis and its octave, or between the interval 1st, and 8th, marked over the notes. The note C, here adopted to represent the basis, and its octaves, may, of course, be transposed to any other note—and, in so doing, the harmonics will also be transposed. Where the pipe is very long, it becomes difficult, if not impossible, to produce the basis or even its octave. The instruments introduced in practice, for the exclusive production of these harmonic sounds, are the trumpet, French-horn, &c., without keys, or pistons.

202. In the above, and following examples, black notes represent octaves to the sounds already produced, and white notes the new sounds—that is, those appertaining to the octave in which they appear. In ex. 76, the only new sound is the 12th or octave 5th, arising from the prime 3—the double octave to the basis, and all the octaves above, being represented by the powers of 2. It is, however, to be borne in mind that, with the exception of the octaves to the basis, no harmonic sound can possibly coincide, exactly, with the tempered system of 12 artificial semitones in the octave. It is also to be noted, that in ascending from octave to octave, a new intermediate sound is obtained between the octaves of every sound, appertaining to the series below—whence it follows, that black and white notes appear, alternately, in every example, except that relating to the basis and its octave.

203. The essentially new sounds to which the octave, ex. 77, gives birth, are the *17th, or double octave to the *3rd, arising from the prime 5, and the ♭21st, or double octave to the ♭7th, arising from the prime 7.

Here the practical interval representing the notes E, G, in the ratio 5, 6, and that representing the notes G, ♭B, in the ratio 6, 7, are both alike—that is, both minor 3rds, then comes the abrupt transition from the minor 3rd, to the wholetone, by which is represented the vibrations 7, 8, answering to the notes ♭B, C. A little attention to this point will show that, by making the ♭B, flatter, the minor 3rd G, ♭B, becomes contracted, and the wholetone ♭B, C, enlarged. By this means a series of intervals is produced, which become gradually smaller, and not abruptly so, as is necessarily the case with the substituted intervals of the semitone scale.

The harmonic 7 is generally looked upon as being out of tune—that is, as being too flat—on account of its being flatter than the artificial interval by which it is practically represented by the semitonic scale. The best excuse (?) that can be made for nature, as being a bad tuner, in this and other instances, is, that in respect to time, she is equally at fault, if the orderly course of the heavenly bodies be judged by reference to clocks and watches.

204. The essentially new sounds to which this octave, ex. 78, gives birth are—(1) the 23rd or triple octave 2nd, arising from the product $3^2 = 9$—(2) the ♭26th, or triple octave to the ♭5th, arising from the prime 11—(3) the ♯26, or triple octave to the ♯5th, arising from the prime 13—(4) the *28th, or triple octave to the *7th, arising from the product 3×5 or $5 \times 3 = 15$.

Here the complaints against nature have for a long time assumed a very grave aspect—and if we enquire into the cause, it is this—that the prime 11, if called upon to do duty as a 25th (the triple octave 4th), is frightfully out of tune, that is, too sharp—and if the prime 13 be called upon to do duty as a *27th (the triple octave *6th), it is frightfully too flat.. Now, though this is unquestionably true, before we proceed to condemn nature, ought we not first to look a little to ourselves—to ask, for instance, whether the 4th be the intervals really intended for the harmonic 11, and the *6 for the harmonic 13? whether, as the harmonic 11 lies about midway between the 5th and 6th semitone—the 6th, and not the 5th, might not be the substituted interval in this case? And, as 13 lies about midway between the 8th and 9th semitone—whether the 8th, and not the 9th, might not be the substituted interval in this case? It is strange, unaccountably strange, that no misgivings, as to his own infallibility, should have entered the mind of any acoustician—so as to have induced him to come forward in the defence of nature, and enter his protest against those who have had the folly, and temerity, thus to impugn her laws and ordinances. These facts, will stand for ages, as monuments of disgrace to all whose names may be handed down to posterity, as having been concerned in the matter.

205. In ex. 78 the octave, divided into 8 *unequal* intervals, is compared with the semitonic scale which is divided into 12 *equal* intervals. Now, it is evident the points pertaining to these two essentially different systems, can never exactly coincide, except as to the starting point and its octaves—and also that at every step in their progress, from first to last, the discrepancy between the two systems, will vary. Moreover, with

nature, the system, in this octave, commences with the wholetone, and ends with the semitone—while, in the artificial system, the intervals are all mean semitones. But let us look to the secret motive that has given rise to the folly of attempting to force from nature, and from one basis only, what is called the diatonic scale—and we shall find that it it has been done in subserviency to a pre-conceived false philosophy concerning the supposed necessity of obtaining all the elements of the diatonic scale from one basis only—never perceiving that if true, the basis of all harmony would thereby be reduced to the drone of the bagpipe—that the ratios of products must invariably refer to the sounds of another system—that the reason the harmonics 11 and 13 have been such a source of trouble to them, is, because in their application, or rather misapplication, to diatonic intervals, they are made to usurp the place of sounds appertaining to a sub-basis—which, except by the substitution of false, instead of true, ratios, is impossible.

206. If the harmonic 11 be represented by the 5th semitone, then, after the introduction of the semitone E, F, in the ratio 10, 11, the resumption of the whole tone F, G, in the ratio 11, 12, becomes imperative—although the ratio 11, 12, is smaller than 10, 11. In like manner, if the harmonic 13 be represented by the 9th semitone, then, after the introduction of the semitone E, F, in the ratio 10, 11, the resumption of the wholetone G, A, in the ratio 12, 13, becomes imperative—although the ratio 12, 13, is considerably smaller than 10, 11. Moreover, since the octave is by nature here divided into 8 parts, and by the semitonic scale into 12, the example shows, clearly, both the cause, and necessity, of these awkward breaks—and also, that nature passes, gradually, and not abruptly, from the interval of the wholetone, to the semitone, and in so doing introduces intervals of the value of about a semitone and a half. § 18, 204.

207. The essentially new sounds to which the octave, ex. 79, gives birth, are—(1) the ♯29th, or fourth octave to the ♯1st—(2) the °31st, or fourth octave to the °3rd—(3) the 32nd, or fourth octave to the 4th, as a ♭7th to the super-basis—(4) the 𝑞 or ♭33rd, or fourth octave to the 𝑞 or ♭5th—(5) the ♯33rd—an unimportant sound in the scale, being the product of $5^2 = 25$—(6) the 34th, or fourth octave to the 6th—another unimportant sound in the scale, being the product $3^3 = 27$, and not the ratio derived from the harmonic 5 to the sub-basis—(7) the ♭36th or fifth octave to the ♭8th—a sound about midway between

the one below and the one above—(8) the ♮36th, or fifth octave to the ♮8th—another sound also about midway between the one below and the one above.

208. In ex. 78, the artificial series of 12 semitones in the octave, is called upon to represent, in the best way it can, the natural series 8—16. In ex. 79, on the contrary, the semitonic series is called upon to represent the natural series, 16—32—and since, in this case, 12 sounds have to do duty for 16—hence it follows, that in 4 instances, one sound must be made to supply the place of 2. Moreover, cases occur, in which sometimes one, and sometimes another, of the artificial series, must be substituted for the same natural sound, § 18.

209. The basal minor 3rd, derived from the prime 19, if ever at all alluded to, is calumniated as one of nature's badly-tuned intervals—and that it is too flat. It was intuitively perceived, by all the system-mongers, that it would never do to grant admission to the prime 19, and kick out the primes 7, 11, 13, 17, under the pretence that they were too high, or too remote, to produce good musical effects—or that they could not be made to amalgamate with the primes 1, 2, 3, 5—or that these, and especially 1, 2, 3, had something divine in them, which being wanting in the higher primes, rendered them profane—and therefore unfit companions to these saintly, and time-honoured, numbers. As to the sounds arising from the primes 23, 29, 31, &c., it has hitherto been the fashion unceremoniously to push them aside as mere mongrels.

210 In ex. 80, the essentially new sounds to which this series gives birth are—(1) the ♭37th, or fifth octave to the ♭2nd—(2) the 37th, or fifth octave to a sort of 2nd—an important sound in the system, because the product of 5×7, or $7 \times 5 = 35$—(3) the ♭38th, or fifth octave to a ♭3rd—the descending chromatic to the diatonic °3rd—(4) the ♯37th, or fifth octave to the ♯2nd—and, as being the result of $3 \times 13 = 39$—the true ♯5th to the super-basis—(5) the ♯38th, or fifth octave to the chromatic, or ♯3rd. Here it is evident that the 1st, 2nd, and 4th, tempered semitones, have to do duty for two, and the 3rd, for three natural sounds.

PART 11.

EXPLANATION OF VARIOUS MATTERS PREPARATORY TO THE INTRODUCTION OF THE MUSICAL RATIOMETER.

211. That the reader may be accustomed to picture to his mind, truly, whatever relates to musical sounds, as forming part of a system—and to reflect upon the system derived from nature, as being immutable and free from defects of any kind—it is important that these matters should first be presented to his view as it were piecemeal, and under a variety of aspects, that he may not find himself perplexed, when called upon to investigate the several parts, as members of one entire system. With this object in view, I proceed to explain the following diagrams and tablet:—

Basis and Primary Harmncs.	1	2	3	5	7	11		13		17		19
Octaves and fractional parts.	0	1	$1\frac{7}{12}$	$2\frac{4}{12}\mid\frac{1}{3}$	$2\frac{2}{12}\mid\frac{5}{6}$	$3\frac{6}{12}\mid\frac{1}{2}$ as a $\flat 5^{th}$	$3\frac{5}{12}$ as an $+3^{rd}$	$3\frac{4}{12}\mid\frac{2}{3}$ as a $\sharp 5^{th}$	$3\frac{3}{12}\mid\frac{3}{4}$ as a $\flat\flat 7^{th}$	$4\frac{1}{12}$ as a $\sharp 1^{st}$	$4\frac{2}{12}\mid\frac{1}{6}$ as a $\flat\flat 3^{rd}$	$4\frac{3}{12}\mid\frac{1}{4}$

212. By the above is represented the basis and primary harmonics 1—19, with the interval produced by each by reference to the octave, and by the semitone as fractional parts of the octave. The basis (1) merely represents the starting point, and consequently no interval—the harmonic 2 represents the octave —the harmonic 3 the 7th semitone above the octave, &c. These intervals are all distinguishable, the one from the other, by the notation—and, in most instances, upon keyed instruments also. The harmonic 29, being peculiarly circumstanced, in respect to its position, in reference to the triune system, is not inserted in the above series. This harmonic can, nevertheless, be represented by the notation, and upon keyed instruments—the interval produced by it being the 11th semitone above the 4th octave. The other harmonics, above 19 are, in practice, all confounded with the above series. § 71.

Basis and Primary Hrmncs.	1	17	37 19	41 5	23 11	3	13	7	29	31 2
Intrvls.	Unison. 1st	♯1st—♭♭3rd	Minor. 3rd	Major. 3rd	×3rd—♭5th	5th	♯5th—♭♭7th	♭7th	♭8th	8th
Semitns.	0	1 | 2	3	4	5 | 6	7	8 | 9	10	11	12

213. By the above is represented the basis and primary harmonics 1—41, with the interval or intervals produced by each by reference to the number of semitones above any given octave. If this diagram be compared with those in § 191, 211, they will be seen to agree in every particular, although each is set out with some specific object in view, which the others are not so well adapted to portray. In the present, for instance, the gradual progress from the basis to the harmonic 2—the corresponding progress of the intervals from the unison to the octave—and of the semitones from 0 to 12, is made conspicuous. Here the peculiar circumstances relating to what may be called the double functions of the primes 17, 11, 13, and the intervals to which they are referred, demands particular attention—and the same may be said of the prime 23, 31, 37, 41, in respect to their position between the lower and more important primes. Here also it may be remarked that, though in respect to the triune system, **29**, as a ♭8th to its bases, is distinguished by the notation from 𝟓, 𝟐𝟗 from 5, and 29 from **17**, they are not so upon keyed instruments.

214. I shall now just touch upon what is called the complement of the harmonic division of a string in which are involved several peculiar circumstances.

Let us here suppose the sound of the open string C, ex. 81, to be produced upon the violoncello—then, if this string be divided into 9 equal parts, as represented by fig. 26, and we touch the node 1, the harmonic produced will be D,—the complement or remaining portion of the string will be 9—1 = 8 times the length of the harmonic portion 0, 1,—whence it follows that the sounds D, D, stand in the relation of basis and harmonic 8.

215. As the numerals below (fig. 26) represent the relative length of the complementary portions of the string, answering to every particular harmonic, those above must severally represent the ratio of the vibrations of those portions—the one being at all times the inverse of the other.

216. If, instead of the node 1, we now touch the node 2, the

harmonic produced will still be D,—but the complement in respect to this point will be E. For, since the fraction $\frac{7}{9}$ cannot be reduced to lower terms, the harmonic sound will remain the same, its vibrations not being affected by touching the node 2 instead of 1. But the complement being now reduced to $\frac{7}{9}$, while the harmonic responds, to the product 9, the sound produced by the complement $\frac{7}{9}$, must be the basis to the prime 7, whence it follows that the sounds E, D, stand in the relation of basis and harmonic 7.

217. If, instead of the node 1, or 2, we now touch the node 3, the sound produced will no longer be D, but G, the complement to which is G, the octave below. The reason the harmonic D is not produced on touching this node, is, simply because the fraction is not now expressed in its lowest terms—$\frac{3}{9}$ being equal to $\frac{1}{3}$—whence it follows that, as the harmonic portion 0, 3, produces G, the complementary portion 3, 9, being twice as long, must of necessity produce G, the octave below. That the sounds G, G, stand in the relation of basis and harmonic 2 is evident. The ratios $\frac{3}{9}$, $\frac{6}{9}$, are pointed out by a star.

218. As these remarks are equally applicable to the circumstances relating to all the other harmonics, and complementary portions, nothing more need be said concerning them. It is important, however, to notice the lines, ex 81, connecting the notes D, G, D, by which it will be seen, that in examples of this kind, the last half of the series is always the inverted counterpart of the first half.

219. Let us again suppose the sound of the open string to be produced upon the violoncello—then if the string be divided into 15 equal parts as represented by fig. 27, ex. 82, and we touch the node 1, the harmonic produced will be B, the complement to which will be ♯C, $3\frac{1}{1}\frac{0}{2}$ octaves below, because the portion of the string 15, 1, is 14 times as long as the portion 0, 1—and, as 14 is the octave to 7, if the note ♯C be understood to represent the basis, the note B, will represent the harmonic 14 to that basis—which is also the harmonic 15 to the open string C, the semitone below ♯C. In other words, the sounds ♯C, B, stand in the relation of basis and harmonic 14.

220. As the ratios $\frac{1}{15}$, $\frac{2}{15}$, $\frac{4}{15}$, $\frac{7}{15}$, $\frac{8}{15}$, $\frac{11}{15}$, $\frac{13}{15}$, $\frac{14}{15}$, are all expressed in their lowest terms, the harmonic produced by touching any one or more of the nodes answering to the numerators of these fractions, will be same—that is B,—but as the ratios $\frac{3}{15}$, $\frac{6}{15}$, $\frac{9}{15}$, $\frac{12}{15}$, when expressed in their lowest terms, become

$\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$—the harmonic produced must, of course, in these cases answer to the prime 5,—and the harmonic 5, to the open string C, is E—and, as the ratios $\frac{5}{15}$, $\frac{10}{15}$, when expressed in their lowest terms, become $\frac{1}{3}$, $\frac{2}{3}$, the harmonic produced must, of course, in these cases, answer to the prime 3,—and the harmonic 3, to the open string C, is G. The ratios $\frac{3}{15}$, $\frac{6}{15}$, $\frac{9}{15}$, $\frac{12}{15}$, are pointed out by *—and the ratios $\frac{5}{15}$, $\frac{10}{15}$ by °.

221. It is evident that in examples of this kind, the complementary, and harmonic sound, must always stand, the one to the other, in the relation of a basis and primary harmonic, through the whole series—and that finally both these sounds must always unite in the same note.

222. It is to be observed, moreover, that all the primes involved in any product, must come out, in the course of its entire harmonic series—also, that the latter half of the series, must always represent the inverted series of the first half.

I have been rather prolix upon this subject, on account of its utility to violin, and violoncello performers—with whom, harmonics have, of late years, become an object of great attention—and who, nevertheless, know little or nothing of the matter, except what they pick up, empirically. If they understood the subject better, they might, now and then, convince certain persons that, to play the violin, &c., *in tune*—that is, so that no one can detect an error, is one of the most astonishing feats that any one can be called upon to perform—and I make no hesitation in declaring that were the violin a newly-invented instrument, on which no one yet had been heard to play, I most certainly should look forward to the probability of any one's succeeding in so doing, for five consecutive minutes, as very doubtful.

223. In the following table, any note from the lowest series upwards, in connexion with the next series on the right, and the next on the left, represents a triune system, in so far as it relates to the notation, and to the quality of those notes, as bases, mediants, terminants, and ♭7ths. Of course, upon these conditions, ♭C becomes the last in the descending, and ♯C the last in the ascending series of key-notes.

224. The letters which stand one above another in the Transposing Tablet, represent, in every instance, a simple system—and every three adjacent simple systems, constitute a triune system. In the series of mediants the upper row of letters apply to the 1st mode, and the lower row to the 2nd, and 3rd mode. In the series of terminants the upper row of letters apply to the 1st, and 2nd mode, and the lower row to the 3rd mode.

225. The Symbolic Tablet offers a key for the explication of the sounds represented by the Transposing Tablet—and, as these symbols are of universal application, they are

TRANSPOSING TABLET,

Representing the notes appertaining to every triune system from ♭C with 7 flats to ♯C with 7 sharps. [§. 224.]

			*	†	‡										*	†	‡			
Basal ♭7ths.	7	♭♭e	♭♭b	♭f	♭c	♭g	♭d	♭a	♭e	♭b	f	c	g	d	a	e	b	♯f	7	Basal ♭7ths.
Terminants.	3	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	A	E	B	♯F	♯C	♯G	♯D	3	Terminants.
	11	♭♭C	♭♭G	♭♭D	♭♭A	♭♭E	♭♭B	♭F	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	11	
Mediants.	5	♭A	♭E	♭B	F	C	G	D	A	E	B	♯F	♯C	♯G	♯D	♯A	♯E	♯B	5	Mediants.
	19	♭♭A	♭♭E	♭♭B	♭F	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	A	E	B	19	
Primary Bases.	1	♭F	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	A	E	B	♯F	♯C	♯G	1	Primary Bases.

The two systems marked *, †, ‡, are practically identical.

Representing the notes appertaining to every triune system from ♭C with 7 flats to ♯C with 7 sharps. [§. 224.]

				*	†	‡										*	†	‡				
Basal ♭7ths.	7	𝄫e	♭♭b	♭f	♭c	♭g	♭d	♭a	♭e	♭b	f	c	g	d	a	e	b	♯f	7	Basal ♭7ths.
Terminants.	3	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	A	E	B	♯F	♯C	♯G	♯D	3	Terminants.
	11	♭♭C	♭♭G	♭♭D	♭♭A	♭♭E	♭♭B	♭F	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	11	
Mediants.	5	♭A	♭E	♭B	F	C	G	D	A	E	B	♯F	♯C	♯G	♯D	♯A	♯E	♯B	5	Mediants.
	19	♭♭A	♭♭E	♭♭B	♭F	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	A	E	B	19	
Primary Bases.	1	♭F	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	A	E	B	♯F	♯C	♯G	1	Primary Bases.

The two systems marked *, †, ‡, are practically identical.

TRANSPOSING TABLET,

Representing the notes appertaining to every triune system from ♭C with 7 flats to ♯C with 7 sharps. [§. 224.]

			*	†	‡										*	†	‡			
Basal ♭7ths.	7	♭♭e	♭♭b	♭f	♭c	♭g	♭d	♭a	♭e	♭b	f	c	g	d	a	e	b	♯f	7	Basal ♭7ths.
Terminants.	3	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	A	E	B	♯F	♯C	♯G	♯D	3	Terminants.
	11	♭♭C	♭♭G	♭♭D	♭♭A	♭♭E	♭♭B	♭F	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	11	
Mediants.	5	♭A	♭E	♭B	F	C	G	D	A	E	B	♯F	♯C	♯G	♯D	♯A	♯E	♯B	5	Mediants.
	19	♭♭A	♭♭E	♭♭B	♭F	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	A	E	B	19	
Primary Bases.	1	♭F	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	A	E	B	♯F	♯C	♯G	1	Primary Bases.

The two systems marked *, †, ‡, are practically identical.

The directions given in § 228 need not be attended to, as, upon consideration, I thought it best to enclose these, which should be cut out for use.

CUT

TABLET OF THE TRIUNE SYSTEM.

		Sub System.	Central System.	Super System.		
13	Basal ♭7ths { 7	a		a	7 { Basal ♭7ths	
	Termi-nants. { 3 / 11	Cut out the paper enclosed within the four corners a, a, a, a.			3 / 11 { Termi-nants	13
17	Mediants { 5 / 19				5 / 19 { Mediants	11
29—30—31	Primary Bascs { 1			a	1 { Primary Bases	17

CUT

TABLET OF THE TRIUNE SYSTEM.

		Sub System.	Central System.	Super System.		
13	Basal ♭7ths { 7	a		a	7 { Basal ♭7ths	
	Termi-nants. { 3 / 11	Cut out the paper enclosed within the four corners a, a, a, a.			3 / 11 { Termi-nants	13
17	Mediants { 5 / 19				5 / 19 { Mediants	11
29—30—31	Primary Bascs { 1			a	1 { Primary Bases	17

CUT

TRANSPOSING TABLET,

Representing the notes appertaining to every triune system from ♭C with 7 flats to ♯C with 7 sharps. [§. 224.]

			*	†	‡										*	†	‡			
Basal ♭7ths.	7	♭♭e	♭♭b	♭f	♭c	♭g	♭d	♭a	♭e	♭b	f	c	g	d	a	e	b	♯f	7	Basal ♭7ths.
Termi-nants.	3 / 11	♭C / ♭♭C	♭G / ♭♭G	♭D / ♭♭D	♭A / ♭♭A	♭E / ♭♭E	♭B / ♭♭B	F / ♭F	C / ♭C	G / ♭G	D / ♭D	A / ♭A	E / ♭E	B / ♭B	♯F / F	♯C / C	♯G / G	♯D / D	3 / 11	Termi-nants.
Mediants.	5 / 19	♭A / ♭♭A	♭E / ♭♭E	♭B / ♭♭B	F / ♭F	C / ♭C	G / ♭G	D / ♭D	A / ♭A	E / ♭E	B / ♭B	♯F / F	♯C / C	♯G / G	♯D / D	♯A / A	♯E / E	♯B / B	5 / 19	Mediants.
Primary Bascs.	1	♭F	♭C	♭G	♭D	♭A	♭E	♭B	F	C	G	D	A	E	B	♯F	♯C	♯G	1	Primary Bases.

The two systems marked *, †, ‡, are practically identical.

SYMBOLIC TABLET OF THE TRIUNE SYSTEM.

[§ 225.]

		Sub System.	Central System.	Super System.		
		56	56	63		
13	Basal ♭7ths	7.	𝟕	7	Basal ♭7ths	—
—	Termi-nants.	3 11	𝟑 𝟙𝟙	3 11	Termi-nants.	13
	Mediants	5 19	𝟓 57 𝟙𝟡	5 57 19	Mediants	11
17						
		64				
29—30—31	Primary Bases	1	𝟙	1	Primary Bases	17
	Secondary-		1	𝟙	-bases	
	Tertiary-			1	-basis	

equally adapted to every triune system. For its practical appli cation, all that is required, is, to apply the symbols of the latter severally, to the notes in the corresponding position of the former. For example, if we take C for the key it will be seen that the sub-basis 1, is F—that the central-basis 𝟙, is C—and that the super-basis 1, is G. Also, that the three mediants, 5 𝟓, 5, refer, respectively, to the notes A, E, B; and the three mediants, 19, 𝟙𝟡, 19, to the notes ♭A, ♭E, ♭B:—that the three terminants 3, 𝟑, 3, refer, respectively, to the notes C, G, D and the three terminants 11, 𝟙𝟙, 11, to the nodes ♭C, ♭G, ♭D —and lastly, that the three basal ♭7ths, 7, 𝟕, 7, refer, respectively, to the notes ♭e, ♭b, f.

226. It will moreover be seen that the note represented by 7, is, in every instance, practically identical with 1—but the small numerals above these symbolic numbers, show, that they are not really identical, and that the ratio of the vibrations of 7 to

1, is as 63 to 64. The ratio represented by 7, 19, and by 7, 19, is also, in every instance, practically identical—but the small numerals above these symbolic numbers, show, that in neither of these instances are they really identical—and that the ratio of the vibrations 7 to 19, and of 7 to 19, is as 56 to 57. Lastly, as 3, 1, and 3 1, are, respectively, identical—for this reason the symbol △, may be used instead of 3 or 1, and ▽, instead of 3 or 1, in all dubious cases, but more particularly where the sound thus represented answers to both there qualities at the same time.

227. Though the prime 7 is the only permanently chromatic harmonic actually represented in the Transposing Tablet, if we bear in mind the intervals severally resulting from 7, 7, 7, it will be easy to ascertain the note represented by any other chromatic harmonic, in agreement with the key, or triune system, to which it relates.

The reason why the prime 7 is the only permanently chromatic harmonic introduced in the Transposing Tablet, is, because this is the only harmonic wherein the letter, or note, is changed otherwise than by means of an accidental sharp, flat, or natural. The harmonic ratios 29—30—31 on the left of the bases—17 on the left of the mediants, and 13 on the left of the ♭7ths, represent the descending chromatics severally pertaining to each series—and the harmonic ratios 17 on the right of the bases, 11 on the right of the mediants, 13 on the right of the terminants, severally represent the ascending chromatic pertaining to each series.

But as the symbols 11,—13, 13, 13—17, 17, are, owing to the imperfection of the notation, and keyed instruments, all practically referable to two different intervals, to prevent confusion, a line, thus, 11, will, for the future, be placed under them whenever they are used as descending chromatics, and for that reason require to be represented in practice a semitone higher than otherwise. For example, in the key of C, the note represented by 11, is ♯A, but by 11, ♭C,—by 13, ♯C, but by 13, ♭♭E—by 13, ♯G, but by 13, ♭♭B—by 13, ♯D, but by 13, ♭F—by 17, ♯C, but by 17, ♭♭E—by 17 ♯G, but by 17, ♭♭B.

228. As it will tend greatly to improve the student to compare those chords one with another which, are practically, but not really the same—and which, in most instances, belong to two different triune systems—the best way to fix his attention to the two required systems will be to cut a hole in two pieces of paper, of the size marked out by the two braces in the Transposing Tablet, § 224, and then to lay one of these papers over each of the two required triune systems. By this means he will separate the letters appertaining to those two systems from the others with which he is not at that time concerned. He can, moreover, mark the denominations, bases, mediants, terminants, ♭7ths, on each side of the hole, in these papers, and sub-system, central-system, super-system, on the top, exactly the same as in the symbolic tablet.

Let the two chords be 1, 5, 3, 7, and **19, 3, 7, 17**—then, if for the first chord we take C, for the triune system, for the last we must take B. In this case, the notes of the super-chord will be G, B, D, F, and of the sub-chord, G, B, D, ♯E, three of which, as to the notation, are the same—every one the same upon keyed instruments—and not one sound the same in reality, nor in respect to its effects. See ex. 31, § 173, wherein the note C is represented as the basal and ♭D as the harmonic keynote.

Those who are in earnest in the study of the science of music, will go to work with these diagrams—but those who are not, will probably adopt the easier course of trying to cheat themselves and others into the belief that they are of no use.

But be it observed that, feeble as they may appear in the eyes of the ignorant and prejudiced, as mere isolated props of the general edifice, this will not be the case when the scattered materials are all put together.

229. With the view to show the practical utility of these tables, I here introduce the symbolic representation of the real and apparent common chord—the chords of the ♭7th—the added 6th—extreme ♭7th—extreme ♯6th, &c.

The *three*, and only three, *real*, common chords are in the 1st mode, **1, 3, 5**—𝟙, 𝟛, 𝟝—1, 3, 5; in the 2nd and 3rd modes **1, 3, 19**—𝟙, 𝟛, 𝟙𝟡—1, 3, 19.

The *two*, and only two, *apparent* common chords are in the 1st mode, **5**, △, 𝟝—𝟝, ▽, 5; in the 2nd and 3rd, modes **19**, △, 𝟙𝟡—𝟙𝟡, ▽, **19**.

The *three*, and only three, *real* chords of the ♭7th are in the 1st mode, **1, 3, 5, 7**—𝟙, 𝟛, 𝟝, 𝟟—1, 3, 5, 7; in the 2nd and 3rd modes, **1, 3, 19, 7**—𝟙, 𝟛, 𝟝, 𝟟—1, 3, 5, 7.

The chord of the added 6th is in the 1st mode, **1**, **3**, **5**, 3—in the 2nd mode **1**, **3**, **19**, 3—in the 3rd mode **1**, **3**, **19**, 11.

The *three*, and only three, *real* chords of the extreme ♭7th are in the 1st, 2nd, and 3rd mode, **17**, **3**, **5**, **7**—17, 3, 5, 7—17, 3, 5, 7.

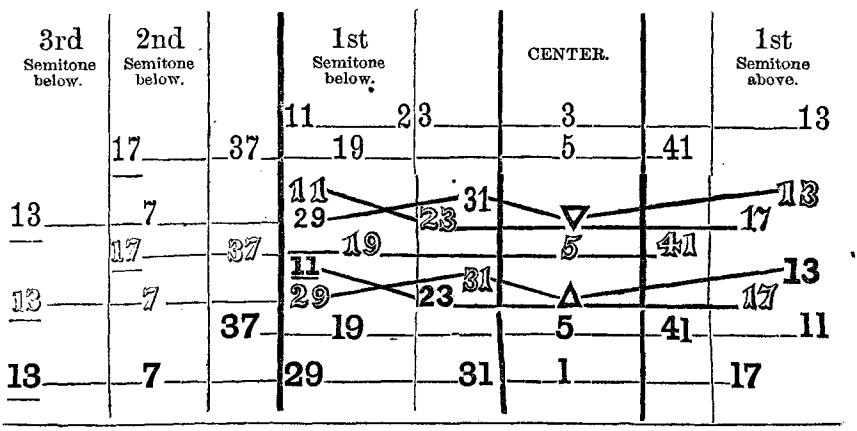
The chord of the extreme ♯6th is in the 1st, 2nd, and 3rd mode, **19**, **3**, **7**, **17**.

To the above may be added the compound chords **1**, **3**, **5**, 1, or **3**, **5**, 1—1, 1, 3, 5, 7—5, 1, 3, 5, 7—19, 1, 3, 5, 7, and many others.

The chord of the extreme ♭7th, is also denominated the equivocal chord, and the chord of the diminished 7th.

It will be seen, that the chord of the extreme ♭7th—**17**, **3**, **5**, **7**—appertaining to the 1st mode, becomes **19**, **3**, **7**, **17**, in the 2nd and 3rd mode, and is then called the chord of the extreme ♯6th.

ALTITUDINAL DISPLAY OF THE THREE CHROMATIC FORMATIONS OF THE SECOND PERIOD OF THE DEVELOPMENT OF THE TRIUNE SYSTEM.



230. Here the position of the primes 1——41, in respect to the triune system, prove that the quality, or peculiar effect, of each is the result of the influence of that position and the mode.

The diatonic harmonics **1**, **5**, △, 5, ▽, 5, 3, being adopted as a center, the chromatic harmonics, on the right, are seen to

ascend to the 1st semitone above—and those on the left, to descend to the 1st, 2nd, and 3rd semitone below. Of those on the right, 41, and on the left, 31 and 23, are in a narrow column by themselves—to show that, being less than a semitone above or below this assumed center, these harmonics cannot be distinguished by the notation, or upon keyed instruments, from the diatonic sounds with which they are linked—and, as 37 is the same in respect to the notation, and upon keyed instruments, as 19, this harmonic is also placed in a narrow column by itself, below 19.

231. In the column for the first semitone above, 11 and 13 are placed at the extremity to the right, a little beyond 17, to show that the sounds represented by these harmonics are higher than 17—and in the columns for the 1st semitone below, 11 is placed at the extremity, to the left of 19, to show that the sound expressed by this harmonic is below 19. Moreover, in the column for the 2nd semitone below, 17 is, for the same reason, placed below 7.

232. The contents, and position of the harmonics, in the 1st semitone below, also show that 29 is confounded in practice with the other, and more important, harmonics which fall upon the same practical interval—and that, by its peculiar position, 23 is, in some instances, confounded with 3, and in others with 11.

233. It is to be noted that 11 answers to the double function of an ascending chromatic to the diatonic harmonic 5, in the 1st mode, and of a descending chromatic, to the diatonic harmonic 3, in the 2nd and 3rd mode, but not so 11, or 11—that both 17, and 17, answer to the double function of an ascending chromatic to the basis 1, 1, or to their octaves 2, 2, and of a descending chromatic to the harmonic 19, and 19, respectively, but not so 17—and lastly that 13, 13, 13, severally answer to the double function of an ascending chromatic to the diatonic harmonic 3, 3, 3, and of a descending chromatic to the basis 1, 1, 1, or their octaves 2, 2, 2.

234. The lines from △, and ▽, direct the eye to their relative ascending and descending chromatics, respectively, as 3 or 1, or as 3 or 1. The mark under 11, 17, 17, 13, 13, 13, shows that the sounds represented thereby, as descending chro-

matics, appertain to the 2nd and 3rd, but not to the 1st mode. § 227.

That the harmonics **11**, 17, 17—**13**, 13, 13, constitute the boundaries at each end of the chain in this wonderful circle, has been glanced at above—and this will still more conspicuously appear by attending to the following explanation of:—

PART 12.

THE MUSICAL RATIOMETER, AND ITS CONNEXION WITH THE TRIUNE SYSTEM.

235. By the term ratiometer is meant a measurer of ratios, or that by which the relative value of ratios, one with another, is measured and compared.

The practical application, or use, of the Musical Ratiometer is, to measure or compare the ratios of the vibrations of musical sounds, one with another, and thereby the intervals by which they are represented in musical notation. To effect this object, the octave is accurately divided into $2^6 \times 3 = 192$ mean proportionals, whereby the space allotted to any given interval is the same throughout the scale—and exactly in proportion to its relative value, or magnitude, in respect to other intervals.

236. In following the course of any one of the series of numerals, from left to right, in a line with the sectional divisions marked "Sub-basis and its harmonics—Central-basis and its harmonics—Super-basis and its harmonics"—they will be found to follow each other in their natural order—each prime being enclosed in a circle.

237. The numbers which appertain to the same simple system, and which stand one above another, apply to the various octaves of the sound represented by the lowest figure in the series—and are the products of that number, multiplied by 2, 2^2, 2^3, 2^4, &c.

238. The bases and primary harmonics appertaining to the super-system, upon being multiplied by 3, are thereby transferred to the central system.

It is important, however, to observe, that the primary har-

monics to the super-basis, are secondary harmonics, or products, and not primes, to the central-basis—and, moreover, that in reference to these harmonics, the central-basis is a secondary, or sub-basis. § 10. What is here said in reference to the basis, and primary harmonics, of the super-system and central-system, is equally applicable to the basis, and primary harmonics, of the central, and sub-system. But since, in transferring the primary harmonics of the super-system, to the sub-system, those harmonics must be multiplied by $3^2 = 9$, they thereby become tertiary harmonics to the sub-basis, and the sub-basis a tertiary-basis to those harmonics.

239. In the upper part of the diagram—on the lower slider* —and immediately above that slider—the bases and primary harmonics 1 to 41, from above, are all brought down, collected together in one line, and thus presented to view, without being encumbered with other harmonics, of comparatively little importance. By this means the relations existing between the elements of the triune system, are seen at once.

But, in order to adapt these ratios to the notation, they must be viewed in connexion, not only with the intervals of the triune-system, as marked below, but also with the specific notes to which they refer, in any given key. To do this, the upper slider must be set to the key under examination, by placing that key-note immediately above the central-basis 1.

240. The actual ratio, in its lowest terms, existing between any two sounds, represented by these primary numbers, will in all cases be found, by tracing them upwards to their source, among the ratios *en masse*, until two consecutive numbers appear in the same plane. For example, it will be seen that, the actual ratio between the harmonics 11, 17, is 33, 34—and that between the harmonics 41, 31, it is 123, 124.

241. To ascertain the ratio between sounds belonging to different triune systems, it is necessary to adjust the position of the under slider consistently with the ratio existing between their respective central bases.

Two triune systems being thus brought into juxtaposition, a *coup d'œil* is presented of all the particulars resulting therefrom—and, where the difference between the sounds, answering to any

* N.B.—Let this slider, for the present, be so set, that the characteristic numerals thereon may all correspond with those above. Then, if the letter C, in the upper slider, be made to stand over both extremities of the scale, the letters and numbers will all answer to the key of C.

given note does nor tally, exactly, with the harmonic ratios, *en masse*, above, reference can at all times be made to the scale of 120 divisions in the octave, and its ratio thus ascertained very nearly.

242. In that part of the Ratiometer which stands above the top slider, the lines which apply to correct intonation—that is to the primary intervals of the triune system, are again introduced. The space assigned to the octave is then equally divided into 12—13, 26, 52—14, 28, 56—15, 30, 60—16, 32, 64—17, 34, 68—18, 36, 72—19, 38, 76—20, 40, 80—21, 42, 84—22, 44, 48—23, 46, 92—24, 48, 96, equal parts.

These divisions prove, to occular demonstration, that, imperfect as is the tempered system of 12 mean semitones in the octave—it is, nevertheless, upon the whole, far better adapted to realize the requirements of the true system, than that in which the octave is equally divided into from 13 to 23 parts—or into any of the sub-divisions of these parts. With respect to those which arise from the subdivision of 12—such, for instance, as 24, 36, 48, 60, 72, 84, 96 equal parts—it will be seen, at once, that, by this means, the effect of certain harmonics would be greatly improved—and also how the matter stands in regard to any particular case. But since the bases, and all the diatonic harmonics, must still remain in *statu quo*, no improvement in regard to these most important sounds, will be effected by any of the means here pointed out.

From what has been said, it follows that no improvement applicable to keyed and fixed-toned instruments, can possibly be made upon the system of 12 mean semitones in the octave—and, deeply as this is to be regretted, it is better distinctly to see that it is so, than to be for ever groping in the dark, in the vain hope of accomplishing that, which, in the nature of things, is impossible.

But though the imperfections inseparably connected with this system cannot be removed from keyed instruments—an unknown state of perfection is yet open to us, in reference to vocal and orchestral effects, whenever it shall become an object of attention, really to study this subject with the view to that end.

243. Should it be desired to know what would be the result, both generally and specifically, of dividing the octave into any given number of equal intervals—this can now be easily ascertained, by merely marking off the proposed number upon a piece of paper—and by placing these divisions in contact with those appertaining to the perfect system. In the two uppermost lines of the diagram, this result is shown in reference to 31, 41, and 53 equal intervals—and also in reference to that vile system of unequal intervals, by which the organ has, for so

many years, been tuned—and by which, generally speaking, it still continues to be tuned.

244. In regard to the "Position of the Central Bases of the Great System," as set forth at the bottom of the diagram—more will be said when this is made an especial object of attention.

245. In that part of the Ratiometer, where the primary harmonics are collected together, **13**, 13, 13—17, 17, **11**, are distinguished by a star marked over them—because in their application to practice, these symbols sometimes represent ascending, and sometimes descending, chromatic harmonics. § 17—20. The cause which gives birth to these changes is as follows:—

246. The ascending chromatics 17, **13**, appertaining to the 1st and 2nd modes, are, by change of position, in respect to the diatonic harmonics, converted into the descending chromatics 17, **13**, in the 3rd mode—because, in this mode, the diatonic series is 1, 11, 19, and not 1, 3, 5, as in the 1st mode—nor 1, 3, 19, as in the 2nd mode. Furthermore, from the same cause—that is, from the changes produced in the position of the diatonic harmonic, by the influence of the modes, 13, of the 1st mode, becomes 13, in the 2nd, and 3rd mode, wherein the diatonic series is 19, **1**, and not 5, **1**, as in the 1st mode—and, lastly, **11**, in the 1st mode, becomes **11**, in the 2nd, and 3rd mode, whenever 19 is treated as a diatonic, and 5 as a chromatic harmonic.

247. It is here important to observe, that though in respect to the intonation, the ascending chromatic **13** is the same as the descending chromatic **13**,—13, the same as 13,—13, the same as 13,—17, the same as 17,—17, the same as 17,—**11**, the same as **11**,—it, nevertheless, so happens that, upon the piano, and other keyed instruments, whenever these sounds are required as descending chromatics, they must, from the necessity of the case, be played a semitone higher than when required as ascending chromatics.

248. Now, the reason for this anomalous proceedure is,

because, when these symbols represent ascending chromatics, the higher of the two semitones is wanted for the resolvent—but when they represent descending chromatics, the lower of the two semitones is wanted for the resolvent, § 18—and it is evident, that the effect of resolving a chromatic note can never be realized by merely striking the same note twice over.

249. Examples 83—88, relate to the harmonics **13**, 13, 13, 17, 17, **11**, which upon keyed instruments are played a semitone lower, respectively, than **13**, 13, 13—17, 17—**11**, and totally differ in their effects—position in respect to the diatonic harmonics appertaining to the mode, being the cause. There are no more than six examples of this kind.

250. Instead of passing at once from **1** to **7**, 1 to 7, 1 to 7, we may, at any time, take the note answering to the ratio 15 as an intermediate sound between the ratio 16, 14 = 8, 7—or, as a substitute for the intermediate ratios 31, 30, 29, between the ratio 32, 28, which is also equal to 8, 7—or, in the case of **1** to **7**, we may introduce the harmonic 17, and proceed thus, **1**, **17**, **7**, in which case the progression involves an enharmonic transition—not in the old, but in the new sense in which that term is used in this work. It may here be observed that a series of harmonics, extending from one octave to another, includes not only all the primes therein actually contained, but the octaves to all the lower primes also.

251. When, in reference to the triune system a chromatic harmonic is placed between two diatonic harmonics, one of which is its relative, and the other its resolvent—if the diatonic relative be below that chromatic harmonic, and the resolvent above it, it thereby becomes an ascending chromatic harmonic—but, if the diatonic relative be above, and its resolvent below, it thereby becomes a descending chromatic harmonic.

252. In ex. 89, bars 1—8, the diatonic and chromatic relatives are united by a slur. In bars 1, 2, 3, 4, the chromatic relative being above its diatonic relative, it thence becomes an ascending chromatic—and in bars 5, 6, 7, 8, the chromatic relative being below its diatonic relative, it thence becomes a descending chromatic.

253. In bar. 4, the diatonic and chromatic relatives **5**, **41**, and in bar 8, 1, 31, are respectively the same upon keyed instruments, and in their notation—they are, nevertheless, very dif-

ferent in respect to their true intonation. In these and other examples, to the symbols above, are added the notes answering thereto, and also to their bases, the key being C. This is done to prevent misapprehension, either in regard to the application of those signals, or the subject under consideration.

254. When, in reference to the triune system a chromatic harmonic is so circumstanced, that neither the diatonic harmonic next below, nor next above, is its relative—it is evident that the progression to any such chromatic harmonic, from its diatonic relative, can only be effected by passing over the diatonic harmonic or harmonics which stand between such relatives. The only instances of this kind are those exhibited in the following table:—

Examples.	Relative Diatonic Harmonics.	Diatonic Harmonics passed over.	Relative Chromatic Harmonics.	Resolvents.
90	1	1	7	5 or 19
91	1	5 . . . 19	7	5 or 19
92	1	5 . . . 19	7	3
93	1	1 . . . 5	13	19
94	1	5 . 19 . 5	13	19
95	1	5 . 19 . 3	13	11
96	1	5	29	19
97	1	5	29	19
98	19	5	17	19
99	19	3	17	11
100	3	5	11	19
101	19	1 or 3	17	11
102	5	19	11	5
103	11	1 or 3	43	11

255. The symbols under the four columns in the above table, severally answer to the four bars in the corresponing example in musical notes. The sounds represented by the symbols and notes which are united by a brace, though in reality quite distinct, are identical upon keyed instruments—but except in

ex. 90, 91, 92, they are not so in respect to the rule herein adopted in regard to the notation.

256. It will be seen upon examination—(1) that, in every example the symbols which represent diatonic and chromatic relatives are characteristically the same—(2) that those in the 2nd column and bar, are characteristically different from those in the 1st and 3rd bar—(3) that the resolvent in the 4th column and bar is always characteristically different from the diatonic and chromatic relative which precedes it.

257. By further attention to this tablet, and the examples, it will appear that ex. 93, 94, appertain to the 2nd, and 3rd, but not to the 1st mode—ex. 95 to the 3rd, but not to the 1st or 2nd mode—ex. 96—98, to the 2nd and 3rd, but not to the 1st mode—ex. 99, to the 3rd, but not to the 1st or 2nd mode —ex. 100 to the 2nd and 3rd, but not to the 1st mode—ex. 102, to the 1st, 2nd, or 3rd mode—101 and 103 to the 3rd mode.

258. In ex. 90—101, and in ex. 103, the sounds represented by the symbols, and musical notes, in the 3rd column and bar, are all descending chromatic harmonics—ex. 102, being the only ascending chromatic harmonic in the series.

259. The changes produced by the influence of the modes do not affect **41**, 41, 41,—**37**, 37, 37,—**31**, 31, 31,—and 29, so as to interfere with their relative position in respect to the diatonic harmonics—but 29, **29**, does fall under the influence of the modes. For, when the resolvent to 29 is 19, it becomes, as it were, disconnected with 7,—and when the resolvent to **29** is 19, it becomes, in like manner, disconnected with **7**, as the preceding examples will show. And since **19**, 19, 19, are resolvents in the 2nd and 3rd modes only—or, in other words, only when diatonic harmonics, the effect of the influence of the modes is obvious.

260. As we are now tracing effects to their causes, should it be asked why 7 is, in every instance, felt as a descending chromatic to 1 or 8, and not as an ascending chromatic to 3 or 6—the answer is, that upon close attention to all the circumstances of the case, it will be found, that in this, as in everything else, our perceptions are in accordance with truth. But let us, as a matter of curiosity, fairly examine both sides of the question, in regard to this rather subtle point.

REAL OR NATURAL PROGRESSION.

	Relatives.			
	Diatonic Harmonics.	Intermediate Sounds.	Descending Chromatic Harmonics.	Resolvents.
Ex. 104	1 or 8	1	7	5 or 12
Ex. 105	1 or 8	5 or 12	7	5 or 12
Ex. 106	1 or 8	5 or 12	7	3 or 11

SUPPOSITIOUS PROGRESSION.

	Relatives.			
	Diatonic Harmonics.	Intermediate Sounds.	Ascending Chromatic Harmonics.	Resolvents.
Ex. 107	3 or 6	5 or 12	7	1
Ex. 108	3 or 6	5 or 12	7	5 or 12
Ex. 109	3 or 6	3 or 11	7	5 or 12

261. In the above table and corresponding music examples, the union between diatonic and chromatic relatives is pointed out by the brace which connects them, both in the real and suppositious progressions. The symbols placed between these diatonic and chromatic relatives, represent those intermediate sounds, one of which is, in every instance, diatonic. In ex. 104, this intermediate diatonic sound is 1 in all the modes. In ex. 105, 5 is the intermediate diatonic sound in the 1st, and 12 in the 2nd and 3rd mode. In ex. 106, 5 is the intermediate diatonic sound in the 1st, and 12 in the 2nd and 3rd mode. With respect to the resolvents, as, with the exception of ex. 107, these also vary with the modes, when one is diatonic, the other is chromatic.

262. In ex. 104—106, the harmonics 7, 7, 7, are treated as descending chromatic harmonics, to their respective relative diatonic harmonics 1, 1, 1, in conformity with universal practical usage—but, in ex. 107—109, they are suppositiously treated as ascending chromatic harmonics, to the diatonic harmonics 3 or 6—3 or 6—3 or 6.

263. Now, if we compare ex. 104 with ex. 107—ex. 105 with ex. 108—and ex. 106 with ex. 109—it will be found that, in ex. 104—106, the chromatic harmonics 7, 7, 7 represent sounds practically distinct from their resolvents—that is, they are so in respect to their notation and upon keyed instruments, whatever may be the mode. But in ex. 107, 7, is the same in practice, as the resolvent 1—in ex. 108, 7 is the same in practice, as the resolvent 12—and in ex. 109, 7 is the same in practice, as the resolvent 12,—wherefore, as these sounds, though distinct in themselves, are, nevertheless, practically identical, it is evident that, one sound only, cannot convey to the mind the idea of the two, the one chromatic, and the other diatonic.

264. Again, as the ratio 8, 7, is less, and consequently the affinity greater, than 6, 7, this is of itself an *a priori* reason why the notes answering to the ratio 8, 7, are relatives, in preference to those which answer to the ratio 6, 7.

1	2	3	4
23 24	31 32	37 38	40 41
23 3	31 . . . 1	37 . . . 19	5 . . . 41
23 3	31 . . . 1	37 . . . 19	5 . . . 41
23 3	**31 . . . 1**	**37 . . . 19**	**5 . . . 41**
5	6	7	8
63 64	56 57	51 52	92 93
7 1	7 . . . 19	17 . . . 13	23 . . . 31
	7 . . . 19	17 . . . **13**	**23 . . . 31**
9	10	11	12
87 88	1 1	1 1	1 1
29 . . . 11	**11 . . . 11**	13 . . . 13	17 . . . 17
29 . . . 11		13 . . . 13	17 . . . 17
		13 . . . 13	

265. In the compartments above, numbered 1—9, are represented those harmonic symbols for which, upon keyed instruments, we have only one sound for two required sounds—the ratios of the vibrations of which are marked above. In the compartments 10—12 are represented those symbols which, though the same in respect to their true intonation, as represented by 1....1 above, are, nevertheless, of necessity, represented by two distinct sounds on keyed instruments, as well as by a different notation. §. 18.

266. In the compartments numbered 1—4, as the symbols are, in every particular instance, both referable to the sub-system—to the central-system—or to the super-system—the difference of their vibrations, as expressed by the small figures at the head of each example, are found by multiplying the symbol **3**, 3, 3, by $2^3 = 24$, the symbol **1**, 1, 1, by $2^5 = 32$,—the symbol **19**, 19, 19, by $2 = 38$,—and the symbol **5**, 5, 5, by $2^3 = 40$. And, since such multiplications affect only the particular octave represented by those symbols, their essential qualities are not thereby changed. In the 5th compartment, the ratio 7, **1**, is as 7×3^2, to $2^6 = 63$, 64. In the 6th compartment, the ratio 7, 19, and **7**, 19, is as 7×2^3, to $19 \times 3 = 56$, 57. In the

7th compartment, the ratio **17**, **13**, and **17**, **13**, is as 17×3, to $13 \times 2^2 = 51, 52$. In the 8th compartment, the ratio **23**, **31**, and **23**, **31**, is as 23×2^2, to $31 \times 3 = 92, 93$. And in the 9th compartment, the ratio **29**, **11**, and **29**, **11**, is as 29×3, to $11 \times 2^3 = 87, 88$.

267. Thus far the examples, in every instance, represent two distinct sounds, for which, upon keyed instruments, we have only one mean tone, as a substitute—while, on the contrary, in the compartments 10—12, we are under the necessity of substituting two sounds, in practice, one of which is a semitone above the other, where only one sound is required. §18.

Such are the evils resulting from the adoption of the artificial system of 12 mean semitones in the octave—but, however, in our aspirations after perfection, or a near approximation to perfection, we may deplore the consequences resulting from this circumstance, we must not overlook the very important benefits which it has been the means of conferring. Nor, would it be wise, to discard it from our service, were that even possible, except by the substitution of another which is better and equally practical.

268 In regard to the harmonic 23, it is a descending chromatic, when 3 is diatonic, as in the 1st and 2nd mode—and an ascending chromatic, when 11 is diatonic, as in the 3rd mode. As a descending chromatic, 23 is not distingished from 3—and as an ascending chromatic, it is not distinguished from 11—either practically, upon keyed instruments, or by the notation. See the Ratiometer.

PART 13.

ANALYSIS OF MUSICAL PHRASES AND PASSAGES EXTRACTED FROM THE WORKS OF THE MOST CELEBRATED COMPOSERS, IN VERIFICATION OF THE TRIUNE SYSTEM, AS BEING THE EXEMPLAR OF WHAT IS TRUE AND NATURAL IN MUSIC.

269. In ex. 110, by Beethoven, bar 4, and even 12, may both be referred to the key of C—but; in both instances, and particularly in the latter, the harmony will be smoother, and the effect better, if the key be changed, as marked in the example. It is constantly to be borne in mind, that because a passage is good, according to one interpretation, and the effects thence

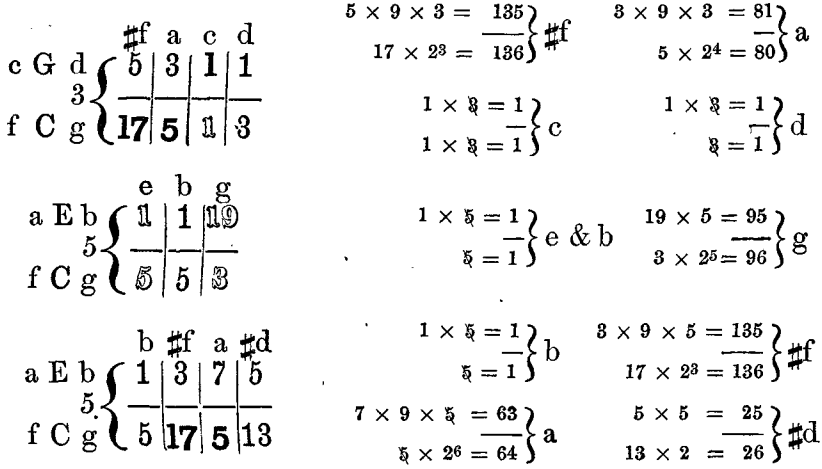
resulting—it does not follow that another interpretation, and its effects, must necessarily be bad. As in language, so in music, very many forms of construction, and modes of expression, are positively bad—there are also many varieties, that are unobjectionable—and among these, some requiring great tact, experience, and taste, to determine which is best.

270. In bar 10, ex. 110, we have the combination **F 1, C 3, A 5**, G 1—and in ex. 111, bar 3, by Mozart, we have **C 3, A 5**, G 1—and as this chord differs from the added 6th, only by the substitution of **G 1**, instead of D 3, it may be called the chord of the 9th. The use of the one, or the other, of these two chords depends upon the circumstances of the case, in regard to the unity of the passage, considered as a whole. This combination has been a puzzle to critics—but the names of the composers, and the beauty of the passages, have saved it from the abuse that might otherwise have been bestowed upon it—the combination F, G, A, C, being neither adapted to fall in with vulgar ideas concerning pretty concords, nor orthodox discords. It is also to be noticed in regard to the chord F, G, A, B, D, ex. 110, bar 14, that if D be 3 and A 9 to the basis G—A is not a major 3rd, in the orthodox ratio 5, 4, either to **F 1**, or F 7—also, that if the ratio of the major 3rd F, A, be 4 to 5—**A 5** is not the perfect 5th to **D 3**, in the orthodox ratio 3, 2. How to settle these conflicting claims, in agreement with the dogmas commonly entertained by theorists, I leave them to determine.

271. That the beautiful and sublime air by Beethoven, and also the air by Mozart, and innumerable others, should, by mere chance happen to involve the primes 2, 3, in their sectional divisions, and subdivisions, and that music should, nevertheless, have no connection with science, and vulgar arithmetic, I leave for the solution of those who have pre-determined that this alliance has no existence.

272. In studying these examples, in order clearly to perceive the nature and value of the intervals involved by the harmonics marked in connection with the notes, it is earnestly recommended to compare them, continually, with the Ratiometer—their relative position will, by this means, soon become familiar—and, ultimately, leave a correct and fixed impression on the mind. Singers, violin, and violoncello performers, have, moreover, a special interest in so doing.

273. The difference between the sounds ♯F, A, C, D, in bar 4, ex. 110, in the key of C, and G—and between the sounds E, B, D—and B, ♯F, A, ♯D, in bar 12, in the key of C, and E, is,



274. In ex. 112—114, will be found a consecutive fifth, which is pointed out by parallel lines thus ═══. Upon this subject more will be said hereafter—in the mean time, the reader is requested to notice—(1) the chord A, C, E, in bar 1, ex. 112, and carefully to attend to the marked difference between the effect of this apparent chord of A minor, and the real chord, in the key of A—(2) the chord C, B, D, F, in bar 3, and to mark the simplicity, and obvious truth, of the method of explaining all combinations, without exception, by reference to the elements of the triune system, to which they belong—(3) the reiterated ♭B 7 in the treble, with ♮B 5, at the commencement of ex. 113. And, should any one doubt the propriety of referring the note ♭B to the prime 7, let him, experimentally, try the effect.

275. Let the ratios 7, 11, 41, ex. 115, bar 2, be examined, in connection with the Ratiometer—and, if not satisfied with this, let the example be put to the test of experiment—in so doing the analysis here given will be found correct, and the above ratio in the highest degree effective.

There are some, who, being gifted by nature with the requisite qualification, will settle the matter in regard to these, and other similar points, at once—that is, by a kind of intuition, or internal perception. Singers, violin, and violoncello performers,

are, therefore, earnestly recommended to try these passages with great attention to their effects, and by reference to the Ratiometer—and if they cannot get to hear the harmony resulting from the union of all the parts, to bear it in mind that the perfection of the harmony is necessarily the result of the perfection of all its separate parts.

To such persons the question in regard to ex. 115 should, therefore, be—does my ear approve of the high chromatic ratio 41 for the note B—of the low ratio 7 for the note F—and the still lower ratio 11 for the note ♭D?

276. In ex. 115, bar 2, by Mozart, we have the combination **11**, **7**, **41**—in ex. 116, bar 2, we have **1**, **19**, 3, which in bar 4 is changed to **1**, **19**, **11**—in ex. 117, we have **1**, **19**, **11** in the treble, with **11** in an ascending progression in the base—and in ex. 118, bar 2, we have a leap from **11** to **19**, which is not a 5th in the ratio 2 to 3.

277. In ex. 119, bar 5 and 6, by Handel, we have a superb specimen of the chord **19**, **11**, **1**, which is not the major common chord of ♭D—in ex. 120, bar 2, we have **11** effectively introduced in a running passage for the voice—in ex. 121, bar 2, by S. Bach, we have **11** and **11** introduced and followed in a style worthy its author—and the same may be said of the combination **19**, **11**, in ex. 122, bar 2, by Beethoven. In ex. 123, by Mozart, we have, repeatedly, **11**, **11**, and **11**, which are evidently introduced as real diatonic sounds, resolutely holding to the fixed intonation which, in this case, appertains to them, and not as chromatic substitutes for 3, 3, **3**, nor as notes of mere grace and ornament in addition to these harmonics.

278. The passage in the 2nd and 3rd bar of ex. 124, is written by Rossini, as if the key were transposed to the semitone above, as in ex 125. That the effect is thereby rendered common, place, and in great measure obscured, no one can deny, whose musical perception is not of the lowest order.

It is due, however, to Rossini, to state, that his compositions afford abundant evidence to prove that these errors proceed from a false theory, and not from want of feeling.

When the head and heart are set at variance, which is always the case when the taste is pure and the theory false, can we wonder at the occasional triumph of what is false over what is true?

279. In ex. 126, by Steibelt, the note ♯C for the oboe is written ♭D. This is done that it may not appear to be at variance with the ♭D in the part for the piano. But, I contend for the independent perfection of the melody of the one, and

harmony of the other—and maintain that neither the truth of the intonation nor the general effect is dependent upon the violation of this principle. Besides, what reason can be given why **13** may not be heard in conjunction with **11**? Or, in other words, why sounds in the ratio 104, 99 may not be heard in connexion?

For herein lies the question, and not that which relates to their names, and notation, as ♯C and ♭D, and the host of preconceived false notions which these conventional signs are sure to call forth. For from the very moment this takes place, the understanding becomes, as it were, deprived of her reasoning power, and cast down into that sphere in which all is disorder and obscurity.

Were any one to interrupt a series of lines, elegantly flowing in various curvilinear courses, with the view to depict some beautiful object, for the purpose of introducing at some particular place the exact form of the square, he would, in all probability, be looked upon with a mixture of pity and contempt—although by so doing he would only be amusing himself after the fashion of many theorists in respect to what they call concords. For here the ratio 99, 104, although impracticable on keyed instruments, is nevertheless rather more than the semitone E 𝟝, F 7, the ratio of which is 100, 105. See the Ratiometer.

280. In ex. 127 by Corelli, if, at the 9th bar, the key be understood as being G, we have the combination **19, 11, 1**—for though the three last crotchets in this bar may be referred to the key of F, by so doing we disturb the unity of the entire piece—and the same may be said of bar 5, wherein the first three crotchets may be referred to the key of D. But it is to be observed that it is not the melodies of the parts, individually considered, that suggest a change in the key at such places, but the harmony *in appearance only*, which, to say the least, is fallacious. The most remarkable combination is in bar 11, wherein we have the super **11** in conjunction with the sub **11**. In this instance **11**, instead of passing to its resolvent **1**, in obedience to the laws relating to it as a chromatic harmonic, is treated as a diatonic harmonic, and with great effect made to proceed to its basis **1**. The note ♯E **11** is of course written ♮F in the original.

Now, though this example introduces the elements, and thereby the peculiar effects of the 3rd mode, for a very short time only, and as a sort of climax to the movement—it is nevertheless sufficient to prove that the emotions it is calculated to inspire, are innate, wherever we find genius, taste, and depth of feeling.

281. The great object of all these examples is to prove that the 3rd mode is not a chimera, since the evidence of its real

existence can be thus strongly supported. As the prime 11 is the harmonic specially concerned in the establishment of this mode, great attention must, of course, be paid to it, whenever it occurs, and to the very low pitch of the intonation expressed by it.

In respect to the difference between the 2nd and 3rd mode, that by which the smallest change is effected is **11**, next in order to this is 𝟙𝟙, and lastly $\underline{\mathbf{11}}$—and hence it happens that, in practice, **11** is of more frequent occurrence than 𝟙𝟙 or $\underline{\mathbf{11}}$. In other words, it will be found that, in practice, we recede from the open, child-like simplicity of the 1st mode, to the mysterious depths of the fully-established 3rd mode, by degrees, and not all at once. That is, practically, the 1st mode may be either pure, or dip occasionally into one or more of the elements of the 2nd mode—and, in like manner, the 2nd mode may be either pure, or dip occasionally into one or more of the elements of the 3rd mode—which, in its plenary state, constitutes the ultimate boundary, or farthest remove from the elements of the 1st mode.

PART 14.

OF THE THIRD PERIOD OF THE DEVELOPMENT OF THE TRIUNE SYSTEM, WHENCE PROCEED THE HARMONICS DENOMINATED SERIAL CHROMATICS AND THE COMBINATIONS THENCE DERIVED.

282. Hitherto chromatic harmonics have been severally referred to, as representing one fixed sound only—but in a more extended and perfect view of the subject, each chromatic harmonic should be considered as the representative of a series—the extreme terms of which are its diatonic relative, and resolvent. The reason for adopting one primary harmonic as the representative of a series, is, because that sound is in every instance the best individual substitute for that series.

283. By serial chromatics is here meant a series of continually accelerating or retarding vibrations, commencing from the relative diatonic harmonic, and terminating with the resolvent—with, or without, the intervention of octaves.

284. The following will serve to convey an idea of this subject. Fig. 28 represents the progression F, ♯F, G, ex. 128, in which the ratio of the vibrations answering to the two extreme terms represented by the diatonic relative F **1**, and the resolvent G 1, is 16, 18 = 8, 9.

Fig. 29 represents the progression G, F, E, ex. 129, in which the ratio of the vibrations answering to the two extreme terms represented by the diatonic relative G 1, and the resolvent E 5, is 6, 5.

285. In the above two figures, the top and bottom line represent the level, as to pitch, of the diatonic sounds denominated relative and resolvent—and the central horizontal line, the fixed chromatic sound. The ascending straight line in fig. 28, and the descending straight line in fig. 29, represent the gradually and *uniformly* varying pitch of the serial chromatic vibrations—and the curved lines, above and below each of these straight lines, represent this variation of the pitch as taking place, *not* uniformly, but either as being more precipitate at the beginning or at the end of the series.

286. In examples of this kind, it is to be understood that the number of serial and fixed vibrations, answering to their respective cycles, are, in all cases, the same—that is, if **17** represents the cycle in respect to the fixed chromatic vibrations, pointed out by the horizontal line, in the parallel ♯F, fig. 28, **17** must also represent the cycle, in respect to the serial chromatic vibrations, pointed out by the straight line, or either of the curved lines, rising from the diatonic relative, and proceeding thence to the resolvent.

287. The straight or curved lines, representing the fixed or serial chromatic vibrations, are also to be understood, either as referring to one entire cycle only, or to more than one—also to any fractional part of a cycle, or to any number of whole cycles with any fractional part of another cycle.

288. I now proceed to take a somewhat microscopic view of the details of a few particulars relating to the vibrations of serial chromatics. In so doing, my object is to enable the reader, after carefully studying the great system (Part 15), distinctly to perceive, that herein, as in everything else, nature is perfect in all her ordinances—that discord, the offspring of confusion, does not originate in her, but in ourselves—not in her simple or combined systems of sounds, but in the grotesque and artificial systems which have, from time immemorial, been set up in opposition to, and, as it were, in defiance of her.

The peculiar circumstances relating to serial chromatics may be classified as follows:—

289. *One graduated series of vibrations answering to a single cycle of the corresponding fixed chromatic*, ex. 130. Here 17 vibra-

tions are supposed to take place in the time of the semiquaver ♯F. During this period, the sound ♯F, as a fixed chromatic, is constantly the same—but, as a serial chromatic, this time will be occupied in the transit from its diatonic relative F to its resolvent G.

290. That the reader may not entertain an erroneous idea in regard to the sufficiency of the time here allotted for 17 vibrations, it may be proper to inform him that this note, ♯F, is the result of 724 vibrations in a second—whence it follows that the time required for 17 vibrations is $\frac{17}{724}$ of a second—that is, rather less than the 43rd part of a second.

291. Since, in respect to their extreme terms, the vibrations of the fixed or serial chromatic harmonic ♯F, to which the example refers, is the same—that is, since those vibrations, and those only, which are coincident with the vibrations of the basis are the same—it follows that, in examples of this kind, no discordant sounds can, at any time, result from the substitution of a serial, instead of a fixed chromatic.

292. A serial chromatic, when judiciously performed, is very beautiful, and ought to be looked upon as being in tune during the whole course of its transit—and also as nature's last and most exquisite finish, for which, the fixed chromatic is, in many instances, but a rude substitute. That the circumstances relating to fixed and serial chromatics are in all respects similar to those relating to diatonic harmonics and fixed chromatics, will be immediately perceived by every one who has paid the slightest attention to this work.

293. *One graduated series of vibrations, answering to two or more cycles of the corresponding fixed chromatic*, ex. 131. During the period of the dotted minim ♯F, the vibrations of the cycle 17 are supposed to be frequently repeated. The pitch of the note ♯F, as a fixed chromatic, is, of course, the same from first to last—but, as a serial chromatic, the whole time is to be understood as occupied in the transit from its relative diatonic harmonic F 1 to its resolvent G 1.

294. As the cycle of the fixed chromatic is here understood to be several times repeated, and as the extreme terms of the vibrations appertaining to every harmonic cycle are coincident with the vibrations of the basis, the conjunction of these extreme terms, with those of one or more relative harmonics, will, of course, generate that basis, at the depth corresponding to the ratio, § 49—57.

But, with respect to a prolonged serial chromatic, the case is in so far different, that, in proportion as the cycle of the two extreme terms is extended, the basis becomes more and more profound—or, in other words, deeper and deeper, but not discordant—that is, without giving birth to sounds at variance with the rest of the harmony.

295. From the above it will again be seen that the circumstances relating to fixed and serial chromatics correspond with those relating to diatonic harmonics and fixed chromatics. For the generated bases of fixed chromatics are, in every instance, deeper in proportion to the greater length of their cycles in respect to the cycles of their relative diatonic harmonics—and, in like manner, the generated bases of serial chromatics are deeper in proportion to the still greater length of the cycles appertaining to their vibrations. It is, moreover, evident that, in numerous instances, these generated bases will be too low for the ear to appreciate, and consequently merely give birth to beats.

296. As a matter somewhat connected with this subject, it will not be out of place here to remark that, in all probability, the passages commonly marked *accellerando*, and *retardando*, are not in their own nature irregular and disorderly—but that, on the contrary, they also represent *graduated* portions of time, upon a larger scale, similar to those relating to the vibrations of serial chromatics—and, consequently, refer to the unity and perfection of some aggregate quantity, or cycle of particulars. For, in this, as in matters relating to certain combinations, hitherto stigmatised as discords, it is to be observed that what is discordant, and consequently at variance with truth and order, when felt and seen either entirely without any regard to unity, or merely by reference to a unity of small and insignificant dimensions, assumes quite another character when felt and seen as a member of a unit of greater dimensions.

297. *Two graduated series of vibrations answering to the single cycles of the two corresponding fixed chromatics*, ex. 132. Here the notes ♯A, ♯F, in the ratio 11, 17, are to be understood as representing one cycle only—in which case the generated basis will be the same, whether the sounds produced be both fixed, or both serial chromatics—or either of them a fixed, and the other a serial chromatic.

But, if in the transit of serial chromatics, the time be sufficient to admit of several repetitions of the single cycle, as in

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ex. 133, the prolonged cycle, thus produced, will give birth to a graduated series .of basal vibrations at a given number of octaves below those answering to the harmonic 17. For as the cycles of the notes ♯F, ♯A, though continually shorter, still remain coincident, it is evident the generated basis must needs rise at the same time, and in the same proportion—and *vice versa* in a descending progression.

That what is here said in regard to the harmonic 17, in connexion with the harmonic 11, is equally applicable to other combinations of serial chromatics, must be evident; for if the cycles be gradually, and in the same proportion, shortened, the bases must also gradually, and in the same proportion, ascend, and *vice versa.*

298. *Two graduated series of vibrations answering to two or more cycles of the two corresponding fixed chromatics*, ex. 134. Here the ascending chromatic ♯F **17**, is united with the descending chromatic ♭A **19**. In a prolonged serial progression, starting from these two points, or from those of their relative diatonic harmonics $\frac{\text{F}\,\mathbf{1}}{\text{A}\,\mathbf{5}}$, and proceeding thence to their resolvents $\frac{\text{G}1}{\text{G}1'}$, the basis will become more and more profound, in proportion to the increased length of the cycles—but never discordant—that is, never at variance with the harmony of the other parts.

299. In order to place these matters in a clear point of view, let it be observed—(1) that as the vibrations answering to the intermediate terms of any graduated scale of serial chromatics must of necessity differ from that of every other series—it is evident no extraneous basal sounds can possibly arise from combinations of this kind—(2) that as the vibrations answering to the intermediate terms of a graduated scale of serial chromatics can in no instance coincide with the isochronous vibrations of any diatonic or fixed chromatic sound appertaining to the same chord, it follows from hence that neither can these combinations give birth to dissonances at variance with the rest of the harmony—(3) that though what may be denominated dissonant knocks, the result of imperfect intonation, or the want of correspondency in respect to the course of the transition, may now and then occur, nevertheless as isochronous vibrations, the admitted sole cause of the generation of basal sounds are herein quite out of the question, their existence as bases is therefore impossible.

PART 15.

Of the great system or system of systems.

300. In the Tablet over leaf, the central triune system appertaining to the great system of C, is given in full—but only the key-note, or central basis, is given to the several sub, and super, triune systems appertaining to the ratios 3, 5, 7, 9, 11, 13, 15, 17, 19.

That two of the three simple systems of the central triune system also appertain to the sub triune system 3, and super triune system 3, is evident, although the character of the symbols will, of course, vary accordingly.

What the particulars relating to each of the triune systems, which constitute the members of the great system are, can, of course, be obtained by transposition from the central system, § 223—and the great system itself can, of course, be transposed from C to any other key.

301. It has been shown, § 91, that though practically the same:—

The ratio of sub 9 differs from super 7, as	9× 7	to 2^6=	63 to	64
,, 7 ,, 9, as	7× 9			
,, 11 ,, 11, as	11×11	to 2^7=	121 to	128
,, 13 ,, 5, as	13× 5	to 2^6=	65 to	64
,, 5 ,, 13, as	5×13			
,, 15 ,, 17, as	15×17	to 2^8=	255 to	256
,, 17 ,, 15, as	17×15			

302. In practice, the sounds appertaining to any triune system will admit of being combined, in any way, not inconsistent with the laws of harmony, to be hereafter explained—but not so, the sounds derived from two or more triune systems. Hence it follows that though the study of the great system is important, in bringing to light the simplicity and sublimity of nature's laws, and the vast system of sounds thence derived—and also as a check to imbecile wanderings, from key to key, in one direction only—that is, from the central to a series of super-systems or sub-systems only, without a due mixture of one with the other—the study of the triune system is, nevertheless, that which demands the greatest attention.

By close attention to their works, it will be found that the

Tablet in which is portrayed the 19 *triune systems of which the great system is composed—the central basis of the central triune system being* C.

CENTRAL SYSTEM.

Basal ♭7ths.	7									56 ♭E	56 ♭B	63 F							7	Basal ♭7ths.
Terminants.	3									C	G	D							3	Terminants.
	11									♭C	♭G	♭D							11	
Mediants.	5									A	E	B							5	Mediants.
	19									♭A	57 ♭E	57 ♭B							19	
Primary Bases.	1	A 19	♭C 17	♭D 15	♭F 13	♯F 11	♭B 9	D 7	♭A 5	64 F 3	C 1 Central-basis.	G 3	E 5	♭B 7	D 9	♭G 11	♯G 13	B 15	♯C 17	♭E 19	1	Primary Bases

SUB-SYSTEMS. SUPER-SYSTEMS.

great masters have been impelled, as it were, by a centrifugal as well as by a centripetal force—or, in other words, by the mysterious influence of this great system, operating from its center, continually, upon the memory, and thereby maintaining the unity of the piece, or of several pieces.

303. As the central basis of the great system constitutes an intermediate link between the series of the sub-bases, and the super-bases of that system—and as no good reason can be assigned for preventing us from roaming with perfect freedom from one triune system to another, within the domain of the great system — it follows that in passing from a sub-system, to a super-system, or *vice versa*, all the immediate products of the ascending and descending series of primes from 1 to 19 are thereby involved. For example, since in the triune system of C we are allowed to pass from the sub-basis F, to its super-basis G, and *vice versa*—and in so doing to involve the ratio $3^2 = 9$ instead of 7—so in the great system we are allowed to pass from any one of its sub-systems, to any one of its super-systems, and *vice versa*—and in so doing to involve the product of the two primes answering to the sub-basis, and super-basis concerned in the particular case under consideration. We are allowed, for instance, in this case also, to pass from F to G, and *vice versa*, in the ratio $3^2 = 9$, instead of 7—from A to E, and *vice versa* in the ratio $19 \times 5 = 95$, instead of $3 \times 2^5 = 96$—from A to G, in the ratio $19 \times 3 = 57$, and also from ♭E to F, in the ratio $3 \times 19 = 57$, instead of $7 \times 2^3 = 56$, &c.

304. But though the tonic transitions which involve the first simple products of the primes 1 to 19 can have place where the great system is concerned, this cannot otherwise be permitted, except in so far as it relates to 9 and 15, these being the only products wanted to complete the series 1 to 19. Moreover, in practice, or at all events upon keyed instruments, the direct tonic transition to 5^2 is not only confounded with the direct tonic transition to 13, but also with the retrograde tonic transition from 5 :—$5 \times 7 = 35$, with $3^2 \times 2^2 = 36$:—$7^2 = 49$, with $3 \times 2^4 = 48$, &c., and we know to a certainty that the ear is not formed to appreciate ratios so very remote, in connexion with others still more so, and wholly unassisted by those that are simple—for, it must be borne in mind that, in every tonic transition, the ratio of the bases is more simple than the ratios resulting from any other part of the harmony—whence it follows that if this ratio be high, those which refer to their harmonics must necessarily be still higher, and, consequently, too remote for the ear to follow.

TABLE

REPRESENTING THE TRUE AND SUBSTITUTED RATIOS OF THE GREAT SYSTEM OR SYSTEM OF SYSTEMS.

Central-bases to the Sub-systems. Central-bases to the Super-systems.

C	*	D	*	E	F	*	G	*	A	*	B	C	*	D	*	E	F	*	G	*	A	*	B	C
1	*	2	*	3	4	*	5	*	6	*	7	1	*	2	*	3	4	*	5	*	6	*	7	1
12	11	10	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12
19^4			19^3			19^2			19						19			19^2			19^3			19^4
17^{12}	17^{11}	17^{10}	17^9	17^8	17^7	17^6	17^5	17^4	17^3	17^2	17		17	17^2	17^3	17^4	17^5	17^6	17	17^8	17^9	17^{10}	17^{11}	17^{12}
15^{12}	15	15^2	15^3	15^4	15^5	15^6	15^7	15^8	15^9	15^{10}	15^{11}		15^{11}	15^{10}	15^9	15^8	15^7	15^6	15^5	15^4	15^3	15^2	15	15^{12}
13^3				13				13^2								13^2				13				13^3
11^2						11												11						11^2
9^6		9^5		9^4		9^3		9^2		9				9		9^2		9^3		9^4		9^5		9^6
7^6		7		7^2		7^3		7^4		7^5				7^5		7^4		7^3		7^2		7		7^6
5^3				5^2				5								5				5^2				5^3
3^{12}	3^5	3^{10}	3^3	3^8	3	3^6	3^{11}	3^4	3^9	3^2	3^7		3^7	3^2	3^9	3^4	3^{11}	3^6	3	3^8	3^3	3^{10}	3^5	3^{12}
2																								2

Central basis to the Central system 1

305. In the preceding table the letters in connection with the stars, on the top, represent the semitonic scale, C being taken as the central key-note. The numerals below represent the intervals of the scale. The stars stand for sharps to the letters and numerals on the left, and for flats to those on the right. The letters on the right of the central-basis to the central-system represent the ascending or super-series of key-notes, and those on the left the descending or sub-series of key-notes. The small numerals on the right refer to the semitones above, and those on the left to the semitones below the central basis. Of course the intervals represented by these, as well as by the larger numerals, remain the same, although the letters above be transposed.

306. In respect to the denomination, or notation, of any particular key appertaining to the great system, the rule should be, to prefer a few flats to a great many sharps, and *vice versa*. For example, if the progression lead to the major key of ♯A, with 10 sharps, write in the key of ♭B, with 2 flats instead—and if to the minor key of ♭A with 7 flats, write in the key of ♯G with 5 sharps, &c.

307. As the series of letters and numerals, above explained, represent music under the trammels of art—that is, in connexion with things conventional and arbitrary, they are separated, by a double line, from matters below, wherein is portrayed the true and substituted ratios to which they refer. The true ratio being represented by the large numerals 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, severally appertaining to each compartment in both the ascending and descending series—and the substituted, or misapplied ratios, by the several powers of those ratios.

308. The large 1 in the center of the system, under the denomination central-basis to the central-system, represents the key-note to a triune system, thus disposed, in respect to the key-notes of the other triune systems, which constitute the great system. The pitch of this central 1 may be anything we please—but this being determined, and thus represented, the pitch, or, in other words, the ratio of the vibrations of all the other key-notes, and consequently of every item belonging to all the triune systems appertaining to the great system, falls under the dominion of this great central unit.

309. The large numerals on the right and left, severally represent the central basis of a triune system, and the relation in which these bases and triune systems stand, in respect to the

central basis of the great system. This is done by reference to the ratio of the length and vibrations of a musical string—for example, if, in respect to the central unit, the numerals in the descending series be understood to represent the direct ratio of the length, those which stand under the corresponding semitone, in the ascending series, will represent the direct ratio of their vibrations—but if, in respect to the central unit, the numerals in the descending series be understood to represent the inverse ratio of their vibrations, the corresponding semitone in the ascending series will represent the inverse ratio of their lengths. The large numerals represent the true ratios of the bases, and triune systems, to which they severally refer—and the various powers of these numbers, expressed by the small numerals, represent false ratios.

310. Let us, for instance, take the number 3 on the left in the descending series, and 3 on the right in the ascending series. The numeral 3, on the left, shows that the length of the string for the note F, the 7th semitone below C, must be $\frac{3}{1}$, and for the note G, the 7th semitone above C, it must be $\frac{1}{3}$—and also that in respect to the ratio of their vibrations, the notes F, G, are as $\frac{1}{3}$ to $\frac{3}{1} = 9$ to 1.

In regard to the prime 5, of the series below, in connexion with the center, and 5 of the series above, the ratios in respect to length of string are ♭A $\frac{5}{1}$, C 1, E $\frac{1}{5} = 25$, 5, 1,—and for vibrations ♭A $\frac{1}{5}$, C 1, E $\frac{5}{1} = 1$, 5, 25, &c. The note ♭A 5 will be seen to stand under the 4th semitone in the descending series, and E under the 4th semitone in the ascending series. Upon examination it will be seen that the numerals and their powers, on the right and left of the central units, are throughout exactly the same, except that the order in which they are placed is reversed.

311. Since the powers of any root are necessarily all products and represent merely the same interval transposed, it is evident no new primary ratio can by this means be produced. For example, though 3^4 is the same on the piano as the 4th octave to 5, they are not really so—3^4 being 81, and the 4th octave to 5 being 80. So in like manner the octave to 5^2, though the same on the piano as 7^2, they are not really so—the octave to 5^2 being 50, and 7^2 being 49, &c.

312. It will be seen by reference to the table, § 305, that the prime 3 and its powers produce a series of sounds answering to every note of the semitone scale of the piano.

This circumstance at one time gave rise to the substitution of the primes 2 and 3, and their powers for all the others—although these substituted powers are, in every instance, out of tune, that is, at variance with the true ratio, and utterly destructive of all correct ideas relating to the truths of science, and the laws of harmony thence derived.

Subsequently to this, as it was impossible the false intonation of the diatonic major 3rd, in the ratio to its basis, 1 to 3^4, could escape observation, the prime 5 was reluctantly introduced in addition to the lower primes 1, 2, 3. In this stereotyped condition everything continued to remain for ages—and if in the interm, a bit by bit reformer made his appearance, and dared to hint a word in favour of the prime 7, the orthodox critics at once took the alarm, and a storm was the inevitable consequence. These facts show the folly and danger of attempting to do battle, while in a helpless and unarmed condition, with an enemy basking in the sunshine of public favour, however false his assumptions, or futile his arguments.

Let it carefully be borne in mind that what is here stated relates only to the calculations and speculations of mathematicians, and by no means to actual practice. In practice the affections have ever been the governing power, and not the misapplied calculations and false philosophy emanating from the head.

313. But to show the folly of thus substituting the prime 3, and its powers, for all the other primes, simply because an artificial system of semitones can thereby be obtained—it will be seen upon inspection that another, and not very dissimilar, artificial system of semitones can be obtained from the prime 17 and its powers—and also from 15 and its powers.

All these artificial semitonic systems are not only at variance, one with another, with the truths of nature, relating to the science of music, but also with the equal temperament, and all the unequal temperaments or systems of tuning the piano or organ ever invented.

314. The reason why the primes 5, 7, 11, 13, 19, do not produce some sort of semitonic scale, is—(1) because in respect to the prime 5, the steps or jumps being by major 3rds, we arrive at a substitute for the octave in 3 steps only, and consequently pass over 9 out of the 12 intervals of the semitonic scale—(2) with the prime 7, the steps being by wholetones, we arrive at a substitute for the octave in 6 steps, and consequently pass over the remaining 6 intervals of the semitonic scale—(3) with the prime 11 we arrive at a substitute for the octave in 2 steps only, and consequently pass over the remaining 10 inter-

vals of the semitonic scale—(4) with the prime 13 we jump at the substitute for the octave in 3 steps only, which though the same on the piano as those relating to the prime 5, are nevertheless widely different in truth—(5) with the prime 19 we arrive at a substitute for the octave in 4 steps, and consequently pass over the remaining 8 intervals of the semitonic scale.

315. As an approximation is made to the octave by some one of the powers of each of the primary ratios from 1 to 19, in order to find the difference between the octaves thus obtained, and the true octave, these powers must be severally ascertained and compared with the corresponding power of the harmonic 2, the ratio of the true octave. For example—(1) $3^{12} = 531{,}441$, and $2^{19} = 524{,}288$, whence it follows that the difference between an octave produced by twelve 5ths, and the true octave is as 531,441, to 524,288—(2) $5^3 = 125$ and $2^7 = 128$, the difference between the octave produced by three major 3rds and the true octave is, therefore, as 125 to 128—(3) $7^6 = 117{,}649$ and $2^{17} = 131{,}072$, the difference between six ♭7ths and the true octave—(4) $11^2 = 121$, and $2^7 = 128$, the difference between two ♭5ths and the true octave—(5) $13^3 = 2{,}197$ and $2^{11} = 2{,}048$, the difference between three ♯5ths and the true octave—(6) $17^{12} = 582{,}622{,}237{,}229{,}761$, and $2^{49} = 562{,}949{,}953{,}421{,}312$, the difference between twelve semitones in the ratio 16 to 17 and the true octave, and—(7) $19^4 = 130{,}321$ and $2^{17} = 131{,}072$, the difference between four minor 3rds and the true octave.

316. In order to convey an idea of the value of the ratio 17^{12} and 2^{49}, and also to show how the truths of science can at all times be depended upon, let it be observed that the vibrations expressed by the ratio $2^{49} = 562{,}949{,}953{,}421{,}312$, if referred to the note C, whose vibrations are $2^9 = 512$ in a second, would involve an uninterrupted period 34,865 years! Furthermore, to show that 17^{12} ought to correspond with 2^{49}, let it be observed that, $17 = 2^4 + \frac{1}{12}$,—that is, the harmonic 17 being the semitone above the 4th octave, 17^{12} must necessarily correspond with $2^4 + \frac{1}{12} \times 12$ or $12 \times 4 + \frac{12}{12} = 49$.

317. Many other particulars may be pointed out in regard to this table, but it is unnecessary to dwell upon every circumstance relating to it, since what has been said, if clearly understood, will answer the purpose for which it is intended. The importance of the discovery of the existence of the great system, consists in the explication which it affords for many of the facts observable in the works of the great masters, but more particu-

larly as it relates to abrupt transitions from one triune system or key to another, while nevertheless the coherence and unity of the composition is clearly felt throughout. Now, it so happens that this *metaphysical* perception of unity and coherence is the result of *physical* unity, § 91.

318. Let, for example, the order of the triune systems or key-notes be C, E, G, C. In this case, if E be the harmonic 5 to C—G the harmonic 19 to E—the ratio of G to C will be $5 \times 19 = 95$,—and if from hence we return to C, the pitch of C will not be the same as at first, but flatter, and the unity destroyed. But if we take the ratio of G from C, that is, from the central basis of the central system, as it ought to be, and not according to the order of their succession—that is, every one from that which precedes it, the ratio of C, G, will be 1, 3, the octaves to which are 6—96, &c., and in this case the unity of the whole is preserved.

319. Again, let the course of the key-notes be D, E, ♯F, D. If the ratio of the vibrations of the two wholetones D, E, and E, ♯F, be both alike, that is both as 7 to 8, 8 to 9, or 9 to 10—the pitch of the key-note D will, upon its re-introduction, in every instance, be changed. For if D, E, be 7 to 8, and E, ♯F, 7 to 8, then D to ♯F will be 7 to 8^2,—or 7×2^3 to $8^2 = 56$ to 64. If the ratio D to E and E to ♯F be 8 to 9, then D to ♯F will be 8 to 9^2, or 8^2 to $9^2 = 64$ to 81. If the ratio of D to E, and E to ♯F, be as 9 to 10, then D to ♯F will be as 9 to 10^2, or 9×2^3 to $10^2 = 72$ to 100. But if we take the ratio of ♯F from D, that is from the central basis of the central system, and not according to the order of their succession, the ratio of ♯F will be 5, the octaves to which are 10—80, &c.—and hence the ratio of D to ♯F will be as 1 to 5, 2 to 5, or 4 to $5 = 64$ to 80—in which case the unity of the whole is preserved.

It is to be remarked that what is here said in regard to the central bases, in a series of triune systems, is equally applicable to the same series of individual sounds appertaining to one triune system.

After what has been said, I may venture to assert that had Handel disregarded, more than he did, the quackery of pedants, and their stiff and silly rules, he would have done, in his own way, some things which Beethoven has done. It is well known that Handel, S. Bach, Haydn, Mozart, and Beethoven have all been abused as innovators. But it is to be observed that this term is susceptible of a two-fold application, the first of which is referable to those who enlarge the field of our enjoyments, at the expense of the pedantic and ignorant law makers who preceded them—in which sense I take the term innovator as implying something rather complimentary than otherwise—in the opposite sense it is applicable to that rather large class of would-be composers, whose every effort appears to be directed against the laws appertaining to nature, or rather against the essential and immutable laws of order which we owe their existence to the author of nature.

320. In the following table will be found the true and substituted ratios of all the key-notes of the great system represented in the table, § 305, each column referring to its respective semitone above and below the central basis of the great system.

1st semitone above and below the Central System.	2nd semitone above and below the Central-system.	3rd semitone above and below the Central-system.
... ...	... ...	$19 \times 2^{31} = 40,802,189,312$
$17 \times 2^{39} = 9,345,848,836,096$	$17^2 \times 2^{31} = 620,622,774,272$	$17^3 \times 2^{23} = 41,213,231,104$
$15^{11} = 8,649,755,859,375$	$15^{10} = 576,650,390,625$	$15^9 = 38,443,359\ 375$
... ...	... ...	... ...
... ...	... ...	... ...
... ...	$9 \times 2^{36} = 618,475,290,624$	... ...
... ...	$75 \times 2^{25} = 563,949,338,624$	... ...
... ...	... ...	... ...
$37 \times 2^{32} = 9,393,093,476,352$	$32 \times 2^{36} = 618,475,290,624$	$39 \times 2^{21} = 41,278,242,816$
... ...	... ...	... ...
Equal Temperament. 9,319,137,300,610	Equal Temperament. 617,080,069,087	Equal Temperament. 40,860,828,500

4th semitone above and below the Central-system.	5th semitone above and below the Central-system.	6th semitone above and below the Central-system.
... ...	... ...	$19^2 \times 2^{16} = 23,658,496$
$17^4 \times 2^{15} = 2,736,816,128$	$17^5 \times 27 = 181,741,696$	$17^6 = 24,137,569$
$15^8 = 2,562,890,625$	$15^7 = 170,859,375$	$15^6 \times 2 = 22,781,250$
$13^2 \times 2^{24} = 2,835,349,504$	... ...	... ...
... ...	... ...	$11 \times 2^{21} = 23,068,672$
$9^2 \times 2^{25} = 2,717,908,992$	... ...	$9^3 \times 2^{15} = 23,887,872$
$7^4 \times 2^{20} = 2,517,630,976$	... ...	$7^3 \times 2^{16} = 22,478,848$
$5 \times 2^{29} = 2,684,354,560$	... ...	... ...
$3^4 \times 2^{25} = 2,717,908,992$	$3^{11} \times 2^{10} = 181,398,528$	$3^6 \times 2^{15} = 23,887,872$
... ...	... ...	... ...
Equal Temperament 2,705,660,282	Equal Temperature ... 179,158,185	Equal Temperament. ... 23,726,527

7th semitone above and below the Central System.	8th semitone above and below the Central-system.	9th semitone above and below the Central-system.
... ...	... ...	$19^3 \times 2^{24} = 115,074,924,544$
$17^7 = 410,338,673$	$17^8 = 6,975,757,441$	$17^9 = 118,587,876,497$
$15^5 \times 2^9 = 388,800,000$	$15^4 \times 2^{17} = 6,635,520,000$	$15^3 \times 2^{27} = 113,246,208,000$
... ...	$13 \times 2^{29} = 6,979,321,856$	... ...
... ...	... ...	... ...
... ...	$9^4 \times 2^{20} = 6,879,707,136$	... ...
... ...	$7^2 \times 2^{27} = 6\,576,668\,672$	... ...
... ...	$5^2 \times 2^{28} = 6,710,886,400$	... ...
$3 \times 2^{27} = 402,653,184$	$3^8 \times 2^{20} = 6,879,707,136$	$3^3 \times 2^{32} = 115,964,116,992$
... ...	... ...	... ...
Equal Temperament. 402,199,863	Equal Temperament. 6,817,831,085	Equal Temperament. 115,572,415,974

10th semitone above and below the Central-system.	11th semitone above and below the Central-system.	12th semitone above and below the Central-system.
... ...	... ...	$19^4 \times 2^{32} = 559,724,432,982,016$
$17^{10} = 2,015,993,900,449$	$17^{11} = 34,271,896,307,633$	$17^{12} = 582,622,237,229,761$
$15^2 \times 2^{33} = 1,932,735,283,200$	$15 \times 2^{41} = 32,985,348,833,280$	$15^{12} \times 2^2 = 518,985,351,562,500$
... ...	... ...	$13^3 \times 2^{38} = 603,906,761,555,968$
... ...	... ...	$11^2 \times 2^{42} = 532,163,627,843,584$
$9^5 \times 2^{25} = 1,981,355,655,168$	... ...	$9^6 \times 2^{30} = 570,630,428,688,384$
$7 \times 2^{38} = 1,924,145,348,608$	... ...	$7^6 \times 2^{32} = 505,298,607,407,104$
... ...	... ...	$5^3 \times 2^{42} = 549,755,813,888,000$
$3^{10} \times 2^{25} = 1,981,355,655,168$	$3^5 \times 2^{37} = 33,397,665,693,696$	$3^{12} \times 2^{30} = 570,630,428,688,384$
... ...	... ...	$2^{49} = 562,949,953,421,312$
Equal Temperament 1,959,109,818,371	Equal Temperament 33,209,539,254,183	Equal Temperament 562,949,953,421,312

321. That the ratios of the vibrations of the central bases

which fall upon the same note and interval in the semitonic scale may be perceived at once, and their relative pitch thus ascertained—they are, in every instance, represented in the same octave, and in their lowest terms. But though these ratios are here all worked out in full, it will be sufficient to look at the first two or three numbers, not only to obtain a generally correct idea to what extent they differ, but also to see that these differences correspond in every instance with the nature of the prime, or the powers of the prime, whence the result has been obtained. The prime 17, for instance, being a little sharper than the mean tempered semitone, the powers of 17 must necessarily work higher and higher, above the mean ratio, as the powers are higher.

The reverse of this takes place with the number 15, and its powers, because the powers of this number proceed in the contrary direction—that is, from the 11th down to the 0 semitone —and not, as in the case of 17, from 0 to the 12th semitone.

322. The results of this Table are obtained as follows—(1) find the highest product involved in the representation of the particular interval and note under examination—(2) ascertain the octave, and (if any) the fractional part of the next octave, represented by this maximum product—(3) ascertain the octave, and (if any) the fractional part of the next octave, severally represented by each of the minor products—(4) deduct the minor products severally from the maximum product, and the remainder will give the octave, or power of the harmonic 2 required, to elevate that minor product to the octave corresponding with that of the maximum product. For example, 15^{11} is evidently the maximum product, represented in the first column. Wherefore—

Ratio.		Octaves.		Ratio.		Octaves.	
As 15	:	$3\frac{11}{12}$	::	15^{11}	:	$3\frac{11}{12} \times 11^{*}$	$= 43\frac{1}{12}$
						17	$= 4\frac{1}{12}$
							39 Octaves.

Whence it follows that in order to compare the ratio 17 with

* The multiplication of intervals corresponds with the powers of ratios.

that of 15^{11}, 17 must be multiplied by 2^{39}, and thereby elevated to the corresponding octave. Again :—

Ratio.		Octaves.		Ratio.		Octaves.	
As 15	:	$3\frac{11}{12}$	::	15^{11}	:	$3\frac{11}{12} \times 11$	$= 43\frac{1}{12}$
As 3	:	$1\frac{7}{12}$	::	3^{7}	:	$1\frac{7}{12} \times 7$	$= 11\frac{1}{12}$
							32 Octaves.

Whence it follows that in order to compare the ratio 3^7 with 15^{11}, 3^7 must be multiplied by 2^{32}, and thereby elevated to the corresponding octave.

PART 16.

Of harmonic, basal, and tonic transitions.

323. In this work the progression from one harmonic to another, in which the basis and tonic is the same to both, is denominated an *harmonic transition.* The progression of one harmonic to another, or from the harmonics of one chord to the harmonics of another chord, by which a new basis is introduced, is denominated a *basal transition.* And the progression from one harmonic to another, or from the harmonics of one chord to the harmonics of another chord, by which a new tonic is introduced, is denominated a *tonic transition.*

324. In ex. 135, the character of the symbols being the same, to both notes, in the same bar, of course the basis is the same—and hence it is that, since the introduction of the second note merely effects a change in respect to the harmonic, such progressions are denominated harmonic transitions.

To the ear, and mind, the sensations produced by harmonic transitions are of the simplest and mildest kind. In harmonic transitions one diatonic harmonic may pass to another, as in bars 1, 3, 5—or a diatonic harmonic may pass to a chromatic, as in bars 2, 6—or one chromatic harmonic may pass to another, as in bar 4—but not a chromatic to a diatonic harmonic. For, as the diatonic harmonic must, in this case, be the resolvent to that

chromatic, a basal, not an harmonic, transition is the necessary consequence.

325. In ex. 136, as the character of the symbols change in passing from the 1st to the 2nd note in the same bar, a different basis is thereby represented; but as the tonic or key-note remains the same, such progressions are denominated basal transitions. A basal transition is also effected by the last of each of the two chords in ex. 137.

Speaking generally, the effect of basal transitions is rather more bold than harmonic transitions.

326. In ex. 138, the tonic or key-note is first C and then E. A tonic transition is therefore effected by the introduction of the note ♯D 5. In examples of this kind the character of the symbols may or may not change.

327. In ex. 139, the tonic is first C, and then B—and in ex. 140 first C, and then A. Hence it follows that the notes G, B, D, ♯E, in the progression from the chord G, B, D,F; and the notes F, B, D, in the progression from the chord F, A, D, are all directly or indirectly concerned in the production of a basal transition. For though, upon keyed instruments, the sounds of the chord G, B, D, F, in the first bar of ex. 139 are the same as G, B, D, ♯E, in the second bar—as is also the case in respect to the sounds F, D, in the key of C, and F, D in the key of A in ex. 140—it will be found that, except the note F, in respect to their true intonation, the ratio of their vibrations are all different —and consequently such as to realize the effect of the tonic transition therein represented.

328. It is, indeed, true, that apart from other sounds, the continuation of the same fixed sounds can convey no idea of a tonic transition, or of any transition whatever. But, notwithstanding this, the very instant the effect is realized by any one or more of the other parts, the fixed note becomes changed in its character, so as to agree with its new associates—and, in so doing, confirms and supports the transition originating with them.

329. To show how wonderfully harmony is adapted to reflect light upon, and, as it were, vivify melody—that is, how the separate melodies, of which the harmony is composed, naturally dissipate the obscurities incidental to each, it will be found, upon close examination, that the exquisitely-refined effects of small intervals, that is, of high ratios, is continually being ideally realized, even by means of such an instrument as the piano—although we know, for certain, that these intervals are not really

produced. Thus, by yielding our wills, or affections, to a species of delusion, we accept the more or less remote approximations to truth in respect to the ratios of vibrations, for the truth itself—and, in so doing, the rude indications of a tonic transition for the fact itself. Moreover, let it also be borne in mind that, were we not so constituted as readily, and freely, to yield our assent to the delusions here pointed out, music, to us, would absolutely cease to exist, since absolute perfection, which alone appertains to truth, is to us utterly unattainable. But we have as yet scarcely described half of the wonders really relating to harmony—for, a little farther reflection upon the subject will also show that, unless the sensuous organ, to which both melody and harmony relate, were vastly more perfect than it is, the most beautiful points relating to melodies of an elevated and refined character, would continually elude our observation, even while listening with the most fixed attention, and thereby render the entire sequence obscure and unintelligible.

330. Thus it appears that perfection in regard to melody only, with such faculties as we now possess, would in a great measure be lost—since for want of other melodies, to furnish the means of comparison, the truths relating thereto could not be discerned. Whereas by harmony, the melodies of which are not too far removed from the truth, a correct idea of the intended effect can be conveyed, and the effect itself, more or less, vividly realized. Our perceptions in regard to melody are, in fact, entirely dependant upon *memory*, and thereby to successive order—but in harmony we have both successive and simultaneous order. And hence its superiority both in respect to distinctness and power.

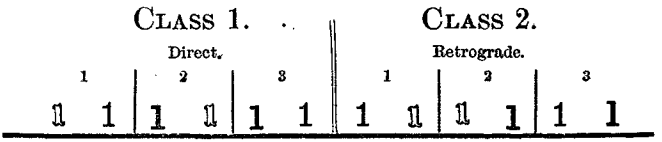
331. With the view to render this subject clear to his apprehension, the reader should endeavour, mentally, to picture the facts relating to both melody and harmony to his imagination, such as they would appear, if represented to the eye, by a corresponding series of dots, in a single line, for melody—and for the melody of each part in harmony, as shown in § 48—55.

In so doing he will perceive the poverty and dubious nature of the former, in comparison with the latter—to which alone appertain diversified and clearly-defined cycles of vibrations, with the unities or unit-sounds thence derived.

I readily admit that to think in this manner is to think scientifically—and if it can be proved that by so doing the affections become deadened and destroyed, my advice is to burn every scientific work in existence—but as I do not assent to this doctrine, or believe that the use of any one faculty is detrimental to the free exercise of another, it is my intention to act accordingly.

332. That no progression can possibly be made which does not involve a harmonic, basal, or tonic transition, must be evident. But since basal transitions are those which constitute the foundation upon which the laws of harmony chiefly depend, I now proceed to the farther explication of this subject.

SINGLE BASAL TRANSITIONS.



333. By the above is represented every possible variety of single basal transitions to which melody alone, or the melody of any particular part in harmony, can give birth. That it is only by reference to the triune system that bases can be applied to the analysis of melody is evident—*two* sounds being required to generate a basis. Class 1 is denominated *direct*, because the last basis stands in the relation of a harmonic to the first—and Class 2, *retrograde*, because the first basis stands in the relation of a harmonic to the last. Class 2 is the reverse of Class 1, and the numbers which correspond are the reverse of each other.

334. The numerals 1, 2, 3, placed over the treble notes in ex. 141—144 refer to the symbols to which they relate in the formulæ, and thereby to the nature of the transition involved in each particular case. As two or more of its harmonics are required to generate any basis, hence it happens that where, by reference to the triune system, only one of the harmonics of that basis is to be found in any chord or combination, that basis cannot be generated.

But, notwithstanding this, as the progression of the parts give to these non-generated bases a peculiar influence, they ought not to be disregarded. In ex. 144, No. 3, for instance, by reference to the triune system, the bases to the notes D, C, in the tenor, is more naturally G, C, than G, F—and, in the application of basal progressions to the rules of melody and harmony, though this last chord may be referred to F only, a more minute analysis may render it necessary to refer it to both C and F. The consecutive 5th, between the bases and harmonic 3, in ex. 143, 144, are here pointed out by thick lines, merely to show that they have not been overlooked.

DOUBLE BASAL TRANSITIONS.

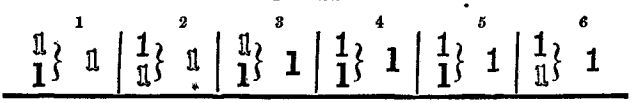
335. The number of varieties of double basal transitions is THIRTY-THREE—and these are divided, by nature, into *eight distinct classes.* These varieties, and the class to which they severally belong, will be seen in conjunction with a series of examples, numbered as before, to correspond with the symbol to which they relate.

CLASS 1.

336. In this class, the bases appertaining to one or more parts of the harmony remain the same—while, in reference to another part, or to other parts, it is changed. In other words, this class commences with *one* basis, and terminates with *two* bases, one of which is the continuation of the first. See ex. 145, 146.

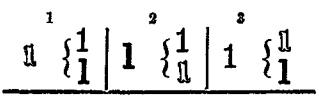
The position of the parts in respect to the symbols representing double basal transitions, is not to be understood as having anything to do with the position of the parts to which they refer.

CLASS 2.



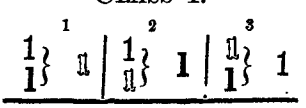
337. This class commences with *two* bases, and terminates with *one* basis—the last being the continuation of one of the two first. The particulars of Class 2 are severally the reverse of Class 1. See ex. 147, 148.

CLASS 3.



338. This class commences with *one* basis, and terminates with *two* bases—neither of the two last being the same as the first. See ex. 149, 150.

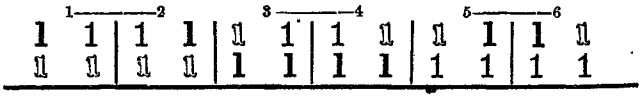
CLASS 4.



339. This class commences with *two* bases, and terminates with *one* basis, which differs from both of the preceding.

The particulars of Class 4 are severally the reverse of Class 3. See ex. 151, 152.

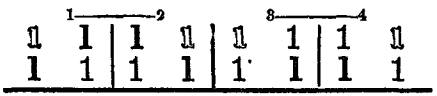
CLASS 5.



340. This class commences and terminates with *two* bases—one of the two being changed, while the other remains.

In this class No. 2 is the reverse of No. 1—No. 4 the reverse of No. 3, and No. 6 the reverse of No. 5. See ex. 153, 154.

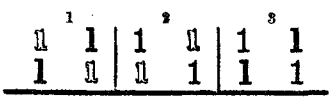
CLASS 6.



341. This class commences and terminates with *two* bases. But though one of the two first is again introduced, since it does not appertain to the same part or parts, a collision between the parts thus opposed to each other is thereby produced.

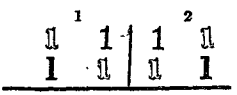
In this class No. 2 is the reverse of No. 1, and No. 4 the reverse of No. 3. See ex. 155, 156.

CLASS 7.



342. This class commences and terminates with *two* bases—the two last being the same as the two first. But since neither the one nor the other appertain to the same part or parts, the collision between the parts thus opposed becomes violent. In this class No. 1 is not especially opposed either to No. 2 or 3—nor No. 2 either to No. 1 or 3—the conflict now, like a civil war, is at home, and consequently of a more interior nature. See ex. 157, 158.

CLASS 8.



343. This class commences and terminates with two bases. But herein the bases appertaining to one or more of the parts produce consecutive fifths to those of another part, or other parts. In this class, No. 2 is the reverse of No. 1. See ex. 159, 160.

344. Ex. 161 represents a combination, wherein the three bases of the triune system are all involved. This sometimes takes place, and its resolution is into the common chord of the central basis.

345. The slightest attention to the above examples will show that as the basal transitions recede farther and farther from simplicity, the effects resulting therefrom become *per se* more and more harsh and disagreeable.

The object of these examples is merely to afford a practical illustration of the particulars relating to each class, that the principle herein set forth may be clearly understood.

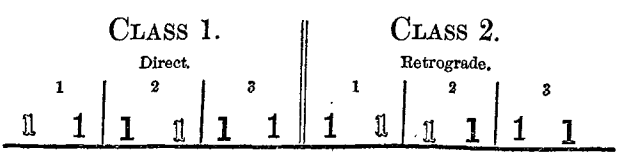
The number placed over each bar refers to the corresponding number in the symbolic formula of the class to which it belongs —and, to prevent confusion, whenever it has been thought necessary to give the harmonic of any particular note, the numerals used for this purpose are small, and placed as near as possible to the note to which they refer. The bases, in capital letters, are also placed below.

PART 17.

CLASSIFICATION AND EXPLANATION OF PRINCIPAL AND SUBORDINATE BASAL TRANSITIONS.

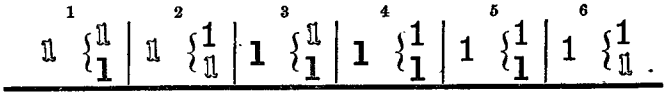
346. That the formulæ relating to basal transitions may be rendered more readily available to the student who may wish to apply this great harmonic principle to practice, they are here presented to view in an uninterrupted series.

SINGLE BASAL TRANSITIONS.

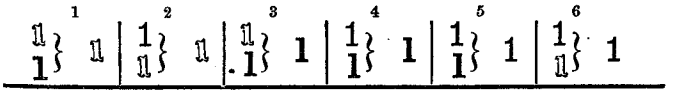


DOUBLE BASAL TRANSITIONS.

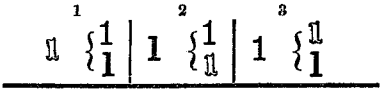
CLASS 1.



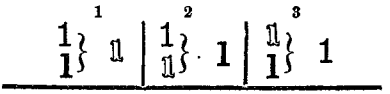
CLASS 2.



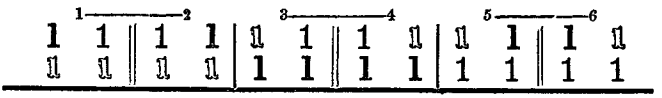
CLASS 3.



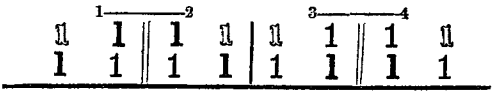
CLASS 4.



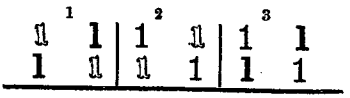
CLASS 5.



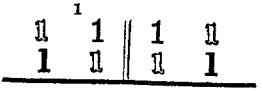
CLASS 6.



CLASS 7.



CLASS 8.



347. PRINCIPAL basal transitions are those progressions of the bases which result from, and relate to, any series of counterpoint harmony, or chords, ex. 21—34, and also to the slower and more simple movements in florid or complex harmony—that is, to the chords which constitute its outline, and to which, by way of simplification, it may be reduced.

In ex. 35, bar 1, the principal basal transition is simply 𝟙, 1 (C, G), except in the upper part, where the basis 1, by the introduction of the note ♭E, is changed to 𝟙𝟡,—and, in ex. 37, bar 1, where the basis 1, by the introduction of the note E, is changed to 𝟝.

348. SUBORDINATE basal transitions are those progressions of the bases which result from, and relate to, the subdivisions of its principal basal transitions, and by means of which those of the principal are diversified, enriched, ornamented, and enlivened.

In ex. 40, during the harmony of the compound chord 𝟙, 1, 3, 5, 7, § 184—in the upper part is introduced the basal transitions 1, 17, 1, 5,—1, 7, 𝟝, 3. In ex. 113, by Cramer, the principal basal transition is that produced by the progression of the chord F 1, G 1, D 3, B 5, to the chord F 1, C 3, ♭A 19, and thence to the chord G 1, D 3, B 5,—whence it follows that in this example the notes ♭B 7, ♭A 19, in connexion with the 1st chord, and the notes ♯F 17, ♭A 19, in connexion with the last chord, appertain to the subordinate, and not to the principal basal transitions. That the sounds of the 2nd chord F 1, C 3, ♭A 19, are not diversified by subordinate basal transitions is self-evident. Furthermore, the subordinate basal transitions which take place during the time of the first chord, when presented in full—that is, by reference to every note, are, 7, 19, 1, 19, 7, 1, and those which take place during the time of the last chord are 17, 1, 19, 1.

349. The laws, that is the formulæ relating to principal and subordinate basal transitions, are the same, but in their application to practice they must be kept distinct, and not to be confounded the one with the other.

That is, in the contemplation of the structure of the outline—or, what amounts to the same thing, in the process of simplification, the elements especially appertaining to subordinate basal transitions must not be mixed up with those of the principal—although in the representation of a series of subordinate basal transitions, appertaining to the same part or parts, the elements of the principal may be united, that the progression of such part or parts may be seen perfect and entire—thus, those of the

principal take from the subordinate whatever belongs to them —while those of the subordinate take from the principal whatever may be required to complete the series.

350. That this two-fold application of these laws adds greatly to their value, as tests of what is relatively pure and impure in harmony, will be seen by the examples which follow. It is, however, to be noted that, though in their application to the intermediate movements which constitute subordinate progressions, there is no difficulty in finding examples of every class, and the varieties of every class, it is not so with those denominated principal—in respect to which examples of some of the higher classes are scarce, or perhaps not to be found.

RULES RELATING TO PRINCIPAL AND SUBORDINATE BASAL TRANSITIONS.

RULE 1.

351. In comparing the bases appertaining to one or more parts with those of another part, or other parts—the principal and the subordinate should be made distinct objects of investigation—and, in so doing, the principal should be examined first, and the subordinate afterwards.

The reason for this injunction is, because we are not in a condition to investigate the subordinate basal transitions until we distinctly perceive what the sounds are which relate to the principal, and have ascertained that these are what they ought to be.

RULE 2.

352. When, besides the principal, single or double subordinate basal transitions are generated by the simultaneous progression of two or more parts, the laws of harmony relating to these parts are the same as for those relating to the principal.

353. But though by the progression of two or more parts subordinate double basal transitions may certainly be allowed a wider range than principal, the higher classes of them ought not to be involved unnecessarily. To introduce them with propriety, it is therefore requisite that, during the period of a principal, the details of a subordinate should be as simple as possible, consistently with the natural flow of the melody of each part. In other words, the principle of unity ought not to be sacrificed for the purpose of rendering some of the details more simple, and, *per se*, more harmonious at any particular place.

354. It is worthy of remark also, that in cases of difficulty, the inexperienced harmonist is much more liable to fall into errors by losing sight of the principle of unity, than from neglecting to pay due attention to the details—while, on the contrary, the great master, in his attention to the former, is sometimes apt to lose sight of the latter. The truth of this is verified by ex. 163, by Beethoven, op. 53, which is given exactly as published, and ex. 164, as it ought to be—or, as it is presumed, more in agreement with the author's general practice.

355. This example is admirably calculated to illustrate the truth of what is stated respecting double basal transitions.

ANALYSIS OF EX. 163.

	1	—	1	—	▽	—	▽	▽	—	▽	—	1	—	1	5
					5		1	3		5	7 or 1		3		
					▽		5	5		1	3		5		1
	1	7			5		3	1		5	5		1		1
Generated Bases.					1		1	1		1	1				
	1	1			5		1	1		5			1		1
					1		1			1	1				

ANALYSIS OF EX. 164.

	1	—	1	—	▽	—	1	▽	—	1	—	1	—	1	5
			5		1		3	5		3	7 or 1		3		
			1	—	▽	—	1	▽	—	1	—	1	—	1	1
	1	7			5		3	1		5	5		1		1
Generated Bases.	1	1			1		1	1		1	1 or $\frac{1}{1}$		1		1

The above analysis will show how crude the harmony of ex. 163 is, in comparison with that of ex. 164. For a long series of double and triple basal transitions can never result from good harmony, or be rendered pleasing to the ear. These errors in Beethoven proceed unquestionably from inadvertency—but they cannot be placed to the account of the engraver, because they are repeated several times. In the above symbolic analysis, the characteristic numerals severally relate to the notes in the corresponding musical example—bars and double bars being also introduced in the analysis. In respect to the generated

basis, placed below the analysis, the sub-medial-basis is represented by **5** or **19**, and the central-medial-basis by 𝟝 or 𝟙𝟡. §.144.

Rule 3.

356. Although a succession of two or three double basal progressions may, occasionally, result from the harmony of three, four, or five parts, without involving the certainty of any error —an uninterrupted succession of them is utterly inadmissable. The above example, as written by Beethoven, will be found to fall under the censure of this rule.

Rule 4.

357. Consecutive octaves must not occur, as it were, undesignedly, so as, in the course of the same phrase, to excite the impression of a temporary interruption of any two, in other respects, distinct parts—yet in order to strengthen the principal parts, such parts may not only be doubled, but even trippled, by octaves, in music for a single instrument—or by octaves, or unisons, in music for more than one instrument—provided it be continued through a whole phrase, or at least during some definite period, and with the view to a particular effect. See ex. 165, 166.

358. With the view to distinguish the principal basal transitions from the subordinate, it will sometimes be found convenient to represent the former by large, and the latter by small, notes, as in ex. 162, in which is first given the plain chords appertaining to the principal basal transitions only, and afterwards the same chords, with the addition of the small notes ♯F, A—♯D, F, which appertain to the subordinate basal transitions, the bases to which are referentially pointed out by means of the harmonics placed near the notes themselves.

Rule 5.

359. Sounds which involve a transition from a principal to a subordinate basis, or *vice versa*, are free from the rules relating either to the one or the other.

360. In ex. 162 the consecutive 5ths, pointed out by the thick lines between the notes $\frac{G}{C}$——$\frac{♯F}{B}$ and $\frac{E}{A}$——$\frac{♯D}{♯G}$ are not really what they appear to be. For, the ratio of the interval $\frac{G}{C}$ and $\frac{E}{A}$

is $\frac{3}{1}$, but the ratio of the interval $\frac{\sharp F}{B}$ and $\frac{\sharp D}{\sharp G}$ is $\frac{136}{45}$. Now, as in these instances, the consecutive 5ths do not represent the same interval—consecutive basal 5ths are not therby engendered—and as the sounds of the upper part are not referable, solely, either to the principal or subordinate bases, the example is free from errors, and therefore agreeable to the ear. In regard to basal transitions, as nothing approaching to a fixed routine can be discovered in analysing the works of the great masters, it would appear that pedantry alone can desire the establishment of stereotype forms in regard to it.

PART 18.

RULES RELATING TO MELODY, HARMONY, AND COMPOSITION.

361. As the rules relating to melody, harmony, and composition, are based upon the ordinary and generally-acknowledged perceptions of mankind in regard to music, they therefore represent the formulæ relating to such matters—in other words, the formulæ relating to the true organization of the fibres and fibrils which constitute its bodily conformation—within which, in really good music, there exists a soul enthroned on high, the angelic influences of which are clearly perceived by some, dimly by others, and it is to be feared, by many scarcely at all. But though the body be but of small value in comparison with the more subtle spirit within, who does not see that there exists a reciprocal dependence of the one upon the other? And further attention to the subject will lead to the discovery that in the more exterior forms are represented the patterns of those which are interior—that the body, in truth, is not merely a covering to the soul within, but its effigy also.

362. In melody, in addition to the obvious necessity of paying the greatest attention to style, and if vocal, to the sense and general spirit of the words—and its adaptation to the purpose for which it is intended—it is requisite, by the introduction of a judicious course of tonic transitions, to avoid sameness and insipidity on the one hand, and want of unanimity on the other.

363. As a melody that will admit of various interpretations in respect to the course of the harmony and key-notes to which it is referable can only give birth to confused and perplexed

ideas and sensations—the habit of submitting every composition to the strict scrutiny of a careful analysis will be productive of great benefit.

364. Let it be carefully borne in mind that a melody in which the key, the harmony involved, and the mode, is every now and then obscure, and incapable of any definite analysis, is faulty, even respect to the most natural, and consequently the most favorable, interpretation that can possibly be given to it.

365. A moment's reflection upon the important truths brought to light by means of the Ratiometer, and other diagrams, will make it evident, *that were there as many distinct sounds in practice as by theory required, the basis to each would be invariably the same.** But since, from the imperfection of all known instruments, this is not the case, the obscurity in which this defect involves the matter will be found almost entirely obviated by attending to the following rules:—

That these rules may be rendered applicable to the works of all composers, it will frequently be necessary to correct their errors in writing the sharp of one note for the flat of another, and *vice versa*.

Rules by which the basis is to be decided, where the same note is referable to two bases.

RULE 1.

366. No note should be made to involve a basal transition, by which such transition is only equivocally indicated. See. ex. 167.

RULE 2.

367. The basal transition from the central to the super-basis is to be preferred to that from the central to the sub-basis. See ex. 168.

This rule may, however, be set aside in a progression of chords, and in those cases, where, by so doing, every part of the harmony may be referred to one basis instead of two—or to two bases instead of three.

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* The key-note and its fifth, or the notes C and G in the key of C, make no real exception to this principle—for, *as harmonics*, they have but one basis each, although at other times they themselves become bases.

RULE 3.

368. The basal transition from the sub to the super-basis is to be preferred to that from the sub to the central-basis. See ex. 169.

RULE 4.

369. The basal transition from the super to the central-basis is to be preferred to that from the super to the sub-basis. See ex. 170.

The Rules which relate to the progression of sounds in melody, and the melody of the parts in harmony, are as follows :—

RULE 1.

370. An ascending chromatic harmonic, in order to resolve upon the sound which the ear anticipates, must ascend to the next diatonic harmonic above. See ex. 171. Here the ascending chromatic, ♯D 13, ascends to its resolvent E 5. Chromatic harmonics, whether ascending or descending, must always be resolved in the same part.

RULE 2.

371. A descending chromatic harmonic, in order to resolve upon the sound which the ear anticipates, must descend to the next diatonic harmonic below. See ex. 172. Here the descending chromatic F 7 descends to its resolvent E 5.

RULE 3.

372. An ascending chromatic harmonic, instead of resolving upon the diatonic harmonic above, according to Rule 1, may proceed to its relative descending chromatic. See ex. 173, 174. Here the ascending chromatic, ♯G 13, instead of proceeding to its resolvent A 5, proceeds to ♭A 19, its relative descending chromatic—and in ex. 174, the ascending chromatic, ♯F 17, instead of proceeding to its resolvent G 1, proceeds to its relative descending chromatic F 7.

In this and the following rule is involved a complete explanation of all those progressions hitherto denominated enharmonic.

Rule 4.

373. A descending chromatic harmonic, instead of resolving upon the diatonic harmonic below, according to Rule 2, may proceed to its relative ascending chromatic. See ex. 175.

Here the descending chromatic, ♭E 19, instead of proceeding to its resolvent D 3, proceeds to ♯D, its relative ascending chromatic.

Rule 5.

374. An ascending chromatic may also proceed to a descending chromatic of the same basis, and a descending chromatic may proceed to an ascending chromatic of the same basis, provided the resolvent to both is the same. See ex. 176.

Here the ascending chromatic, ♯D 13, proceeds to the descending chromatic F 7, the basis to both being the same, and the resolvent to both, namely, E 5, being also the same. Here also the descending chromatic, F 7, proceeds to the ascending chromatic, ♯D 13, the basis to both being the same, and the resolvent to both, namely E 5, being also the same.

Rule 6.

375. When a melody is made to produce the effect of harmony consisting of a certain number of parts, those notes, which in harmony would form a distinct part, should be treated as such, and the chromatic harmonics resolved accordingly. See ex. 177.

Here D 3 is the resolvent to the ascending chromatic, ♯C 17, —the lower D 3 the resolvent to the descending chromatic, ♭E 19—and E 5 the resolvent to the descending chromatic, F 7.

376. That the following rules relating to harmony may be seen in co-ordination with those faculties of the mind and body upon which they depend, and which constitute the foundation upon which they rest, it may be expedient here to premise—that, as with the eye, so with the ear, some of the objects of its attention are direct, and some indirect—the particulars selected by the mind as the direct object of its attention, is seen, as it were, in light, or heard perspicuously—and, in proportion as the mind becomes absorbed in the contemplation of these objects, in the same proportion it is withdrawn from others, which are thereby, as it were, more or less removed from the center to the circumference, or from the light to the shade. For, in regard to

this, it must be borne in mind, that it is not the eye which sees, or the ear which hears, but the mind, or soul, by means of these organs.

If these particulars be kept constantly and steadily before the mind's eye, it will be readily admitted that it must necessarily be an object of importance how things are presented to the mind, and that it is equally so whether this be done through the medium of the eye or ear—that things heterogeneous be not confounded with things homogeneous.

377. In the union of two or more melodies, as in harmony, it is of importance that the more homogeneous melodies be distinctly and unequivocally united in couples, as duets, that the author's intention may not only be seen upon paper, but so distinctly perceived in the performance, as to prevent the auditor from forming to himself duet conjunctions, derived from materials more or less heterogeneous in their nature and quality.

This will be understood by attending to what follows:—

378. Two parts give birth to one duet only—and this, generally speaking, is nearly as good when inverted as in its original form—consecutive 4ths, in two parts, being nearly as bad as consecutive *5ths*.

379. Three parts give birth to three duets. Thus, from the three parts, $\begin{matrix}3\\2\\1\end{matrix}$ can be obtained the duets $\begin{matrix}2&3&3\\1&1&2\end{matrix}$. If the parts $\begin{matrix}2&3\\1&1\end{matrix}$ form the principal duets, the parts $\begin{matrix}3\\2\end{matrix}$ will form a residual duet—and if the parts $\begin{matrix}3&3\\1&2\end{matrix}$ form the principal duets, the parts $\begin{matrix}2\\1\end{matrix}$ will form a residual duet.

380. Four parts give birth to six duets. Thus, from the four parts $\begin{matrix}4\\3\\2\\1\end{matrix}$ can be obtained the duets $\begin{matrix}2&3&4&3&4&4\\1&1&1&2&2&3\end{matrix}$. If the parts $\begin{matrix}2&4&4\\1&1&3\end{matrix}$ form the principal duets, the parts $\begin{matrix}3&3&4\\1&2&2\end{matrix}$ will form residual duets. If the parts $\begin{matrix}3&4&4\\1&1&2\end{matrix}$ form the principal duets, then the parts $\begin{matrix}2&3&4\\1&2&3\end{matrix}$ become, as it were, cast into the shade, and

form the residual duets. And, again, if $\begin{matrix}2 & 4\\3 & 1\end{matrix}$ form the principal duets, the parts $\begin{matrix}2 & 3 & 4 & 4\\1 & 1 & 2 & 3\end{matrix}$ will form four residual duets.

381. It is here to be observed that the highest and lowest parts, that is, the parts $\begin{matrix}3\\1\end{matrix}$ in a trio and the parts $\begin{matrix}4\\1\end{matrix}$ in a quartet or chorus, ought almost always to be the principal duets, because these parts very seldom fail to command especial attention.

The Rules which relate to harmony are as follows :—

RULE 1.

382. Harmony being a combination of melodies, the melody of each part ought to conform to the rules which relate to melody.

RULE 2.

383. Every part, or melody, must be referable to the same triune system, or key—whence it follows that the transition from one key to another must take place, simultaneously, *i.e.*, in every part at the same time.

RULE 3.

384. In the harmony of four real parts, wherein consecutive 5ths, or even consecutive 4ths, are not only found to exist virtually, but conspicuously also—that is, between the same two parts, and so as to divert the attention from the stronger affinities existing from the duet combinations of other parts—such harmonies require correction, either by a different arrangement of the parts, or by reducing the four parts into three only.

Where consecutive 4ths and 5ths virtually exist—the great point is, not to draw attention to them, as if they existed by design.

RULE 4.

385. In choruses, where all the parts are sung by several voices, and the peculiarities of individual voices thereby neutralized, it is evidently a matter of no moment whether consecutive 5ths or consecutive 4ths exist between the same parts or not—unless the duet thus formed, from some cause or other, be rendered conspicuous, and thereby divert the attention from the

stronger affinities existing from the more pure and consequently more beautiful combinations of other parts.

The case is the same in a chorus of instruments, and also evidently the same in respect to the organ, &c.

386. In ex. 178 the union of any two of the parts forms an agreeable duet combination, except that in which consecutive 5ths or 4ths occur. In proportion as the parts are multiplied, this disagreeable combination is not perceived, because in this case it cannot become a specific object of attention. Wherefore:—

Rule 5.

387. In what may be termed a double duet combination, wherein each duet is good in itself, and both adapted to form a quartett, or chorus, in four parts—no objection ought to be made against the union of this double duet, on the ground that another duet can thence be formed, which in itself is not agreeable.

Rule 6.

388. If consecutive 5ths or 4ths be allowed to exist at all, the only difference that can possibly arise from varying the position of the parts is, that by so doing the attention is directed to the stronger affinities of the more pure duet combinations, and by the same means diverted from the weaker affinities of those which are comparatively impure. The principal basal transitions may also be doubled by unisons or octaves, while the subordinate are either entirely or partially omitted. This takes place in ex. 179, between the tenor and treble parts. Consecutive octaves or unisons are not avoided, but merely in appearance evaded, by the union of relative diatonic and chromatic progressions. See ex. 180.

Rule 7.

389. Diatonic sounds may be strengthened, at all convenient opportunities, by doubling the bases, basal 5ths, and basal 3rds. But as the distinction between the 1st and 2nd mode depends upon the nature of the basal thirds, the omission of these intervals should be avoided if possible.

Rule 8.

390. Compositions in which a series of tonic transitions are involved should, in addition to the central system, always consist of both super and sub-systems, that the buoyancy of the one may be duly balanced by the gravity of the other. Moreover,

the super and sub-systems, which are only remotely related to the central system, ought not to be dwelt upon too long, or introduced to the exclusion of those in which the relation is more simple, and consequently, more homogeneous with that of the central system.

The means to be employed in order to determine the key to which any phrase or portion of music is most naturally referable, in difficult cases, are as follows:—

391. It is difficult, if not impossible, to lay down an infallible rule by which the key may be decided by a purely scientific principle, without listening to the effect and attending to the impression produced on the feelings—but it should constantly be borne in mind that this difficulty is occasioned solely by the ambiguity of the notation, respecting the exact sounds they are meant to indicate. Now, in consequence of this defect, it happens that whoever undertakes the analysis of a piece of music presenting any difficulties, finds himself at the onset involved in a complete dilemma—for if his first attempt be to determine the course of the tonic transitions, which is what it ought to do, he finds it impossible to proceed a single step while he remains unacquainted with the course of the basal transitions, and if he begin with attempting to settle the course of the basal transitions, he discovers that they are absolutely dependant upon the course of the tonic transitions. Hence it is evident that there exists no absolutely certain criteria whatever by which the key and the bases can be determined, either from the consideration of the notes or the intervals they produce, capable of universal application to practice. This is, indeed, a fatal blow to those who have no better foundation to rest upon than the ordinary notation, and one that necessarily puts a stop to all further proceedings. But it is otherwise when means are employed for correcting these imperfections. I therefore proceed to point out what appears to me the best and most certain criteria by which the key may be determined—in doing which it must be understood, however, that I can only address myself to those who have ears and feelings, and possess, moreover, some knowledge of the practice.

RULE 1.

392. The best general rule for deciding the key in any passage or phrase is to fix upon those sounds as the diatonic

harmonics, which are obviously treated as such, and that note as the key-note upon which a perfect and satisfactory close can be made by the shortest course possible.

Rule 2.

393. If the mode be major, let it be changed to the minor, and if minor, let it be changed to the major, *so as to satisfy the ear.*

Then, as the sounds which require alteration are the mediants to the bases, the bases themselves will be discovered by the same means, and consequently the central basis or key-note among the rest. Of all the means that can be devised this is decidedly the best—and few indeed will be the examples which, after such a test, will leave the mind in doubt.

Rule 3.

394. Ascertain whether, in reference to any supposed key-note, the chromatic harmonics be correctly resolved—for if so, in all probability the key has been correctly determined. But if not, either the assumed key is not the key, or the notation of the chromatic sounds incorrect, and perhaps their resolution also.

395. It will, in general, be found that the most frequent errors arise—(1) from mistaking a super-basis for a central-basis —(2) from mistaking the mere appearance for a real retrograde transition to the minor 3rd below. For example, the mere appearance for the real key of A minor, while the key is in truth C major—(3) from mistaking the mere appearance for a real retrograde transition to the major 3rd below. For example, the mere appearance for the real key of ♭A major, while the key is in truth C minor.

396. In giving examples to the preceding rules, the object has been to prevent misapprehension in regard to them. The example chosen for illustration has, therefore, uniformly been the shortest and simplest possible, in preference to a more lengthy, elegant, and elaborate specimen.

As to the bases, as marked below—they, of course, are to be taken for what they are, and not as specimens of what would form the most melodious accompaniment to the part or parts above.

397. That these rules, in connexion with what has been made known in regard to the 6 single and 33 double basal transitions

—and concerning principal and residual duets, will be found of great service to the student who is not utterly destitute of musical genius or taste, I think no one can deny—and, if not adapted to enable him to rival the works of genius in the absence of that divine influence whence alone such works proceed, as I make no pretension so to do, no one has any right to complain. The object of this work is to furnish the *light* to direct and purify, not the *heat* or motive power itself. Hitherto we have been indebted to the Creator alone for ever real great poet and great composer, whether hereafter persons of dull and obscure perceptions, and of vulgar coarse minds, can be made to work some cunningly-contrived *composing machine*, and thereby excite feelings and passions in others, to which they themselves are insensible, remains to be proved—but, in my opinion, this will never come to pass. But though genius is not likely to be superceded by the mere stiff observance of rules, the want of a scientific basis is nevertheless a great barrier to its free, ample, and speedy development.

Let the student who is in earnest to become a good and original composer, go through a considerable variety of exercises, beginning with the shortest, and proceed gradually to the longest, and in so doing let him determine beforehand the course of the basal transitions through which he will pass—the time he will dwell upon each—and the passions he will endeavour to excite, &c. All this should be done with the view to acquire facility and power. And though analysis is the opposite to synthesis, he who desires to acquire skill in the art of construction must submit the productions of others to a minute and careful dissection—the secrets of construction being, as it were, laid open and exposed to view in the process of dissection. This, and this only, is the course to be pursued by any one who desires to become a composer in the legitimate sense of the word.

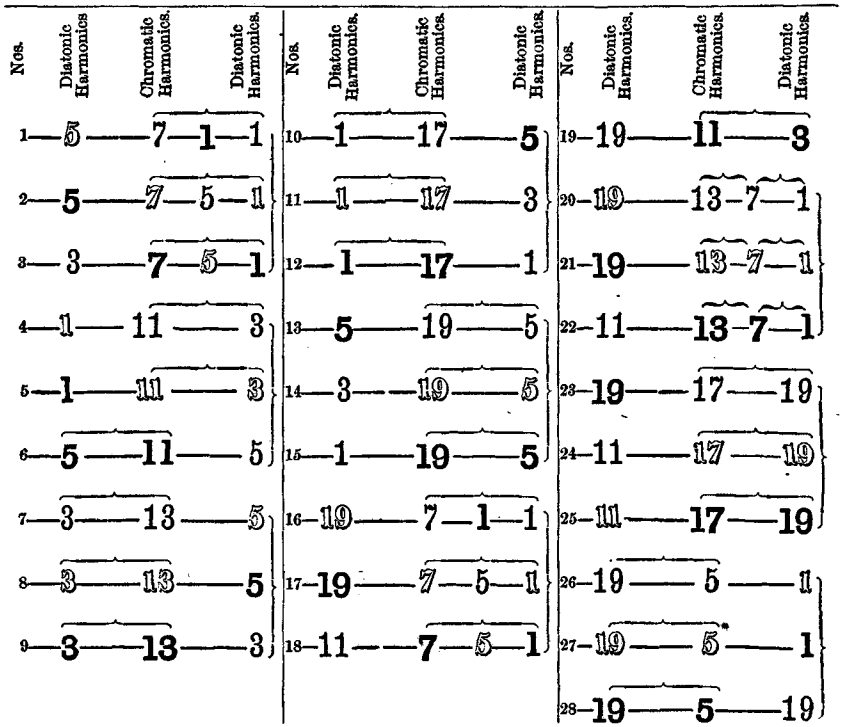
PART 19.

On position. Why the triune system is referred to its central and not to its sub-basis. And on compound vibrations.

398. The term *position*, generally speaking, refers to the pitch, and thereby to the situation of every diatonic and chromatic harmonic in respect to the triune system; but when the object is to draw attention to some peculiarity in the position of any chromatic harmonic in particular, with the view to account for the facts relating to the resolution which the ear demands, the attention is then especially directed to its position in respect to the diatonic harmonic next above and below, and not merely to the triune system as a whole.

399. The specialities relating to chromatic harmonics in respect to their diatonic relatives and resolvents are three. (1) A chromatic harmonic is termed *ascending*, when its relative is the next diatonic below, and its resolvent the next diatonic above. (2) A chromatic harmonic is termed *descending* when its relative is the next diatonic harmonic above, and its resolvent the next diatonic harmonic below. (3) A chromatic harmonic is also termed *descending* when its relative is neither the next diatonic above, nor the next below, but two removes above, and its resolvent the next diatonic harmonic below. The supposititious case of a diatonic harmonic two removes *below* its relative chromatic, has no theoretical and consequently no practical existence. § 260.

Table showing the position of the chromatic harmonics 5, 7, 11, 13, 17, 19, *in respect to their diatonic relatives and resolvents.*



400. Of the harmonics at the two extremities of every horizontal brace, one is diatonic, and the other its relative chromatic. The sub-basis 1, and the harmonics 5, 5, placed under the center of the brace at Nos. 1, 2, 3, 16, 17, 18, severally represent that diatonic harmonic, over which the diatonic on the right passes, in its progress to the chromatic harmonic, and thence to its resolvent below. The resolvents, as shown by the position of the braces, stand sometimes to the right and sometimes to the left—to the right when preceded by an ascending chromatic harmonic, and to the left when preceded by a descending chromatic harmonic. § 399. Of these examples:—

Nos. 1, 2, 7 are especially applicable to the 1st mode.
Nos. 3, 4, 5, 9, 11 do. 1st and 2nd mode.
Nos. 16, 17, 19, 20, 21, 23, 26, 27, 28 do. 2nd and 3rd mode.
Nos. 18, 22, 24, 25 do. 3rd mode.
Nos. 6, 8, 10, 12, 13, 14, 15, are applicable to all the modes.

Table showing the position of the chromatic harmonics 23, 29, 31, 37, 41, 43, *in respect to their diatonic relatives and resolvents.*

Nos.	Diatonic Harmonics.	Chromatic Harmonics.	Diatonic Harmonics.
1	5	31	1
2	5	31	1
3	1	29–5	31–1
4	19	29–5	31–1
5	19	29–5	31–1
6	5–7	29–5	31–1
7	5–7	29–5	31–1
8	3–7	29–5	31–1
9	19	13–7–29–5	31–1
10	19	13–7–29–5	31–1
11	11	13–7–29–5	31–1
12	5	23	3
13	1	11	23	3
14	1	11	23	3
15	19	11	23	3
16	1	43	11
17	1	43	11
18	19	43	11
19	3	37	19
20	1	37	19
21	19	17–3–37–19
22	11	17–3–37–19
23	11	17–3–37–19
24	5	41	1
25	5	41	1
26	5–41–7–43–11–5
27	19	5–41	19

401. Of these examples:—

Nos. 1, 2, 6, 7, 25 are especially applicable to the 1st mode.
Nos. 19 do. 2nd mode.
Nos. 11, 16, 17, 18, 22, 23 do. 3rd mode.
Nos. 8, 13 do. 1st and 2nd modes.
Nos. 4, 5, 9, 10, 15, 20, 21, 27 do. 2nd and 3rd modes.
Nos. 3, 12, 14, 24, 26 are applicable to all the modes.

402. An example, in musical notes, of most of the particulars set forth in the above Tables, will be found in ex. 41 to 74. In the 1st Table, for instance, the example answering to No. 3 is 47—that answering to No. 16 is 53. In the 2nd Table, the example answering to No. 6 is 54—and that answering to No. 9 is 55.

403. That the particulars here pointed out in reference to the laws relating to position are abundantly sufficient to establish the truth of the doctrine laid down concerning it, § 399, and

also to prove the universality of its influence, as the cause of the effects peculiar to harmonics, both generally and specifically, no one, I presume, will venture to deny. Nevertheless, its application to practice can only be dimly perceived through the medium of such detached examples as those of 41 to 74—and though good examples, consisting of entire phrases, will tend greatly to strengthen the conviction that it is unquestionably true, the student is, nevertheless, earnestly requested not to shrink from the trouble of entering into a close investigation of one and all the particulars presented in these two Tables, that his ideas relating thereto may be rendered clear and familiar.

404. If in the idea of a system of sounds the mind be restricted to that of a simple system, the basis to every successive system must be looked upon as the parental sound, whence alone all the other sounds are derived. But if because the sounds appertaining to three simple systems are constantly united in practical music, we think proper to unite these three simple systems into one, whereof the two extremes are correlatives to the central system, and the affinity of both the nearest possible to that central system—it is evident that the individual root to neither of these branch systems can constitute the center to the original simple system—and the fact that our ears will not admit of this in practice, proves the folly of that philosophism which attempts to make it otherwise. This may be illustrated as follows—the mathematical idea of a center is that of a point without magnitude—but the practical idea of a center is that in which a relatively small circumference is included, the center to which is also the mathematical center. Moreover, a little reflection will show that, in so doing, the true mathematical center is not disturbed or transferred to any point of the periphery relating to the area included in the idea of the practical center.

To assist the imagination in regard to this matter, let the speck in the centre of fig. 33 represent the mathematical center, both to the larger and smaller circle by which it is surrounded —and the points *a*, *b*, the two attached centers appertaining to the triune system. Now, though in a practical point of view, the whole of the area of the smaller circle, including these three centers, may be looked upon simply as a center in respect to the area of the larger circle—nevertheless, when reference is especially made to the mathematical center, it is evident the area of the smaller circle, with its two centers, *a*, *b*, is then rejected.

That the case in regard to the three bases of the triune system is in every respect similar to this, is evident. For though practically, it is useful and expedient to look upon the simple central system in conjunction with its two correlative simple systems as three distinct systems, it is evidently incorrect to do so when reference is made to it individually, as a triune system, or to its central simple system only. To sum up the whole in few words, we must not mix together, and thereby confound, the two essentially distinct ideas relating to the mathematical and practical center, whether in relation to space, or to the three bases of the triune system.

405. In figs. 3—12, and again in figs. 30—32, the entire length of the string is represented by the two points A, B, and the nodal points by the numerals as before. The vibrations represented by figs. 3—12 are all *simple*—by fig. 3 the basis only, by fig. 4 the harmonic 2 only—but the vibrations represented by figs. 30—32 are *compound*—by fig. 30 the basis in conjunction with the harmonic 2—by fig. 31 the basis in conjunction with the harmonic 3—and by fig. 32 the basis in conjunction with the harmonic 5.

406. By attending to the circumstances relating to simple and compound vibrations, it will be seen that the node 1, which is absolutely at rest in fig. 4, is only relatively at rest in fig. 30.

Now as the vibrations of the harmonic 2 are 2 to 1 in respect to those of its basis, it follows that during the semi-vibration of the basis from the point 1 above (fig. 30) to the point 1 below, the vibration relating to the harmonic 2 will have made a complete revolution. If then the dotted line above the center line be understood to represent the form of the string at starting, the dotted line below will represent its form after a semi-basal and a complete harmonic vibration. On the contrary, if the plain line represents the form of the string in one position, it will also do the same in the other.

407. In respect to fig. 31, the case is similar, except, that in this instance, as the harmonic makes 3 vibrations while the basis makes 1—and consequently 3 semi-vibrations, while the basis makes 1 semi-vibration—it follows, that in every instance in which the harmonic is an odd number, the line, whether dotted or otherwise, representing the string in its elevated position, will assume the opposite form in its depressed position.

408. As the harmonic 5, fig. 32, vibrates five times while the

basis vibrates once, it is evident that five semi-harmonic vibrations must also go to one semi-basal vibration.

409. It is commonly supposed that the vibration of the basis may be simultanously accompanied with a variety of harmonic vibrations; but a moment's reflection will show that this is impossible, since the nodes of one harmonic must necessarily interfere with those of another, in every case, except that in which one harmonic is a sub-multiple of another. For example, the harmonic 2, and 3, cannot possibly vibrate simultaneously, either with or without the basis—but the harmonic 2 and 4, 2 and 6, 3 and 6, 3 and 9, &c., may do so, because 2 is a submultiple of 4 and 6—and 3 is a submultiple of 6 and 9.

410. Fig. 34 presents a magnified pattern of the form of the vibrations of a given point of a string,* during the production of the basal sound in conjunction with its harmonic 3. In this figure, the thin outer circle represents the course of this given point of the string answering to the basis only, and the thick elliptical form within, the course of the same point, as the basis and the harmonic 3.

Now, since it can be shown that the deflection from the simple basal circle in respect to any given harmonic is in the inverse ratio of the square of that harmonic, it follows that in this instance the relative diameters of the basal and harmonic circles are as $3^2 = 9$ to 1. The diameters of the smaller circles which accompany the perephery of the larger, and which are here represented twelve times over, and each time divided into 4 equal parts, are all drawn in agreement with this ratio. This will appear by reference to the line with the divisional points 1—9, commencing from the center of the large circle. But since the vibrations of the basis, and its harmonic 3, are here understood as acting conjunctively—the elliptical line can represent neither the one nor the other simply, apart from the effects produced from the other. This being the case, it follows that if the point on the left at 9 is that from which the string is supposed to start, it must touch the corresponding point 3 times in the course of its basal circuit, in agreement with the 3 straight lines drawn at equal distances from the center to the circumference—and hence, also, it follows that this particular

* What is here said is applicable to every part of a vibrating string, except its nodal points, but more particularly to the central position between the nodes.

point of the string must, in the course of its revolution, coincide in a regular and orderly manner with the points answering to the 4 quarters of the smaller circle, and in so doing produce the elliptical thick line by which the real course of the string, under such circumstances, is represented.

411. What is here said in regard to fig. 34 is equally applicable to fig. 35, except that herein is represented the vibrations answering to the basis and its harmonic 5—in which case the deflection of the string at the given point is as $5^2 = 25$ to 1—and the points answering to the 4 quarters of the harmonic circles visited, in regular order, 5 times during the course of the circuit appertaining to the basis.

412. By attending to fig. 34, it will, moreover, be seen that, in the production of the harmonic 3, the string has been deflected, or undergone collapsion, $3—1 = 2$ times—for the harmonic 5, the same thing has occurred $5—1 = 4$ times, and consequently for the harmonic 6 it will collapse $6—1 = 5$ times—for the harmonic 7, $7—1 = 6$ times, &c., universally.

413. A little consideration will also make it evident that the accompaniment of harmonic sounds, in conjunction with the basis, proceeds from want of the power to maintain the distended position in full vigour and activity. That this is fully verified by experience, must be well known to every one conversant with such matters—vibrating strings being disposed to yield harmonics in conjunction with the basal sound, in proportion to their increased length, smallness of diameter, and want of tension. After what has been pointed out, it is almost needless to remark that, in the production of harmonic sounds in addition to the basis, every portion of the string concerned in realizing the effect, instead of having farther to travel, has less space to pass over than for the production of the basis only—and moreover less and less as the harmonic produced is lower—and, upon examination, this also will be found to agree with experience.

414. The arc of the harmonic 3, in respect to its basis, being as 1 to $3^2 = 9$—the *motion* appertaining to its individual vibrations must, of course, be also as 1 to 9. But as its vibrations are 3 to 1, its relative force or loudness is in all probability 1 to 3—and upon the same principle that of the harmonic 5 as 1 to 5.

PART 20.

ACCENT.

415. That extra force which, in the performance of a musical phrase, is given to some notes more than to others, is called accent.

The term accent ought, properly speaking, to have only a relative signification—so that the very same notes which are accented in reference to those which are weaker, are un-accented in respect to those which are stronger.

The point here contended for is, that *time*, *pitch*, and *force*, in due subordination and co-ordination to the laws of proportion, are all required in the correct performance of a piece of music of real merit. The particulars of these laws in regard to time and pitch are all portrayed with sufficient accuracy, but not so in regard to force—but this does not disprove the real existence of similar varieties in respect to accent, or degrees of force, in a correct and impressive performance.

416. By means of accents, the phrases and sub-phrases become duly separated the one from the other, the effects of which conduce to brilliancy, perspicuity, and the perception of order. Moreover, accent being thus opposed to monotony, imparts to monotony itself a charm which, by way of variety and contrast, is sometimes wanted.

417. Many persons try to persuade themselves and others that what they call emphasis is distinct from accent; but when called upon to point out in what the difference consists, they fail so to do. But since in this, as in many other instances relating to distinctions without a difference, there is generally something to which a tendency to mislead may be referred, I shall here endeavour to point out what that something is, in respect to the synonymes, accent and emphasis. In the first place, then, it is a known fact that if any one listen attentively to the ticking of a watch, under the superinduced impression that every alternate tick, every third tick, or every fourth tick, is louder than the others, and therefore accented, it will appear to continue so to do, although it is evident that in this case the notion is purely imaginary. But the mind, having been predisposed, or having, by an effort of the will, predisposed itself into the belief of a periodical accent, is thereby also predisposed to flow on and continue in the same state, until resolutely forced out of its reverie by the perception of a different order of things. Just

so it is in regard to accent—an imaginary as well as a real impression, is quite sufficient to retain the mind in any one fixed state or idea—but when it becomes necessary to awaken its semi-dormant faculties to some fanciful, lively, or bold and unlooked for change, it is evident that this will require the exertion of more or less force, because in this case it is required to establish a new law of order in opposition to some other. Now, in a conflict of this kind, the force employed is commonly called *emphasis*—but when a force, exactly similar in kind, whether the same in degree or not, is employed, *not* in the way of opposition to any other force, it is then called *accent.*

To sum up all, in few words, as accent and emphasis both relate to degrees of force, or energy, and not to pitch, time, or quality of tone, it is evident they are both ultimately referable to the same idea, and consequently mere synonymes.

The particulars adverted to respecting the proportions of time which the phrases and sub-phrases bear to one another, and also the peculiar order in which the various accents are disposed, and, as it were, balanced, one against another, are commonly classified under the term rhythm.

418. In some examples which follow, the degrees of accent are represented by means of large and small dots, placed under the notes to which they refer—the large dots being placed where the greatest force or strongest accents are required—and the small dots where the accent should be weaker.

By this arrangement the small dots represent a mean force between two extremes—namely, unaccented notes, as distinguished by the absence of any dots, and strongly-accented notes.

419. In respect to the composer, the first point for consideration is to place the bars in the positions best adapted to show where the regular recurrence of the accent is most frequent, and consequently most orderly, and then to mark the places where this monotonously uniform series is interrupted, changed, and varied.

420. The symbols commonly made use of for this purpose, and which serve as guides for the performer, are > ∨ *fz*, *sf*, *ff*, &c.—to which may be added the slur, the effect of which, in such cases is, to displace the accent, and throw it forward to another part of the bar.

421. It is is evident that the object of the bar is to divide the music into equal portions of time, so that the accent may, *in the*

majority of instances, but not always, coincide with it. Hence it follows that the bar is an instrument in the hands of the composer, whereby he is enabled to direct the performer concerning the *general course* of the accent—the symbols above enumerated being employed to mark the aberrations from the monotony of the bar-accents, which cultivated taste continually demands, by way of relief, and for the sake of variety and expression.

422. In the performance of music, though not expressed in written or printed characters, the first note of every phrase should be more or less accented—and if it be required pointedly to distinguish the sub-phrases, then the first note of each of these must also be more or less accented.

423. That the first note of every phrase should be always more or less strongly accented, no one can deny who pays attention to the subject by reference to his own feelings in respect to the effect required—although this may not appear to be the case to those whose minds are stuffed with a jumble of confused ideas relating to the bar, as if music existed for the sake of the bar, and not the bar for music. In short, the very fact of a note's being the first of a phrase—that is the first after a repose—or in consequence of a real and perceptible separation from that which has immediately preceded it, renders it accented—since the weakest sound, in respect to silence, may be said to be accented—and the degree of force in this case, as in every other, should of course be regulated consistently with the general spirit of the piece.

Where grandeur, sublimity, or elegance is required, the sub-phrases ought not to be too pointedly, or too frequently, accented—while, on the contrary, in pieces of a light and playful character, it imparts brilliancy and vivacity. When, for instance, one of the three notes in triplets are accented, which ought not to be so, it imparts to such passages a certain common-place and vulgar effect—and it is worse still, when, under similar circumstances, in common time, every alternate note is pointedly, or even in the slightest degree, accented. In general, it requires more care to avoid falling into a regular doggrel course of accents, than to mark them in their proper place. The various and contrasted effects arising from the well-timed use of accentuation, from its most violent extremes, through every gradation down to that which is scarcely perceptible—and from that to the most perfect equality, without any

accent at all, is one of the greatest beauties in music, and one that is seldom acquired.

424. In melodies relating to the binary or common measure, any portion consisting of 2 equal parts or members, will admit of being accented on either, or both of these members—or both members may be passed over without being accented.

425. In ex. 181 we have an alternate series of comparatively strong and weak accents, and in the intervals between these sounds, others, which being the weakest of all, are said to be unaccented. These varieties are here and elsewhere represented by the larger and smaller dots, as stated above. But although this effect is pointed out by the position of the bars—with respect to the author the bars are subservient to the intention or desired effect, and not *vice versa*—while to the peformer they serve to prevent misapprehension in respect to the feeling, and correct reading of the author.

426. In order to change the course of the accents, by substituting the weak for the strong, and *vice versa*, the position of the bars must be changed, and the additional note at *, ex. 182, must be introduced, that the strong accent may still fall upon the last note, and thereby complete the series of eight instead of seven periodical strong accents. Thus, by means of the additional note at *, the close is, as it were, postponed, that it may fall in with the periodical series of strong accents. In ex. 181, the last note must be a semibreve, but in ex. 182 it becomes a minum, that the course of the major accents may, in both cases, continue to flow on evenly and without interruption. Furthermore, in ex. 181, the time appertaining to the last bar must, for the reasons already assigned, be in some way or other occupied before the repetition can properly commence—but in ex. 182 the circumstances of the case are not the same, and hence the repetition can take place without delay. Should the minor accent at *a*, in either of these two examples, be changed to the major, and thereby produce three consecutive major accents—the minor accent at *b*, must also be changed to the major, that three consecutive major accents may again take place, by way of response to the three former. In so doing, it will be seen that the three consecutive major accents in ex. 182, are in both instances, as it were, pushed forwards, in respect to the air, so as to agree with the orderly series pertaining to that example.

The reader must not suppose that ex. 182 is offered as an improvement upon the original by Handel—it is introduced merely

to show the circumstances relating to accent by reference to the same melody.

427. The slightest attention will show the great difference between the melody, ex. 183, and the former by Handel—and that in this, after the first and second notes of each phrase, the sounds are meant to die away gradually, sometimes without any accent at all, and sometimes with a series of minor accents. To play this melody with an equally strong accent upon the first note of every bar, in obedience to the rule commonly laid down, would have the effect of converting a beautiful air into a vulgar tune. Although the first note in each phrase is marked with the dot of the minor accent, it may be played as strong, or nearly so, as the following note, with the large dot—nevertheless, the second note must not be weaker than the first, otherwise it becomes either positively or relatively unaccented.

In the prestissimo movement the major accent falls upon the middle, and not upon the first part of the bar, simply because herein the bar contains twice the original quantity.

428. In ex. 184, the first note begins with the major accent upon the second (the weak) member of the bar—and to the words "that was slain," and again to the words "by his blood," we have three consecutive major accents.

429. In the varied and tasteful accentuation of the melody, ex. 185, the major accents fall upon the second member of the 1st and 5th bars only—while the minor accents take place at the interval sometimes of a crotchet and sometimes of a quaver, the latter being pointed out by the marks between the minor dots. In consequence of this procedure the accents fall sometimes upon the 1st and 3rd, and at others upon the 2nd and 4th members of the bar. In this example there are four phrases, as pointed out by the slurs above, in each of which the first note is accented in conformity with the rule § 419—and it will be perceived that these accents cannot be omitted without destroying the rhythm and sense of the passage. For though it is not necessary that these notes should be strongly accented, they, nevertheless, require to be accented in some degree. If the notes in the 3rd and 4th, and in the 7th and 8th bars be accented, like those in the 1st and 2nd, and in the 5th and 6th, the general effect is decidedly vulgar. When played as here marked, the accents in the 1st and 2nd bars are contrasted to those in the 3rd and 4th—as are also the 5th and 6th to the 7th and 8th bars—yet the unity of the whole is preserved, because

the accents of the 1st and 2nd bars correspond with those of the 5th and 6th, and the accents of the 3rd and 4th bars with those of the 7th and 8th. Furthermore, the 1st and 2nd phrases may be united into one, as may also the 3rd and 4th, but not the 2nd and 3rd—because the 2nd bar is evidently connected with the 1st and the 3rd with the 4th. The 1st, may, however, be united with the 4th, and thereby form one short but complete phrase.

430. In melodies relating to triple measure, any portion consisting of 3 equal parts or members will admit of being accented on any one, or any two of these members—or the three members may be all accented, or all unaccented.

431. In ex. 186, there are only two major accents, both of which take place on the *second* member of the bar. The minor accents should be all very faint.

In ex. 187, from the same sonata, there are no major accents at all—and the minor accents should not be too strongly marked.

432. In the first portion of ex. 188 there should be no accents in the treble, except the two first, notwithstanding its being marked $<$; and in the base, only one major accent, and this takes place on the second member of the bar. The second portion presents a series of major accents on the third member of each bar.

In ex. 189 there are no major accents, and the minor accents fall, first upon the second, and afterwards upon the first, member of the bar. By means of the three consecutive accents, pointed out by the slur over the three dots, the force of the 1st is, as it were, pushed on to the 3rd, in order to change the position of the series from the 2nd to the 1st member of the bar.

433. The accents, as marked in the above examples, are not to be understood as representing the only good reading or interpretation of which the several passages are capable. It is, however, presumed that the course here pointed out is, in every instance, consistent with the composer's views, in so far as this can be ascertained.

434. In melodies relating to quintuple time, the portions consisting of 5 equal parts or members, will admit of being accented on any one, two, three, or four of these numbers, provided these accents flow on in an uninterrupted series from first to last, but not otherwise. Or, these five members may be all accented, or all unaccented.

There are, therefore, *six* varieties of accentuation, applicable to quintuple time, of which the following is a representation—(1) . . . . . (2) . . . . . (3) . . . . . (4) . . . . . (5) . . . . . (6) . . . . . In these examples a period is in no instance interrupted, and thereby split into two, as in the case of (1) . . . . . (2) . . . . . &c., which represent an alternate series of triple and common, or of common and triple measures, which in itself is of a nature widely distinct from the quintuple measure—and in practice inadmissable.

435. The reason why the accents should not be interrupted in quintuple measure is, because the unity of the phrase is thereby destroyed, and the mind misled and impressed with the idea of two distinct and unequal portions, instead of one only. Authors, therefore, who have written, alternately, a bar of three crotchets or quavers, and the next with two, as specimens of quintuple measure, have totally misunderstood its nature.

436. It continually happens, in the harmony of two or more parts, that the parts which proceed with notes of longer duration appear to come to a close before the others—and it is the same whether these slower notes be held on, or played staccato, with or without the intervention of rests. Now, the cause of this apparent irregularity is simply owing to the circumstance that, in respect to time, a phrase is understood to extend from end to end, that is, from the commencement to its last extremity but the accent only from the commencement to the last accent —that is, from the moment of the first to the last act of percussion. See ex. 190. Here as the last base note is held on to the end, both parts terminate at the same time, although the treble runs on after the last note in the base has been struck. In ex. 191, though the last accented note in the base is not held on, the mind is nevertheless kept in suspense—and, as it were, occupied, even in respect to the base itself, until the treble is also brought to a close.

437. Portions of 5 measure notes, that is of 5 bars, appertaining to a long phrase, do not properly relate to quintuple measure—because herein the bar, that is the bar measure, does not constitute the phrase—and though the phrase is divided into 5 parts, the bar is not. Moreover, in this case the phrase approximates more to that which corresponds to a sentence; the progress to the completion of which being marked by the subdivisions of its 5 primary divisions, are therefore perceived to

develope itself gradually, and in an orderly manner, from first to last. See ex. 192.

438. Ex. 193 is a beautiful specimen of a passage consisting of 7 bars, followed by another also of 7 bars. These bars are then repeated without interruption, and thus give rise to a series of $4 \times 7 = 28$ bars. Such examples, though of rare occurrence, serve to prove that musical phrases are not of necessity confined to the powers and products of the numbers 2, 3, or of even 2, 3, 5.

439. There are but few examples of quintuple measure in existence, and, I believe, none at all of septuple measure. As to measures of 11, 13, and other primary divisions of a phrase, the thing is quite out of the question. For though these primes can have place as subdivisions of the members of a phrase, because in this instance the entire series is passed over with considerable, if not with extreme, rapidity—the case is otherwise when the time of the entire period is in any considerable degree increased, because, in this case, the mind is naturally, and, I may also say, irresistibly led to form such groups into a confused admixture of twos and threes—that is, of portions derived from common and triple measure, and in so doing entirely lose sight of the primary divisions which it is intended to represent. Another consideration is, that since nothing conclusive and definite can be expressed, during the interval of any primary ratio, till the entire group is completed, the time occupied by a phrase involving a high prime, would exact an effect of attention, alike painful to the hearer and performer. The law of nature in this is therefore the same as in the case of vibrations—for although the lower primes can be used in the absence of the higher, the higher cannot be used without the lower.

440. It is evident that what has been stated above is perfectly consistent with what we experience in regard to the ratios of vibrations, and the ratios which relate to time in respect to the notes, which being comparatively slow, and consequently more external, are therefore much more simple.

The same remark is also applicable to the ratios of chromatic harmonics, in respect to those denominated diatonic, &c.

From the extreme rapidity of the vibrations of sounds, especially in the treble, we can easily discern a ratio in which the entire cycle is repeated several times in a second; but had we to wait sixty times as long, that is, minutes instead of seconds the attention would get weary, and either become dormant

from sheer fatigue, or be diverted to other objects. In proof of this, let any one at all conversant with such matters, ask himself —why in respect to the measure we are almost entirely confined to the primes 2 and 3? For where, except as curiosites, do we find instances of the bars being divided into 5 or 7 equal parts? And as to 11, 13, 17, and the higher primes, they never have been, and never can be, practically introduced as the primary divisions of the bar.

Now, whoever is attentive to the operations of nature, will perceive that, in respect to her general outlines, she is uniformly more simple than in her subdivisions—and that it is so with the organized substances of the animal, vegetable, and mineral kingdoms. Moreover, in music, as in other things, neither the lower nor the higher primes are cast aside, simply because they cannot be introduced where they would be out of place, useless, and inconsistent. The prime 2, for instance, is not rejected in the structure of the human frame, because a man has but one head; nor the prime 3, because he has only two legs, two arms, two hands, two feet, two eyes, two ears, &c.—for many of the members of his body are divided into 3, others into 5, 7, &c., parts. But let us return to music, and show, by way of example, the consequence of having a bar, the primary division of which is into 5, 7, &c., equal parts.

A bar divided into 5 parts is liable to be mistaken, and especially at first, for the barbarous admixture of an alternate series of the primes 3 and 2—and certain authors have even looked upon it in this light, marked the measure thus, $\overset{5}{\frac{3}{4}\,\frac{2}{4}}$, and then written in conformity with this idea. But this hobbling from the prime 3 to 2 has nothing to do with the genuine quintuple measure, or any other division of the bar resulting from a prime, since, in order that any primary division may be perceived as such, it must not be represented as being compounded of others which are lower. In triple measure, for instance, the bar ought never to be represented, felt, or thought of as $\overset{3}{2,\,1}$—that is, as being compounded of two bars, and the principle here pointed out is equally applicable to all other primes.

From what has been said it will now be seen that, instead of

a genuine septuple time, arising from the protracted period of the prime 7, the feelings of the auditor would continually lead him to form to himself an uncouth admixture of minor portions, such as

7 7 7 7
3, 3, 1—3, 1, 3—2, 3, 2,—3, 2, 2, &c.

The only way to prevent this in respect to the comparatively low prime 7 would be the uniform occurrence of a strong accent at the commencement of every bar—in which case, if the time allotted to the bar was short, the attention might be kept from going astray during the anxious intervals between these strongly-accented notes. But, in respect to the higher primes, even this expedient must fail, as the intervals between these crashing accents would be too long for their regularity to be perceived, and their recurrence anticipated. As a matter somewhat connected with this subject, it may be expedient here to remark, that ignorance in regard to these things has led some to imagine that the higher primes can be used without the lower, and that the near approximation of some very high prime to a basis, or one of its octaves, is by nature adopted instead of that basis, as an harmonic to another basis. In other words, that nature, finding herself in a great strait in regard to certain sounds of the diatonic scale, is driven to the miserable expedient of resorting to a contemptible sophistry. The points especially alluded to are these—commencing with the false assumption that the key-note (C for instance) is, *or ought to be*, the basis to all the sounds of the diatonic scale, a difficulty arises in respect to the notes F, A, which it is admitted are neither of them an harmonic to C—and that so far is this from being the case with the note F, that, on the contrary, F is the basis to C, and C the harmonic 3 to F. In this dilemma what is to be done? The scale fabricator is, of course, infallible, and therefore refuses to listen to the imperious decrees of nature—his motto is—if C be not the basis to F, it ought to be. Knowing, however, that the claims of nature still command attention, here and there, an apologetic sort of theory has been kindly submitted to her consideration by way of defence, to the effect that though she cannot, in strict truth, obtain F, the true fourth of the scale, as an harmonic from the key-note C—she can, nevertheless, by means of a very high prime, get as near to F as she may think proper, if she will but agree to sign the treaty of peace thus condescendingly presented for her acceptance. But, in so doing, she must overlook the

matter relating to time, in respect to the cycle appertaining to that prime—as to whether it be a year only, a century, or a century of centuries, before we may again anticipate the pleasure of recommencing the series—and also agree to forego asking the awkward questions—(1) Why that prime in particular? (2) What use is to be made of the lower primes which also hover about the same note F? (3) Is the note strictly in tune according to that high ratio? (4) Why not, by the adoption of a still higher prime, get still nearer to the true sound? &c.

PART 21.

IMITATION. FUGUE. CANON.

441. Passages which resemble each other in their general form and construction, but in which the intervals are not necessarily the same, produce what are called *imitations*—and pieces in which imitations abound, are continuously carried on, and skilfully mixed up together, are denominated *fugues*. And when the interval by which the imitations are produced are presumed to be precisely the same as that of the original type, or pattern, first presented to notice, the composition is denominated a *canon*.

The term fugue, from the Latin word *fuga*, signifies to *fly*—because in such compositions, whether vocal or instrumental, one part leads off with some determined succession of notes, after which the parts appear to be continually flying, or running after each other.

442. The form, melody, type, or pattern for imitation, first presented to notice, is called the *subject*. Two or more subjects are also frequently united in the same piece—sometimes in the same, and sometimes in the inverted form of the original—sometimes with notes of the same, of half, and of twice the original length.

443. From what has been said it will be seen that imitation, fugue, and canon do not relate to matters in which nature, or the laws of nature, are immediately concerned, as is the case with melody and harmony. Imitation is, indeed, more or less perceptible in all good compositions, however free the style may

be—and hence it follows, that no check is offered to the free inspirations or fancy of any author by the mere *ad-libitum* introduction of imitations. But the case is otherwise with fugues and canons, which are esteemed and admired in proportion to the closeness of their texture, strict adherence to artificial restraints, and purity of harmony.

444. But though fugues and canons are not the fruit of any higher or more complete development of the laws of nature, the study and practice of such compositions is nevertheless of great importance, on account of their tendency to keep the mind from going astray, and thereby to preserve unity and impart strength and power, and also as affording the means of displaying masses of harmony in varied and perspicuous positions. Under judicious management, free from the contamination of a certain twang, peculiar to scholastic compositions, the result of bad harmony, the effects produced are refreshing, bold, vigorous, and inspiring. But the snarl proceeding from the continuous entanglement of a long series of perplexed harmonies pertaining to no one key in particular, presents an alloy but too frequently found in such compositions.

It is of the highest importance to bear it in mind that however interesting a dry, tasteless composition of this kind may be as an object of curiosity to *look* at, and investigate—considered as music, that is, as something addressed to the ear, it is valueless.

445. Now, though there really are specimens of fugues, in which taste and genius, the offspring of the affections, is found united with admirable contrivances, the offspring of the intellect, and both together flowing down a rich stream of pure harmony, it must be admitted, that in the majority of instances, this is not the case. Wherefore, let the student who would write for the ear, and not for the eye, constantly bear in mind the following precepts:—

(1) Never waste time upon an unmeaning or uninteresting subject.

(2) If two or three subjects are to be united in the same piece, take care that they so differ in character that one is adapted to aid and enliven the other.

(3) Never suffer a composition of this, or indeed of any kind, to toil through its whole extent without distinct and perceptible breaks—wherefore, as in a good painting, be careful to maintain a very broad, grand, and bold outline.

(4) If the composition be of a grave cast, let it not sink into drowsiness and insipidity—if of a lively cast, let its sportive character be based upon something simple and comprehensible.

(5) Be very careful not to spoil what may called the melodic flow resulting from the harmony of the severed parts, by the constant maintenance of holding notes, and thereby of compound harmonies, where a more simple harmony would be better, and afford the mind an occasional relief from its otherwise unceasing efforts to disentangle the intricacies incidental to such harmonies.

(6) Look very carefully to the harmony, and let nothing operate as a temptation to suffer it to remain impure. Recollect, that to maintain the subject form, at the expense of the harmony, is to do honour and reverence to a mere conceit, at the expense of the laws derived from nature.

446. In respect to canons, except those in the octave, and except where, by the intervention of breaks, by means of rests, for the purpose of preventing objectionable portions of the subject from coming together, *their existence in conjunction with pure harmony is impossible.* That is, in every other instance, except those above enumerated, either the canon is in two keys at the same time, in which case the harmony cannot be good, or the intervals of the subject melody are not exactly the same, in which case the so-called canon is not a canon. That this is the case I now proceed to prove by reference to that celebrated canon, and time-honored harbinger to good dinners, '*Non nobis Domine.*' See ex. 194.

447. In this example, if G be the key of the treble and base, for the intervals to be exactly the same, and the assumed canon a real one, the tenor must be in D. But, upon the supposition that all the parts are in the key of G, then the notes in the tenor, which, in respect to the melody, or subject, severally answer to the notes in the base and treble, are—

		*		†			‡	§	
Tenor - - -	▽ D	5 E	5 F	△ G	3 A	5 B	1 or 7 C	♯C	▽ D
Base and Treble	△ G	3 A	5 B	1 or 7 C	▽ D	5 E	7 or 19 F	5 ♯F	△ G

in regard to which it is to be observed that the note E in the

tenor at *, does not truly respond to the note A in the base and treble, because the ratio of the 5th, in this instance, is $\frac{80}{27}$, instead of $\frac{81}{27} = \frac{3}{1}$. At †, we have the single fixed sound, G △, in the tenor, in answer to the two sounds, C 1, C 7, in the base and treble—G △ being the correct response to C 1, but not to C 7. At ‡, though C 7 is the correct response to F 7, it is not so to F 19—nor C 1 to F 7, or F 19. At §, the true response to ♯F 5 in the base, is ♯C 5 in the tenor; but ♯C 5 appertains to the key of D, and not to the key of G. How the matter stands in respect to the individual sounds of the melody of this canon will be seen by the following example, wherein the parts are set one over the other, and the harmonics marked, where the melody in the tenor does not correspond with the base and treble, and also where the melody in the treble is at variance even with the base. See ex. 195.

448. Now, it is to guard against the necessity of getting out of the key, or rather of being obliged to render the duality of the key-notes apparent, and thereby of exposing the sophistry of the doctrine hitherto set forth and entertained concerning canons, and the strict laws to which they are subject, that the introduction of the note ♯F in the treble and base is avoided, that ♯C may not be called for in the tenor. That all this is very clever in its way, I do not dispute, although I still adhere to the opinion that nature presents a more noble, honorable, and useful field for study than the clap-trap manœuvres relating to any cunning artifice whatever. Such matters have no real connexion with science, not even with its outermost husk or shell—and the evil consequences which invariably attend every dereliction from the path prescribed by nature, must be felt with regret by every one conversant with this subject, and the history of the lives of those whom it most especially concerns.

449. But let us return to the canon, ex. 195. In the 2nd, 5th, 13th, and 18th bars, E 5 is made to respond to A 3, to which, as has already been shown, it is not a perfect 5th. In bar 3, the second note in the treble is properly C 7, but in the base the corresponding note *must* be C 1, and cannot be C 7. In bar 10 we have ♯F 7 in the treble and base, the response to which, in the tenor, is C 1, and not C 7. In bar 16 we have C 1, but more corecctly C 7 in the treble, the response to which, in the base, if we look to the sounds above, ex. 194, bar 19, is C 1, and not C 7.

450. Considered merely as a specimen of harmony, the less

that is said about this canon the better. It is hardly fair to compare such an antiquated production with the choice specimens of more recent times. It has, however, like most canons, the merit of being short; and though I cannot conscientiously become an admirer of pedantic forms, I can appreciate the beauty of a simple, natural, and consistent melody, and fairly admit its claims to all these essential and most valuable qualities.

451. As it would be inconsistent with the object of this work to attempt to point out the numerous particulars relating to fugues and canons, in repect to their details, and in connexion with suitable examples—and as this sort of information is best obtained from observation and the investigation of the works of authors celebrated for such productions, the student is earnestly recommended to adopt this course, and in so doing to analyse several of those works, not superficially, but critically, first in respect to their general construction, as fugues or canons, and then in respect to the purity of the harmony. The errors he may detect in so doing will operate as a warning to himself, and thus afford the most salutary lessons he can possibly obtain.

PART 22.

FURTHER REMARKS UPON ANALYSIS AND COMPOSITION.

Use of the Transposing Tablet.

452. To show the value and importance of being able, upon every occasion, to recur to first principles, and also to show the simplicity and immutability of nature's laws, I here propose to point out the mode in which the Transposing Tablet, page 78, and the Symbolic Tablet in the next page, may be rendered practically useful. But since, in order to render the Transposing Tablet fully available for practical purposes, it frequently becomes requisite to have two copies, to the intent that while one copy is set to one triune system, another may be set to another triune system, in the manner described in § 224, and seq., I here present the student with duplicate copies, detached from the work, for that purpose.

453. The questions he is now required to answer, by reference

to the Transposing Tablet, are—(1) What is the note C in the key of C? The reply to which is—The key-note, or central-basis, and also the harmonic 3 to the sub-basis. (2) How is this note symbolically represented? Answer—By 𝟙, **3**, or △. (3) Do these symbols represent precisely the same, or different sounds? Answer—Precisely the same sound. (4) What is the note D in the key of C? Answer—The harmonic 3 to the super-basis G. (5) How is it symbolically represented? Answer—Invariably by the harmonic 3. (6) What is the note E in the key of C? Answer—The harmonic 5 to the central-basis C. (7) How is it symbolized? Answer—By 𝟝. (8) Does the note E in the key of C stand as the representative of any other quality? Answer—Yes, it also represents the sub-medial terminant to the sub-medial basis A **5**—and the central-medial basis to the central-medial terminant B 5. (9) What have you to say concerning the note ♭E in the key of C? Answer—That it is the harmonic 19 to the central-basis C, the symbolization of which is 𝟙𝟡—that it is also the sub-medial terminant to the sub-medial basis ♭A, **19**, and the central-medial basis to the central-medial terminant ♭B 19, § 128, and seq. (10) What is the note F in the key of C? Answer—It may represent either the sub-basis **1**, or the harmonic 7 to the super-basis G. (11) But, are the sounds F **1** and F 7 identical? Answer—No, F **1** is sharper than F 7 in the ratio 64, 63. (12) By what means do you determine which of these two sounds is the required sound in any given case? Answer—By attending to the rules in § 366 and seq. (13) What is the note G in the key of C? Answer—It is the super-basis and harmonic 3 to the central-basis. It is, therefore, symbolized by 1, 𝟛, and also by ▽. (14) But do the symbols 1, 𝟛, ▽ represent the same sound? Answer—They do. § 106. (15) What are the notes A and ♭A in the key of C? Answer—A is the harmonic **5**, and ♭A the harmonic **19** to the sub-basis F; A **5** is also the sub-medial basis to the sub-medial terminant E 𝟝—♭A **19** the sub-medial basis to the sub-medial terminant ♭E 𝟙𝟡. Moreover, A **5** is also the sub-medial basis to the central-medial terminant B 5—and ♭A **19** the sub-medial basis to the central-medial terminant ♭B 19, § 144. (16) What are the notes B and ♭B in the key of C? Answer—B is the harmonic 5, and ♭B the harmonic 19, to the super-basis G. B 5 is also the central-medial terminant to the central-medial basis E 𝟝—and B 19 the central-medial terminant to the central-medial basis ♭E 𝟙𝟡. B 5

is, moreover, the central-medial terminant to the sub-medial basis A 5—and B 19 the central-medial terminant to the sub-medial basis ♭A 19.

The reader must remember that though to all appearance we have been fixing his attention to the mere consideration of individual notes, this is not strictly true, since, according to the true science, the adoption of the spelling principle is the only sure and certain method of obtaining a correct knowledge of chords and of harmony generally—whence it follows that he who has correctly analyzed the elements of which any chord, or combination of sounds, is composed, has, in so doing, analyzed the chord itself.

454. If, however, instead of the notes C, D, E, ♭E, F, G, A, ♭A, B, ♭B, in the key of C, reference had been severally made to the same sounds as the key-note, the 2nd, major 3rd, &c., of the scale, what has been said above would have been found equally applicable to every key. This must be evident to every one—wherefore, let the student himself apply the principle to practice, by shifting the paper with the square hole from key to key, over the Tablet, page 78, until all that relates to its operation becomes clear and familiar to him.

455. As a farther application of the Transposing Tablet to practical purposes, let the student proceed to put to the test, one by one, all that has been said respecting the bases of chords in subserviency to different triune systems, in § 93—117, and in so doing let him continually contemplate the same apparent chord by reference to *two* Transposing Tablets at the same time, the paper covering being set to the two keys under consideration. The very sight of the various positions which such apparently similar, but actually dissimilar, chords will in such cases assume, in respect to different triune systems, will be highly instructing, and have a powerful tendency to dispel the mists of error and prejudice, should anything of the kind exist.

456. Pursuing the course already begun at § 453, I now ask (17) What is ♯C in the key of C? Answer—The harmonic 17 to the central-basis C, symbolized as ♯C 17, or the harmonic 13 to the sub-basis F, and therefore symbolized as ♯C 13. (18) What is the ratio of the vibrations of ♯C 17, ♯C 13? Answer—51, 52. (19) What is ♭D in the key of C? Answer—The harmonic 11 to the super-basis, and therefore symbolized as ♭D 11. (20) What is the ratio of the vibrations of ♯C 17, ♭D 11? Answer—34, 33. (21) What is ♯D in the key of C? Answer—

The harmonic 13 to the super-basis G, symbolized as ♯D 13. (22) What is ♭E in the key of C? Answer—The harmonic 19 to the central-basis symbolized as ♭E 19—or the harmonic 7 to sub-basis, symbolized as ♭E 7. (23) What are the ratios of the vibrations of ♭E 19, ♭E 7—♭E 19, ♯D 13—♭E7, ♯D 13? Answer—57, 56,—38, 39,—112, 117. (24) What is ♯F in the key of C? Answer—The harmonic 17 to the sub-basis, symbolized as ♯F 17. (25) What is ♭G in the key of C? Answer—The harmonic 29 to the super-basis, symbolized as ♭G 29. (26) What is the ratio of the vibrations of ♯F 17, ♭G 29? Answer—272, 261. (27) What is ♯G in the key of C? Answer—The harmonic 13 to the central-basis C, or the harmonic 17 to the super-basis G. (28) What is ♭A in the key of C? Answer—The harmonic 19 to the sub-basis F. (29) What are the ratios of the vibrations of ♯G 13, ♯G 17—♯G 13, ♭A 19—♯G 17, ♭A 19? Answer—52, 51,—39, 38,—153, 152. (30) What is ♯A in the key of C? Answer—The harmonic 11 to the sub-basis F. (31) What is ♭B in the key of C. Answer—The harmonic 19 to the super-basis G, or the harmonic 7 to the central-basis C. (32) What are the ratios of the vibrations of ♯A 11, ♭B 19—♯A 11, ♭B 7—♭B 19, ♭B 7? Answer—176, 171,—22, 21,—57, 56.

457. What has been said in reference to the sounds ♯C, ♭D—♯D, ♭E—♯F, ♭G—♯G, ♭A—♯A, ♭B, is, of course, alike applicable to every key, if, instead of these specific notes the ♯1st, ♭2nd—♯2nd, ♭3rd—♯4th, ♭5th—♯5th, ♭6th—♯6th, ♭7th—that is, if the intervals which correspond therewith be understood. It is also obvious that there are several other purposes to which the Transposing and Symbolic Tablets are applicable, which it is needless here to particularize. Enough has already been shown to convince any unprejudiced person that by their aid the foundation may be laid, in an exceedingly short space of time, for the reception of truth, and of truth only, in matters relating to music—to the branches of which it is impossible to assign any limits—and which is, nevertheless, adapted to the apprehension of all but mere infants. Let us hope that the impediments which have been placed, and still stand in the way of, children, will shortly be removed—and that the time is not far distant before it will be distinctly perceived, and universally admitted, that *spelling* is as applicable to music as to language—only ten times easier.

458. As I am now especially treating upon the analysis of

music, and having in my "New Analysis" distinguished the three simple systems of the triune system, the one from the other, by the three primary colors, it may be desirable to speak of it here as a mode of analysis by analogy. My reason for not using colors in this work is on account of the great difficulty and expense. My mode of applying colors is blue for the basis and harmonics of the sub-system—red for the central—and yellow for the super-system. Consequently, simple chords are thereby represented by one of the three primary colors, and compound chords by the various intermixtures of these colors. For instance, the chord 1, 3, 5, is a red chord—the chord of the added 6th, **1, 3, 5**, 3, a green chord, green being a compound of blue and yellow—the two apparent major and minor common chords 5, **3** or 1, 5—**19**, **3** or 1, 19, purple chords, purple being a compound of blue and red—the two apparent major and minor common chords 5, 3 or 1, 5—19, 3 or 1, 19, orange chords, orange being a compound of red and yellow—and **5**, 5, 3, a brown chord, brown being a compound of red, blue, and yellow. For a specimen of this mode of analysis, see ex. 196—201, wherein the letter B represents the color blue—R, red—Y, yellow—p, purple—o, orange—g, green—b, brown.

The following may serve to show the position which the compound colors, representing the compound chords, occupy in respect to the three simple or primary colors,

Y	Y	Y
o		
R	g	R b
p		
B	B	B

Though this symbolic method of distinguishing the sounds of the triune system is attended with many advantages, and particularly in its application to chords, I do not vouch that the analogy is true to nature.

459. By comparing ex. 196 with 198, and ex. 197 with 199, it will be seen that the mutations incidental to the mode, and to transitions from diatonic to relative chromatic, do not affect this analytic mode of analysis—and as the symbolic analysis to these examples is given in full, and does not vary with the key, it will be found of great benefit to acquire the habit of playing from them in all the keys, the time of the notes, &c., being the same as in the example above.

460. Ex. 201 is presented as representing the feelings of Mozart more accurately than ex. 200—and upon examination

it will be seen that, although the chromatic harmonics involved in ex. 201 are higher than in ex. 200, as the harmony in ex. 201 consists entirely of simple chords, which is not the case in ex. 200, the latter example is really simpler than the former.

Laws which regulate the human ear and mind, in regard to music, and to which many of the rules relating to practice are ultimately referable.

461. The ear, that is the mind by means of the ear, invariably resolves every problem relating to vibrations, consistently with its simplest interpretation, and consequently never puts a forced and unnatural construction upon anything. The ear perceives all ratios in their lowest terms, and this is not only without any mental effort, but entirely free from any consciousness of the very extraordinary and wonderful nature of the operation in the performance of which it is engaged. The ear invariably interprets imperfect, that is, false ratios, according to the meaning of true ratios for which they are substituted—marking, at the same time, with pain, all the discrepancies existing between truth and error, both in detail and in the aggregate. In making this assertion it is, of course, to be understood as being restricted to those cases only in which the difference between the imperfect and true ratio is not too great.

462. Any sound heard *per se*, without mental reference to any other sound, is adopted as a key-note, whether the sounds that follow confirm this idea or not—in like manner, any common chord, heard *per se*, without mental reference to any other common chord, is by the ear, and mind, adopted as the common chord to the *central* basis, whether the chords or sounds that follow confirm this idea or not. The reason is, that this note (the key-note), and this common chord (the common chord to the central basis) is in a state of equilibrium; and since nothing has yet occurred to destroy the peaceful serenity of this state, or to show that this is not the very center of all attractions and of all converging influences, the affirmative side of the question is, as it were, assumed, in preference to the negative. Wherefore, a series of sounds must be either unmeaning, or point to a simple or triune system.

463. As every chromatic chord determines the diatonic chord which is to follow, the key is thereby ascertained, in as far as these two chords are concerned, wherefore, if these two chords be sufficient for the purpose, the key is fully determined—but

if not, something more is required before this point is entirely freed from doubt and obscurity. For :—

464. Two major common chords, the basis to one of which is a 5th to the other, cannot, with certainty, determine the key, for the very simple reason that, in respect to the triune system, one may be the super and the other the central-basis, or one may be the central and the other the sub-basis. But :—

465. Two major common chords, the basis to one of which is a major 2nd to the other, determine the key with certainty, because in this case one must be the sub, and the other the super-basis, whereby the central-basis is also known.

466. Two common chords, the basis to one of which is neither a 5th, nor yet a major 2nd to the other, can belong to no individual triune system whatever, and are therefore irrelevant to the matter at present under consideration.

467. Two minor common chords, the basis to one of which is a 5th to the other, determine the key. For, as the super-basis demands the major common chord in the minor, as well as in the major mode, consequently, as neither are referable to the super-basis, they must be referred to the sub and central-bases, whereby the super-basis is also known.

468. Two common chords, one minor and the other major, determine the key. In this case, the major common chord appertains to the super-basis, whereby the sub and central-bases are also known.

469. A succession of common chords, some major and some minor, having no common bond of unity, can at best present only a pretty piece of patchwork, but to call this music is a misnomer.

470. Wherever, and by whatever means, the three bases of the triune system are clearly developed, of course the key is fully determined—and, in this case, but not otherwise, the foundation is laid for something which *may* truly be called music.

Further remarks upon the 3rd mode.

471. As from the circumstance of its gradual introduction, and, generally speaking, short duration, some persons may still entertain doubts respecting the reality of a 3rd mode, it will not be out of place to make a few observations relating to the subject. Now, although the three modes are equally indebted to nature for their elements and structure, it does not follow that they should therefore present equally voluminous fields of

matter, in the way of practical examples—or that because the more primitive formations of the 1st or 2nd mode is more consonant with our ordinary states of mind and feelings, and therefore of more frequent occurrence, the deeper and more mysterious emotions pertaining to 11, 111, 11, as diatonic harmonics, ought not to constitute a distinct mode—or that they ought not to do so, simply because such emotions are comparatively of rare occurrence, or because we are not constitutionally adapted to feast, for any great length of time, upon such excitable matter, without having recourse, every now and then, to something of a milder nature, by way of relief and repose. So far, indeed, are the points here alluded to from affording any real grounds for denying the existence of a third mode, that on the contrary, they serve rather to confirm the fact—for, were it otherwise, that is, if no reasons could be assigned why, in the order of their development, the 3rd mode should not have constituted the primitive, or first, and the 1st and 2nd subsequent formations, would not our entire ignorance of the cause, in respect to the order of their real development, afford a just ground for censure, and have a latent tendency to create doubts concerning other points of importance? The order of development is the same with individuals as with communities, the simple elements are perceived prior to the more complex—simpler combinations prior to the more elaborate—the elements of the diatonic system prior to the chromatic—and those of the 1st mode prior to the 2nd and 3rd. Nor does it ever happen that any new accession to what we had previously acquired is obtained otherwise than as an additional gift from nature, and consequently not in the way of exchange. Moreover, the first gifts of nature are invariably such as serve as a substratum for the reception of others—the diatonic, for instance, as a substratum for the chromatic—the 1st mode as a substratum for the reception of the 2nd, and the 1st and 2nd for the reception of the 3rd—and in no case the former in exchange for the latter.

472. If we watch attentively the course pursued by nature in the development of her three diatonic or rudimental systems, it will be found to consist in the construction of the major or 1st mode, by means of the primes 1, 2, 3, and of 5 exclusively—of the minor or 2nd mode, by means of the primes 1, 2, 3, and of 19 in connection with 5, and consequently not of 19 exclusively —and of the 3rd mode, by means of the primes 1, 2, and of 11, 19, in conjunction with 3, 5, and consequently not of 19 or 11 exclusively. In the formal representation of these three different

arrangements of the initiatory elements of the triune system, the object is to mark the differences between them, so that the elements peculiar to each may be distinctly perceived; but it is not from hence to be inferred that, in practice, they are in all cases kept distinct the one from the other, since this, in point of fact, is seldom the case. It is, indeed, true, that passages in the 1st mode are seldom tinged, as it were, with the harmonic 19, used diatonically, and never with 11; but, notwithstanding this, **19** is sometimes introduced instead of **5**, but never 𝟭𝟵 instead of 𝟱, nor 19 instead of 5. But in passages in the 2nd and 3rd modes, this rigid adherence to the elements peculiar to either of those modes will seldom be found. In examples in the 2nd mode, for instance, **5** is continually taken instead of **19**, and still more frequently 5 instead of 19, but never 𝟱 instead of 𝟭𝟵, except at the final close, and with the view to a particular effect. Now, if, practically, the diatonic elements peculiar to the 2nd mode, are scarcely ever to be found, except in conjunction with those of the 1st, we must not be surprised to find that the elements peculiar to the 3rd mode are also equally subject to the admixture of those which, in point of form, belong to the 2nd and 1st modes. Experience and observation, in reference to practice, will also bring to light the fact that the 3rd mode is frequently announced by the introduction of **11** only, without 𝟭𝟭 or **11**—and thereby, without interfering either with the mediants or terminants of the triune system—but never by the introduction of 𝟭𝟭, or **11**, without 11. A moment's reflection will also show, that in proceeding from **11** to 𝟭𝟭, and thence to **11**, we penetrate deeper and deeper towards the foundation of the system, and, in so doing, farther and farther from the open simplicity of the 1st mode. That is, the key being C, though by the introduction of **11**, instead of 3, we change the super-terminant D into ♭D, by so doing we neither displace the mediants A or ♭A, E or ♭E, B or ♭B, nor the bases F, C, G, nor the terminants C, G. But by the introduction of 𝟭𝟭, the central terminant G is made to give place for ♭G—and, by the introduction of **11**, the sub-terminant C is made to give place for ♭C.

That this constitutes the absolute terminus to this mode of procedure is evident; for were we to advance one step farther, the note to be changed would be F. But in the key of C, F is not a terminant—and to change the sub-basis F to ♭F, the semitone below, is to change the key, which has nothing to do with matters relating to the mode or manner in which the sounds appertaining to the same key may be treated in practice.

Position of the diatonic harmonics in respect to the next diatonic harmonic above and below.

The letters which stand over the symbolic numerals serve as a practical illustration, and refer to the key of C.

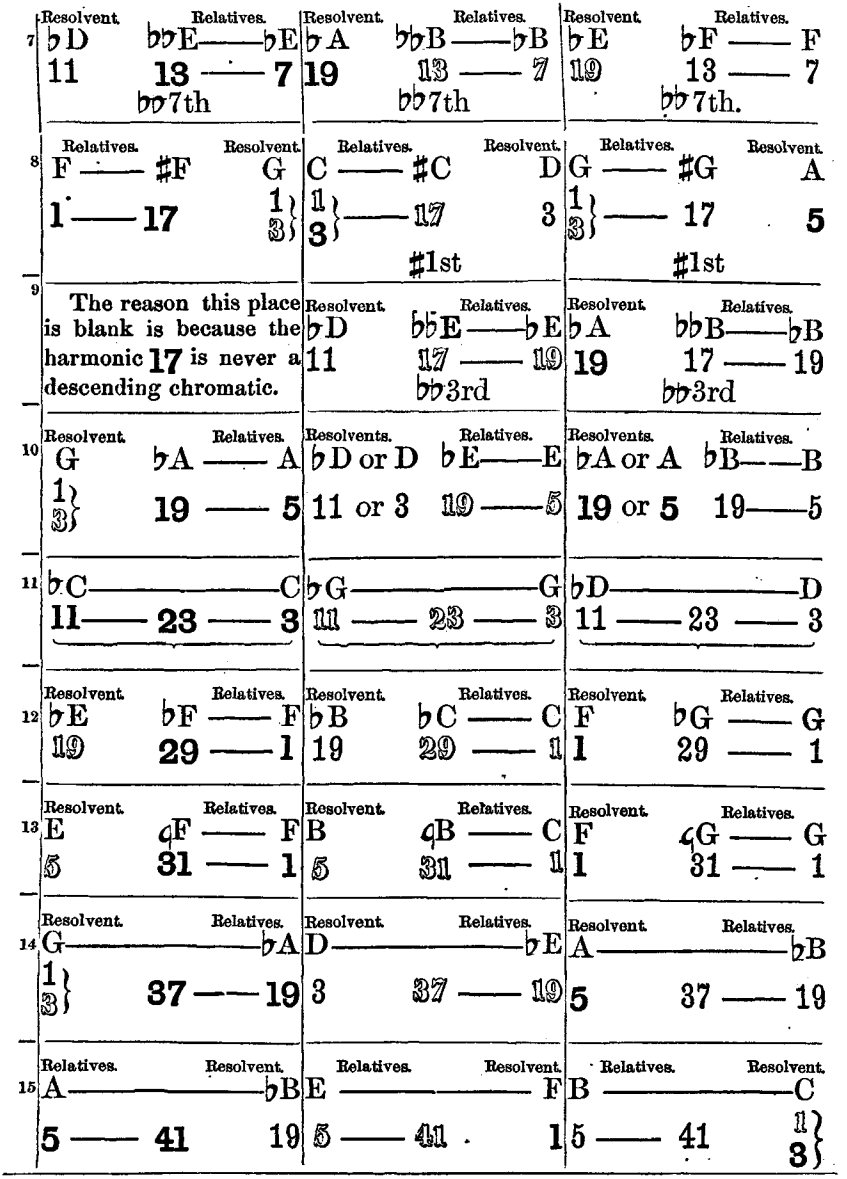
♭E or E . . F . . ♭G or G 𝟭𝟵 or 𝟱 . . **1** . . 𝟭𝟭 or $\frac{1}{3}$	♭B or B . . C . . ♭D or D 19 or 5 . . $\frac{1}{3}$ . . 11 or 3	F . . . . G . . . . ♭A or A **1** . . . . $\frac{1}{3}$ . . . . **19** or **5**
The reason why there are two blank places here is, because the sounds **3**, 𝟙, and 𝟛, 1, are identical.		C . . . . D . . . . ♭E or E $\frac{1}{3}$ . . . . 3 . . . . 𝟭𝟵 or 𝟱
♭B . . . ♭C . . . ♭D or D 19 . . . **11** . . . 11 or 3	F . . . . ♭G . . . . ♭A or A **1** . . . . 𝟭𝟭 . . . . **19** or **5**	C . . . . ♭D . . . . ♭E or E $\frac{1}{3}$ . . . 11 . . . . 𝟭𝟵 or 𝟱
♭G or G . . A . . ♭B or B 𝟭𝟭 or $\frac{1}{3}$ . . **5** . . 19 or 5	♭D or D . . . . E . . . . F 11 or 3 . . . . 𝟱 . . . . **1**	♭A or A . . . B . . . . . C **19** or **5** . . . . 5 . . . . $\frac{1}{3}$
♭G or G . . ♭A . . ♭B or B 𝟭𝟭 or $\frac{1}{3}$ . . **19** . . 19 or 5	♭D or D . . . . ♭E . . . . F 11 or 3 . . . . 𝟭𝟵 1	♭A or A . . . . ♭B . . . . C **19** or **5** . . . . 19 . . . . $\frac{1}{3}$

473. If we examine attentively the above examples, it will be seen that in no instance is any diatonic harmonic referable to the same basis as that of the next above on the right, or the next below on the left. §78. If, for instance, the diatonic harmonic in the center refers to the sub-basis, those next above and below are referable either to the central or super-basis. If the diatonic harmonic in the center refers to the central-basis, those next above and below refer either to the sub or super-basis. And lastly, if the diatonic harmonic in the center refers to the super-basis, those next above and below refer either to the sub or central basis.

Position of the chromatic harmonics in respect to the next diatonic harmonic above and below, and in the 3rd example, to the second diatonic harmonic above, and the first or second below.

1	Relatives. ♭C——C 11——3*	Resolvent. ♭D 11	Relatives. ♭G——G 𝟙𝟙——3	Resolvent. ♭A 19	Relatives. ♭D——D 11——3	Resolvent. ♭E 9
2	Relatives. ♭A——A 19——5	Resolvents. ♭B or B 19 or 5	Relatives. ♭E——E 𝟙𝟡——5	Resolvent. F 1	Relatives. ♭B——B 19——5	Resolvent. C {11 3}
3	Resolvents. ♭D or D 11 or 3	Relatives. ♭E——F 7——1	Resolvents. ♭A or A 19 or 5	Relatives. ♭B—C 7—11	Resolvents. ♭E or E 𝟙𝟡 or 5	Relatives. F——G 7——1
4	Resolvent. ♭B 19	Relatives. ♭C——C 11——{11 3} ♭5th	Resolvent. F 1	Relatives. ♭G——G 𝟙𝟙——{1 3}	Resolvent. C {11 3}	Relatives. ♭D——D 11——3
5	Relatives. A——♯A 5——11 ×3rd	Resolvent. B 5	The reason why there are two blank places here is, because the harmonics 𝟙𝟙 and 11 are never ascending chromatics.			
6	Relatives. C——♯C {11 3}——13 ♯5th	Resolvent. D 3	Relatives. G——♯G {1 3}——𝟙𝟛 ♯5th	Resolvent. A 5	Relatives. D——♯D 3——13 ♯5th	Resolvent. E 5

* It must be recollected that 3 is chromatic when 11 is diatonic, and 5 chromatic when 19 is diatonic.



474. In the above examples, when the relative diatonic is on the left, and the resolvent on the right, as in examples 1, 2, 5, 6, 8, 15, they are *ascending* chromatics—and, when the relative

diatonic is on the right, and the resolvent on the left, as in examples 3, 4, 7, 9, 10, 12, 13, 14, they are *descending* chromatics.

The position of the chromatic harmonic **7**, 7, 7, (example 3) is peculiar, the relative diatonic being neither the next note above nor the next note below, but the second note above. Hence, in the descent from the one to the other, the intermediate diatonic harmonics are, in every instance, passed over. It is, moreover, to be observed, that **7**, 19,—7, 19—and **7**, **1**, are not only identical upon keyed instruments, but the same in respect to their notation also.

In example 11 the harmonic 23 is a descending chromatic when 3 is diatonic, and an ascending chromatic when 11 is diatonic—the resolvents to 3 and 11 being the same as when the harmonic 23 is not concerned.

Upon the bases generated by slides.

475. With the view to remove all embarrasment from the mind concerning the basal sounds generated by means of a slide in one part, and a fixed sound in another—or by the union of two or more slides in conjunction with one or more fixed sounds—let it be remembered that, as the basis generated by any two or more sounds is in all cases the result of the coincident vibrations answering to the respective cycles of these vibrations only—and consequently *not* of the intermediate vibrations lying between them—hence it follows that, in as far as any generated basis is concerned, it matters not whether the intermediate vibrations are a series of isochronous or of accelerating or retarding vibrations, provided their coincidences be the same. It is moreover to be noted that, in such instances, the generation of a basis, foreign or extraneous to the given sounds, is absolutely impossible, except from faults in the execution of those sounds. Furthermore, upon this principle also, a slide may include any interval, however large, and yet be referable to a single basis only.

Directions for harmonizing symbolically in 2, 3, 4, *&c., parts.*

476. In all exercises of this kind use a pencil, and begin by ruling an horizontal line for each part—then unite the several parts by a brace, as in § 355. To this the generated bases below may be added or omitted at pleasure. Make down strokes for the bars and double bars, and, if required, introduce rests, pauses, &c. The measure may also be stated at the beginning,

in the margin—and in order to assist in obtaining a practical illustration of any particular example, the central basis of the triune system, or the central bases of the triune systems appertaining to the great system may be marked at their proper places. Moreover, though it is desirable to perceive, as readily as possible, the symbols truly appertaining to the melody and harmony, whatever may be the key—it is, nevertheless, attended with many advantages habitually to think and write by reference to the key of C. By so doing every species of basal transition will uniformly present itself to the mind under one specific form, which is at once true to nature and highly suggestive of many other particulars. The severance of symbolic representations from a notation varying in subjection to a diversity of keys, will be found to afford a relief to the mind that can scarcely be overrated.

477. If the treble be the given part, the base should first claim attention—for, except in fugues, wherein the subject or subjects are introduced by turns, first in one part and then in another—and except where the lowest part is intentionally confined to a few holding notes—the base, in connexion with the treble, should form a good and agreeable duet. In the separate parts of a duet, trio, or quartett, &c., owing to the imperfections incidental to temperament, and the notation, it is quite impossible, at all times, to render the *general* course of the harmony, that is, the chords produced by the melody of *all* the parts, clear and distinct, by means of *one* part or melody only—but in two parts these obscurities ought not to exist. In writing for the voice, the natural compass of the several voice parts must at all times be attended to—all chromatic notes properly resolved in the *same* voice part, and all unnatural and unsingable skips and jumps avoided. If, while engaged with the base part, any given note, or series of notes, present themselves to the imagination as deserving attention, let them be written down at once, without delay or hesitation.

478. The three bases, in the order of the symbols **1**, 1, 𝟙, and 𝟙, **1**, 1, and occasionally by **1**, 𝟙, 1, may follow each other in the same part—but 1 does not naturally proceed to **1**, except in consequence of the repetition of the same phrase. What is here said in reference to the order of the three bases, is also applicable to the three terminants **3**, 𝟛, 3, and to the three mediants **5**, 𝟝, 5—**19**, 𝟙𝟡, 19—but it has nothing to do with

chromatic harmonics of any kind. At the termination of the various phrases and sections, the basis of the chord should, *in general*, occupy the lowest part—but, at the final close, this law becomes imperative.

479. If the base be the given part, the treble should first claim our attention—in reference to which, all that has been said concerning the base is equally applicable. In general, however, a little more freedom may be allowed to the treble voice, in regard to the shake, turn, appoggiatura, and what are called grace notes. At the termination of the various phrases and sections, the treble may also end upon the harmonic 5 instead of 1, but seldom upon 3, and never upon 19.

480. The two extreme parts (the base and treble) being finished, the next object of attention is, to get the very best, that is, the most singable, natural, and flowing 3rd part—or, in other words, a 3rd part, which, in connexion with the base and treble, forms a good trio.

It is a matter of no consequence whether this 3rd part falls within the compass of the alto, or tenor voice, provided it be written in its proper place—but it is a matter of considerable importance that the whole be written for the *same* voice, where this can be done without straining that voice too high, or letting it down too low. But, should the compass of the melody be really too great for either the alto or tenor part, singly—this wide-spreading melody must either be judiciously divided between those two parts, or give place to another of smaller dimensions.

The terminating sounds of the various phrases and sections appertaining to intermediate parts may represent any harmonic of the chord to which they belong.

481. Having in the trio just finished, if correctly and tastefully done, already taken possession of the most important elements that can really be said to belong to the fullest harmony, it now remains to fill up all *elementary* vacancies, as far as with 4 parts this can be accomplished, consistently with the easy and natural flow of the melody of those parts. For, experience will soon show, that even in 4 parts the natural course of the melodies will frequently lead to the doubling of certain parts, and the consequent omission of others. In most instances, it is better to let this take place, rather than interrupt the easy and graceful flow of the melody of any one part, for the sake of

a harmony replete at every point. This is more especially the case where there are instrumental accompaniments, by which all omissions can be easily filled up.

Having proceeded thus far with the 4th part, it now remains to finish this part by doubling first one, and then another, of the *diatonic* intervals pertaining to the other parts. Now, as this is in a great measure a matter of taste, all that can here be said concerning it is—avoid, as much as possible, doubling the mediant **5**, which, in the other part, proceeds to 5—the mediant 5, which, in the other part, proceeds to **1**—and the mediant 5, which, in the other part, proceeds to 1. For, by so doing, either the course of these two parts must be the same, and thereby produce consecutive octaves—or the course of the mediant **5**, 5, or 5, in the 4th part, must be unnaturally, and therefore offensively, turned aside. In a phrase of a very bold and marked character, it is often better to double one or more of its most prominent features, in the unison or octave, rather than persist in maintaining the previously-assumed number of parts—and, it is but the extension of this principle, which sometimes renders a melody in unisons or octaves only, so very magnificent—and in which, even the idea of an additional part is repulsive to the feelings.

482. If the alto or tenor be the given part, the other parts had best be added conjunctively. The reason for recommending this mode of procedure is, because the alto and tenor are subordinate parts, and, as such, not well adapted to suggest either a good base alone, or a good treble alone, although either of them may be forced into a sort of make-weight alliance with a really good base and treble.

The student must now understand that if he has filled up the harmony in 4 parts, in obedience to the given rules, he has accomplished an object of much more importance than he can at present fully comprehend, although something yet remains to be done.

483. In proceeding to write additional parts, after the 4th part, the first object should be to fill up all *elementary* vacancies, and then to begin to double the other parts, subject to the same precautions as when writing the 4th part. Six parts may, however, be regarded as 3, especially where there are instrumental accompaniments, whereby all *elementary* vacancies may be filled up.

The best use that can be made of a 7th part is, after reducing 6 of these parts to 3, to employ the 7th part, in filling up *elementary* vacancies—in strengthening weak places—and in adding

force to bold and prominent melodic phrases. Where the oral, and not the visual perception of distinct parts is aimed at, 8 parts should be regarded as the mere duplication of 4. For, in spite of what any one may say to the contrary, I venture to affirm, that after 4 parts, the mind begins to be confused and distressed in its attempts individually to follow them, and at length perceives nothing but the general effect, with, perhaps, an occasional perception of the two extreme parts, the base and treble. But, as the silly conceit of writing a multiplicity of parts is, happily, now out of fashion, it is only necessary to observe, that what has here been said is not meant to be applied to quartetts, quintetts, &c., wherein the melodies are mostly of a varied, and sometimes of a very florid nature—and for the production of which, instruments of distinctive characters, and varied qualities of tone, are employed.

484. What has hitherto been said upon the subject of symbolical harmonization, has been with the view to the maintenance of a limited number of distinct parts, not written for the eye, but for the ear, the individual melodies of which are more or less perceived, as such, by every one, but unquestionably more so by educated ears than others.

Now, as this perception is to many the source of great mental enjoyment, with the view to cause these effects to stand out in a large orchestra, I propose, in addition to a set number of such parts (never more than 4), to have them, as it were, based upon a substratum of massive harmony, whence no other effect is intended to be produced than that of a vast combination of sounds, sympathizing with the declared and more decided averments of the chorus of *specialities*.

The rules to be attended to for writing the several parts of such a chorus are—(1) Constantly to render the compass as extended as the different voices will permit. (2) To confine the melody of every individual part within a very small compass. (3) Never to suffer the parts to cross each other unnecessarily. (4) Totally to disregard the existence of consecutive octaves or fifths between any two parts. (5) To be very careful not to spoil the purity of the harmony by the admixture of extraneous elements. (6) To resolve all chromatic harmonics in the *same* part. (7) Not to forego the opportunity to introduce 11, as a sub-basis to the harmony 1, 3, 5, 7, whenever, by so doing, the effect is rendered more rich and powerful.

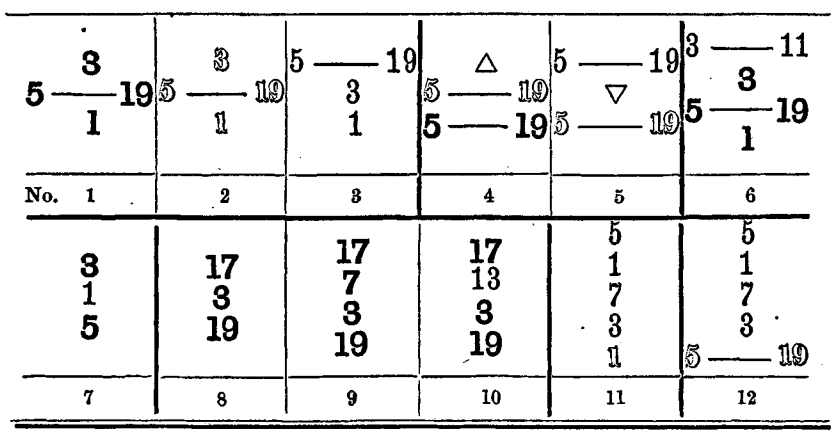
In the execution of this kind of chorus, the conductor should

keep the voices subdued, except in *fz* notes, at the climax of grand crescendo passages, and at the momentary bursts of extraordinary expression and force.

485. As a help to the practical exercise of harmonizing symbolically, in 4 or more parts, I here subjoin a representation of the 12 chords mostly in use, which will be found to constitute more than 999 out of every 1,000 parts of all the combinations to be found relating to *vocal* music—to the harmonies denominated *simple counter-point*, and thereby to the harmonies resulting from melodies of the most florid description, and most intricate and elaborate construction.

In his early efforts at harmonizing, let the student keep the following list of 12 chords constantly before him, think and write in the key of C only—experiment both mentally and practically upon these chords, two or three at a time, as he proceeds—and when finished to his entire satisfaction, transpose the whole in the key required.

Symbolical representation of the 12 *chords mostly in use.*



No. 1 represents the *real* common chord of the sub-basis—No. 2 of the central basis—and No. 3 of the super basis. Nos. 4 and 5 represent *apparent* common chords, which pertain to the 1st mode, when, upon keyed instruments, and according to the notation, the chord is *minor*—and to the 2nd mode, when, upon keyed instruments, and according to the notation, the chord is *major*. By omitting the harmonic **3**, in No. 6, the *apparent* common chord is also represented by the harmonics

$\begin{matrix}3\\5\\1\end{matrix}$—and by changing 1 to 17, by $\begin{matrix}3\\5\\17\end{matrix}$, and by by changing 3 to 11, by $\begin{matrix}11\\19\\1\end{matrix}$—but when viewed in reference to all its elementary sounds, No. 6 represents the so-called *chords of the added 6th.* No. 7 represents a chord hitherto but little noticed, and which might be called the *chord of the added 9th.* Nos. 8, 9, 10, represent *chords of the extreme ♯6th.* No. 11 is the same as No. 3, with the addition of 1. No. 12 is the same as No. 3, with the addition of 5 or 19.

In the chords 1, 2, 3, the basis may, of course, be changed to 7 or 17—or pass from 1 to 7, or from 1 to 17. The following are the letters or notes to which the above symbols refer, the key being C:—

C A——♭A F	G E——♭E C	B——♭B D G	C E——♭E A——♭A	B——♭B G E——♭E	D——♭D C A——♭A F
No 1	2	3	4	5	6
C G A	♯F C ♭A	♯F ♭E C ♭A	♯F ♯D C ♭A	B G F D C	B G F D E——♭E
7	8	9	10	11	12

On Composition.

486. It has sometimes been hinted to me that any one acquainted with the "True Science of Music" should be able to analyze everything. Now, if by the analysis of anything and everything, is meant a satisfactory explanation and defence of whatever is *good*, in ancient as well as modern compositions, I entirely agree with the principle contended for—but if it is meant also to include all sorts of crude productions, and that by my peculiar mode of analysis these things are to be made to appear otherwise than what they really are, in subserviency to sinister objects and prejudices, I protest against the justice of the demand. Truth is eternal, and by no means dependent upon their adaptation to the works of any particular age or master—and it is in the nature of things that dull and feeble

lights should suffer eclipse, and appear to be deprived of the little brilliancy they may really possess, when seen in connection with objects more luminous than themselves.

487. From all one can learn upon the subject of composition, it appears that much, very much, depends upon judicious early drilling, and a long and continuous course of efforts, earnestly directed to that end. But though this is unquestionably requisite in order to swell the germ or seed within to the largest dimensions, and ripen it to perfection, I am convinced that the seed itself, as to its kind, is not to be overlooked. What might have been done with such geniuses as Palestrina, Talis, and Bird, &c., I do not pretend to determine, although I feel convinced that, under the most favorable circumstances, they would not have been rivals to Handel, Mozart, or Beethoven.

488. The student who is desirous to succeed in the art of composition is earnestly recommended to enter into a close examination of the structure of a considerable number of melodies, and in so doing to note down all their peculiarities. He should pay great attention to the means employed to vary their terminations, so as, when required, to lead *from* or *to* the harmony of the tonic—also how these terminations may lead from one triune system to another—how the measure may be changed from common to triple, or *vice versa*—how, while maintaining a general resemblance, the character may be changed from an adagio or adante, to an allegro or vivace movement, &c.—also how it may, with good effect, be inverted—introduced upon different degrees of the scale—characteristically harmonized—and, lastly, what new effects may be obtained by syncopation, &c. Moreover, while engaged in the study of composition, he must not forget that as a single note does not constitute a melody, neither should a single chord be confounded with the true idea of harmony, *namely*, the union of two or more melodies. He must also recollect that though it is indeed true that melody may be said to exist independently of harmony, harmony cannot exist independently of melody, and consequently that in this sense melody is the foundation of harmony, and not *vice versa*—nevertheless, since by the aid of the memory, harmony virtually exists in melody, while melody, in actuality, exists in harmony, it is sheer folly to attempt to set up the one against the other, since the absence of either is the destruction of both.

Melody and harmony being thus united, it is evident that

they should be studied in conjunction, that is, by fixing the attention alternately, first upon the one, and then the other. The study of harmony should consist not merely in the continual effort to analyse the works of others, and in correcting their errors, if any, but also in harmonizing all sorts of melodies, and especially in variously harmonizing the same melody several times over. Then, as these exercises become easy and familiar, efforts should be made to write variations to several simple tunes, taking great care to reject everything commonplace and vulgar.

489. In writing entire tunes, and especially pieces of any considerable length, the student will do well to sketch an outline of the tonic transitions he may choose to make, before he enters upon the work itself. It is common, for instance, when commencing with the 1st mode, to effect a direct tonic transition to the harmonic 3 (as from C to G), on or before arriving at the first double bar—and in proceeding thence to the close, to effect the retrograde tonic transition (as from G to C), back to the original key. It is also common, when commencing with the second mode, to effect a direct tonic transition to the harmonic 19 (as from C in the 2nd, to ♭E in the 1st mode), on or before arriving at the first double bar—and in proceeding thence to the close, to effect the retrograde tonic transition (as from ♭E in the 1st, to C in the 2nd mode), back to the original key. But notwithstanding the frequency and propriety of the observance of the particulars here pointed out, there is no reason why we should slavishly adhere to them, as the only models of what is good and effective, as numerous examples by the greatest masters will testify.

By comparing the series of tonic transitions of various pieces, by the same and different authors, one with another, I have endeavoured to ascertain whether anything like a fixed routine could be found to prevail among them—but I could discover nothing of the kind. Nothing, in short, which could give the slightest sanction to the idea of what, did it exist, might be called the law of progression. Why, then, should we attempt to impose restrictions upon the composer, where, to all appearance, nature has left him free to roam at pleasure? How are we to reconcile the idea of such a law with the idiocracy of different individuals? If the course we ought to pursue was prescribed by law, would not all our efforts tend to the production of the same result, and thereby prove to demonstration that

we were mere machines? Nature, it is true, has recently been drilled into the execution of portraits and other pictures, but in so doing she merely lends her aid in copying her own original productions. For, it is to be observed, that painting, as an imitative art, in verification of its excellence, directly or indirectly, always refers to something superior to itself—but not so music. The compositions of individuals are not the representatives of material existences above and superior to themselves, however the case may be in respect to immaterial essences.

Finally, a painting lives in its way, for a few centuries only, and then falls into decay; but music demands an execution apart from its production (*i.e.* its composition), and hence becomes more and more alive, and the spirit of its author more and more familiar to us, in proportion as the arts and sciences are rendered subservient to its use. Hitherto it has been nurtured in the dark; what may be the effect of even the dawn of sunshine, time only can make known to us.

PART 23.

ANALYSED EXAMPLES, WITH STRICTURES.

490. Having at length fully stated the chief points for study relating to my views of the science of music, I now introduce a variety of analysed examples from various authors, including Beethoven's Moonlight Sonata, and Sonata Appassionata complete. And that the peculiarities of these examples may not be overlooked, I have added a comment upon each, with the view to draw attention and prevent misapprehension thereto. These examples, with the commentary attached to each, are therefore to be looked upon as a series of lessons adapted to call to mind, confirm, and establish, all that has hitherto been said upon the various subjects connected with this work.

491. In ex. 202, bar 1, we have the combination 11, 5, 13,—in bar 4, 13, 1, 19, 11,—the same in bar 7, with the addition of 3. In bar 8, we have **11, 13, 7, 1, 1,** the true intervals and effects of which are very imperfectly realized upon keyed instruments, and not easy for the voice.

492. In ex. 203, during the repeated occurrence of the note ♭B, the key is C—but F the momentthe note ♯B takes place in the base, and which holds on with power and effect. This example would be a sad poser to those whose ideas concerning the key are determined by appearances in respect to the notation.

493. The first chord in the four portions of ex. 204, is the chord of the diminished or extreme ♭7th. The peculiar circumstances relating to this chord are treated of § 114—117.

494. In ex. 205—207, out of the 6 duet combinations appertaining to 4 parts (§ 397) all are good except that of the 1st and 4th parts in ex. 205, 206, bars 1, 2, 3,—and that of the 2nd and 3rd parts in ex. 206, bars 1, 2, 3. The reason why the duet combinations of these parts are bad, is, because solely by their means, neither the key nor the mode can be determined with certainty. Nevertheless, this does not render them unfit for being united with other parts, whereby these important *desiderata* are supplied.

In regard to the consecutive 5ths pointed out by the thick lines, as the 1st chord in every bar is only an *apparent*, and not a *real* or basal 5th, there are no real consecutive 5ths—and, moreover, though the double octaves in the base, as in ex. 207, are not actually given in ex. 205, 206, nevertheless, as these are the notes really anticipated by the ear, and as the pre-conceived idea of them is not set aside simply by the introduction of another note appertaining to the same harmony—they virtually do exist to the imagination, and thereby divert the attention from the notes which are made to stand in their place.

It is, moreover, to be constantly borne in mind that to write in 4 or more parts, so that every possible duet combination involved therein shall be equally pure, clear, and beautiful, is an absolute impossibility—and hence it follows, that when the defects of one duet combination are rectified by another, and the aggregate perfection the highest possible, only the hyper-critic will venture to assail it.

The harmony in ex. 207 is the same as in ex. 205, 206.

495. The analysis of the chords in ex. 208, 209, will show that the only real chord of the ♭7th is upon G in the base, and that this can be nothing else. The chord ♭B, D, F, ♭A, in ex. 209, though the same as to the notation and upon keyed instruments as the chord of the ♭7th, in the ratios 1, 5, 3, 7, is, by reference to the corresponding chord in ex. 208 shown to be otherwise—for surely no one will pretend that these examples

differ, except in so far as relates to the mode. It is, moreover, to be noted that, as in ex. 208, bar 2, B 5 is not the medial basis to A 5—neither in ex. 209, bar 2, is ♭B 19 the medial basis to ♭A 19, but *vice versa* in both instances, A 5 being the sub-medial basis to the super-medial harmonic B 5—and ♭A 19 the sub-medial basis to the super-medial harmonic ♭B 19.

In ex. 208, bar 3, as A 5 is a primary medial basis to E 5, so in ex. 209, bar 3, ♭A 19 is a primary medial basis to ♭E 19.

The notes F, A, C, D, in ex. 208, bar 4, and F, ♭A, C, D, in ex. 209, bar 4, represent the chord of the added 6th to the sub-basis F.

496. Ex. 210—217 are examples in 2, 3, and 4 parts of the resolution of 7 in to 5,—7 into 5,—7 into 3—and $\frac{7}{17}$ into $\frac{3}{1}$.

If the progression and resolution of 7 into 3, in connection with one or more other parts, be less frequent than from 7 to 5, or from 7 to 5, it is because in such progressions it is usual to change 1 into 17. Now, as the position of 17 is intermediate between 1 and 1, it is therefore adapted to create the sensation whereby 1, the basis to the following chord, is anticipated. Moreover, another modification of 1 takes place, when, instead of 17, we have 1, which, in reference to the sub-basis, is $1 \times 9 =$ 9 or 18, and thereby introduce the chord 5, 1, 1, as in ex. 110, by Beethoven, and ex. 111, by Mozart.

497. As ex. 218, 219 differ only in respect to the octave of the lowest part, they are, theoretically speaking, both alike—nevertheless, while ex. 218, is very agreeable, in continual use, and sanctioned by theorists, ex. 219 is decidedly offensive, and therefore rejected in practice. The cause assigned for these facts is, because, in ex. 218, the ear is not diverted from the series of major and minor 3rds appertaining to the 1st and 2nd parts, to the series of 4ths appertaining to the 2nd and 3rd parts. To this the ear in some measure, but not entirely, assents—because, though the duet combination of the 1st and 2nd parts is very superior to that of the 2nd and 3rd parts, the duet combination of the 1st and 3rd parts is unquestionably the best. But, if the ear be so exquisitely sensitive to nice and delicate distinctions, it is evident too much attention cannot be paid to its decisions in any case. And since this is a point which peculiarly appertains to my views in respect to duet combinations, it follows from hence that this mode of investigating the effects of harmony, deserves the closest attention, and ought

not to be rejected merely on account of its novelty. In ex. 218, the duet combination of the 1st and 2nd parts is put forth with greater brilliancy than in ex. 219, their coincident vibrations being twice as frequent.

498. Ex. 220 is an admirable specimen of sublime, yet simple and pure harmony. Among other particulars the basal transition $\begin{matrix}19—1\\3—5\\7—3\\1—7\end{matrix}$ to the words "on him," in the 11th and 12th bar, wherein all the parts descend simultaneously, ought not to be passed unnoticed.

499. In ex. 221, the two key-notes or central bases are obviously D, C—but the question is, what is the ratio of C to D in this instance? Now since D is the first, and also the final key-note in this chorus, and consequently the central basis of the central system of the great system, it appeared to me that the ratio of the key-notes D, C, should be 8, 7, and not 9, 8, and still less 10, 9, and upon tuning the portion in D upon one piano, and the portion in C upon another, in agreement with these ratios, I found this to be the case. The effect of each of these ratios was also put to the test by an appeal to the ears of several other persons, and in every instance the ratio 7, 8, was declared to be the most effective, and most congenial to the general character and spirit of the chorus.

500. As the chorus "He trusted in God," which immediately precedes ex. 222, is in the key of C, the key-notes through which this example passes are therefore most naturally referred to the great system of C—and in this case the *direct* tonic transitions involved are to G 3, and E 5,—the *retrograde* to F 3, and A 19,—and in addition to these we have the tonic transition to D, either as the super 9 or sub 7, and to B, either as the super 15 or sub 17.

That the first chord in bar 1 should be $\begin{matrix}\sharp D\ 11\\C\ 3\\\flat A\ 19\end{matrix}$, and not $\begin{matrix}\flat E\ 19\text{ or }7\\C\ 3\\\flat A\ 19\end{matrix}$ and still less the common chord to ♭A, is evinced by the chords which follow. Such is the unity of the members of this example, if we admit the existence of the great system—and, be it remembered, if we deny its existence, and the influence thence derived, no one can determine what the relation between the key-notes really is, or show by what law the several parts of the

harmony cohere the one with the other. With respect to the two specimens of the 3rd mode, as represented by the note ♭E 11 in the key of D, and by the note ♯C 11 in the key of B,—let any one, after fixing his attention to these notes, endea vour to sing them perfectly in tune, and so as to give full effect to the words "but there was no man, neither found he any to comfort him," and if he does not feel himself, as it were, impelled to sing them very flat, and particularly so in reference to the note G 1, which stands between the reiterated note ♭E 11, and also in reference to the note E 1, which, in like manner, stands between the reiterated note ♯C 11, I regret his want of discernment, for it is questionable whether a more powerful, beautiful, and pointedly obvious example of the kind is anywhere to be found.

501. Ex. 223 is chaunted daily in the cathedrals, and by changing A 1 into ♯A 17, it is modernized and turned into the chord of the extreme ♯6th. As to the consecutive 5th, it does not produce a bad effect, and is easily evaded, as in ex. 224. Ex. 225, Nos. 1, 2, 3, 4, 6, severally present two distinct melodies and a harmony perfectly pure and unequivocal. No. 5 really presents but *one* melody, the upper part being the same as the lower transposed to the 5th above. The harmony is therefore unsatisfactory. But though No. 5 is not good as a duet combination, it nevertheless constitutes the *very best* fourth part in a quartett—and in this case, neither the harmony nor the bases of the two chords are in the remotest degree equivocal. No 7 (leaving out the note G) is a bad substitute for No. 1,—and since the note A, in its descent to ♯F, may be supposed to suggest, and in practice give birth to the note G, in this case the hidden consecutive 5th becomes real. No. 8 is a bad substitute for No. 4. No. 9 is an objectionable substitute for No. 5, because suggestive of the consecutive octaves which follow. Nos. 1—6 refer to the 3rd and 4th chords in ex. 223, and Nos. 7, 8, 9 to the two last chords in ex. 224, whence it appears that the best harmony is that produced by the 3rd and 4th chords in ex. 223, notwithstanding the consecutive 5th.

502. The best way to approach ex. 226 is to look to the parts separately in the first place—and if in so doing nothing is seen to suggest a tonic transition, it is, to say the least, questionable whether we ought to give our assent to its existence at all. At all events, the mere apparent existence of one or more

common chords ought not to be received as evidence of the fact, since nothing can be more fallacious.

If we now begin to examine the base, *per se*, what is there to be seen in the slightest degree at variance with the key of D? And as the same remark is equally applicable to the treble, *per se*—and also to the middle part or parts, *per se*—how can the key be changed?

By following up this mode of analysis a little farther, it will be found that a good singer, whose musical perceptions are not of a very low standard, will be sure to render the note ♮C very flat, while the note E, in the base, requires to be rather sharp than otherwise.

Now if we proceed to analyse the 1st and 2nd chords in bar 2 by spelling these chords by reference to D (the real key), and F (the apparent key), we shall find that the harmonics to which they are severally referable are as marked in the example, and consequently that the difference between real, and these two apparent common chords is as follows:—

$$19\left\{\begin{array}{c|c|c} 5 & 3 & 1 \\ \hline 3 & 1 & 7 \end{array}\right. = \text{Keys} \left\{\begin{array}{c|c|c} \text{F—}95 & 513 & 57 \\ \hline \text{D—}96 & 512 & 56 \\ \text{E} & \text{G} & \text{C} \end{array}\right.$$

$$19\left\{\begin{array}{c|c|c} 11 & 5 & 3 \\ \hline 19 & 3 & 7 \end{array}\right. = \text{Keys} \left\{\begin{array}{c|c|c} \text{F—}1 & 95 & 57 \\ \hline \text{D—}1 & 96 & 56 \\ \text{F} & \text{A} & \text{C} \end{array}\right.$$

In the first of these examples the sounds are all different, and in the last, one only remains fixed. Of course it will be perceived that, in refering to the voice, an appeal is really made to experiment, though not of so rigid and inflexible a nature as when suitable instruments are employed for that purpose.

503. The beautiful harmonic progression, ex. 227, and every other similar example, should be listened to with great attention during the examination of the symbolic analysis by which it is accompanied. This will lead, by degrees, to a perception of what is true and false in respect to harmonics generally—but more particularly with those which stand out conspicuously and boldly. And that this is always the case in respect to the sensations, wherever it also happens to be so in

regard to position, will be more or less apparent in proportion to the attention given to the series of primes marked in the Ratiometer, "Bases and primary harmonics of the triune system," our ears being at the same time kept open to their effects.

The many beautiful examples of consecutive 5ths, in passages which defy the possibility of amendment, ought to operate as a check against their indiscriminate condemnation, and also as an incitement to attend to the circumstances relating to them.

504. The analysis of ex. 228, and the continual introduction of the harmonic 11, first in connexion with the note ♭E in the key of D, and then with the note ♭A in the key of G, will show that this splendid example can only be explained upon the principle of a 3rd mode. The harmonic 41, which occurs more than once, will not, I trust, be overlooked by the student, since, in the works of Handel, it is rather remarkable, and may serve to prove that his musical perceptions in regard to such sounds were more acute than many are disposed to admit.

505. The chords in ex. 229, 230, are not confined to such simple combinations as, judging from appearances, we may be led to suppose. For instance, the second chord in ex. 229, bar 3, is chromatic, the basis to which is ♯G, and not E. In bar 6 the chord is not the real, but only the apparent common chord of ♯F minor, whereby is generated the three bases D, ♯F, A,—*i.e.*, the basis D, by the sounds ♯F, A,—the basis A by the sounds A, ♯C,—and the medial basis ♯F, by the sounds ♯F, ♯C, § 109. The first chord in bar 7 is the added sixth 1, 5, 3, **5**.

In bars 11, 12, the sounds C, E, G, and G, B, D, do not represent the real major common chords of C and G, but the two apparent major common chords which appertain to the minor mode of E, and which, therefore, generate three bases each.

In bars 17, 18, the key is still C and not G, consequently the last chord in bar 17 is **17**, **5**, **3**, 3, and not 5, 3, 7, 1 respectively, § 110. In ex. 230, the last chord in bar 2 is the same as that in ex. 229, bar 3. In bars 5, 6, the key may be either B or D,—if B, the chords are 1, 5, 3, and 1, 19, 3; but if the key be D, they are 5, 17, 5, and **5**, **3**, 5, respectively. The sounds B, ♯F, ♯C, will be sharper in the key of B than in the key of D, in the ratio 96 to 95—the sound ♯A 17 in D will be sharper than ♯A 5 in B in the ratio 323 to 320—and the

sound B 5 in D, will be flatter than B 11 in B, in the ratio 95 to 96—the sound D being the same in both keys.

506. In example 231, the last chord in bar 2 is the added 6th, D 1, F 19, B 3, and not the common chord of B minor. In bar 4, we have the chord G 7, ♭B 11, E 3, ♯C 5—and in bar 5, the chord ♯F 5, A 3, D 1, followed by the chord of the extreme ♯6th, ♮F 19, A 3, ♯D 17, which is a strange and unaccountable progression, if the basis of the last chord be B, and the key nobody knows what; but not so when these chords are both referred to the basis ♮D, and the last shown to be a chromatic modification of the first. Who, in the face of this example, can doubt, or venture to call in question, the truth of what has been said in § 118—122 concerning this chord? Such examples as these may serve to convince that Handel was *practically* better acquainted with the chromatic, than those who neither understand it, nor him, are disposed to admit. And that the same may be said of S. Bach, is evinced by ex. 232, bar 1, wherein will be seen the chord C 1, F 7, ♭A 11, D 3—and in bar 2, the chord of the ♭7th to the sub-basis, C 1, ♭B 7, ♭E 19, G 3, followed by the chord of the ♭7th upon the superbasis, D 1, A 3, C 7, ♯F 5.

507. Ex. 233 is introduced merely to impress C upon the mind as being the key-note, prior to the introduction of ex. 234 or 235. Now, though ex. 234 presents nothing to lead to the idea that the key is not still C, this is not the case with ex. 235, wherein, in appearance, it is G. But ex. 235 presents only the more modern form of ex. 234; and if so, the key is C and not G, and ♯F the harmonic 17 of ♮F, and not the harmonic 5 to D. In this case, also, the note C is the harmonic 3 to ♮F, and not the harmonic 7 to D, and A the harmonic 5 to ♮F, and not the harmonic 3 to D. The ear being impressed with the minor mode of C, through the agency of ex. 236, no change of key will appear to take place by the introduction of ex. 237—but if, in consequence of the introduction of the note ♯F, in ex. 238, G be the assumed key-note, ♭A becomes refractory in respect to its assumed basis D. But the true science is beset with none of these difficulties. The key being understood as C, ex. 234 is perceived as the rude diatonic form, of which ex. 235 is the chromatic—and ex. 237, as the rude diatonic form, in the minor mode, of which ex. 238 is the chromatic. Moreover, by the true science we see at once the reason why, in a descending

accompaniment to ex. 235 and 238, the note F must be ♮, and not ♯—and also why the ear will not suffer us to proceed with the maestoso movement in A, as in ex. 239, but persists in its being D, as in ex. 240. Nevertheless, if, after the double bar, at the end of bar 6, Handel had introduced his maestoso movement in A, as in bars 7, 8, 9, &c., and not in D, as in ex. 240, his practice would have been in keeping with the notions entertained by theorists, according to whose doctrine the key in bars 4, 5, 6, really is A, the note ♯G in bar 5 being the harmonic 5, and B the harmonic 3 to E. But, fortunately for us, Handel never did anything so ridiculous, and, what is more, no one else has ever done so, although the passage, under the guise of modern fillagree work, is of frequent occurrence with all authors. Passages to which ex. 237 or 238 is the outline and foundation, and in which the mode is minor, are frequently made use of as an introduction to the direct tonic transition to 19, wherein the mode is major—as, for instance, from ex. 237 or 238, wherein the key is C minor, to ♭E, the harmonic 19 to C, the mode then being major.

Nothing can be better adapted than these examples to show the fallacy of trusting to appearances in matters relating to harmony, or to draw attention to the value and importance of the application of the spelling principle, and its superiority over that which is dogmatical, correct only by chance, and by way of conjecture. What veteran theorist of the old school would venture upon a fair competition with a child of ordinary intelligence, 10 or 12 years of age, and after two or three months' instruction in this mode of spelling? If after naming the key, and the basis to which the notes in ex. 234, 235, 237, 238, are severally referable, such a child were unexpectedly required to state why, in every instance, the last chord must be major, he would be at once prepared to answer—"because it is the 7th of the key." And if asked why A, in ex. 235, is changed to ♭A in ex. 238? he would instantly reply—"because A, in the key of C major, being the harmonic 5 to the sub-basis F, must be changed to ♭A 19 when the mode is minor." And, again, if asked why in a descending passage in connection with the chord A, C, ♯F, as in ex. 235, or ♭A, C, ♯F, as in ex. 238, must the note F be ♮, and not ♯?—he would without hesitation reply—"because ♯F, being an ascending chromatic, *must* ascend to its resolvent G—but ♮F, if F 1, *may* descend, and if F 7, *must* descend." But the veteran dogmatist dare not be thus

explicit; for turn, which way he will, he is beset with difficulties. Should he dispute the point with his juvenile opponent, and persist in the key being G, in ex. 235, and in having D for the basis to the chord A, C, ♯F, he will be called upon to account for the change to ♭A in ex. 238; and if, upon the assumption of the same dogma, he persists in making A the key in ex. 239, bar 4, 5, 6, he will have to explain why the continuation of the same key in bars 7, 8, 9, should produce an effect so singularly offensive, and idiotically ridiculous—and also, why Handel, and a host of others, both before and after his time, should, by way of introduction to a new movement, get from the right into a wrong key, merely for the sake of having to turn back to the original. But let us suppose our theorists to yield the point concerning the key—in this case, he immediately lays himself open to an attack concerning the chords A, C, ♯F, ex. 235—and ♭A, C, ♯F, ex. 238—that is, concerning their bases, &c., and the matter above alluded to, relating to ♮F in descent, in connection with ♯F in ascent—and, finally, to the test of various experiments. Now, in a dilemma like this, the best, shortest, and most honorable course is, at once to admit having hitherto been misled and mistaken in matters relating to the science—or, if there be private reasons against such an open avowal of the truth, let such persons proceed silently, and in an orderly manner, to extirpate the seeds of error as fast as they make their appearance.

508. The numerals in ex. 241, on the top of each score, refer to the bars—sometimes to the upper and sometimes to the lower series of four parts, which, together, make up Handel's score of eight parts. The numerals before the brace refer to the parts of the harmony. That portion of the melody of the upper part, in bars 8, 9, 10, 11, is bad under every point of view. For (1) the key has hitherto been F; and if by assumption it is still to be F, the note ♭E must be 7, or 19, neither of which ought to rise to F 1. (2) If the key be ♭B, or ♭E, the notes ♭E, F, in either of these keys, form a strange interpolation between the notes which precede in F, and those which follow in C. (3) If C be the assumed key-note, then the antecedent to the note F should be ♮E, and not ♭E. For when the key of C is preceded by F, if F be major, the ear expects C to be major also—and even when this is not the case, as in the present instance, the progression from E to F, wherein the note E forms one of the parts of the common chord, or chord of the

♭7th, the ear still demands ♯E, to which both reason and experience assent. Furthermore, if the antecedent to G be ♯F **17**, the antecedent to F ought certainly to be ♯E 5, and not ♭E 19.

509. I now proceed to point out the errors and incongruities which these apparently trifling faults in the melody of the parts give birth.

Ex. 243. (*a*) This is a bad progression, because the ear is misled in respect to the key, which is C, and neither ♭B nor ♭E. (*b*) Here ♯F **17**, instead of resolving upon G, drops down to D. (*c*) This jump from bar 9 to 10 is neither consistent with the assumption of ♭B as 7, in the key of C, nor adapted to express a tonic transition from the common chord of ♭B, in the key of ♭B or ♭E, to the key of C. (*d*) This is a strange jump from one key to another, if ♭B be the basis to F—and a false progression if the ♭B be 7. (*e*) Here ♭B 7, instead of resolving upon ♭A, as in the 3rd part, takes a false progression, and rises to C 1. (*f*) Here, again, we have a strange jump from one key to another, adapted neither to the idea of a change of key, nor yet to the continuation of the same key. (*g*) Here the note ♭E should be ♯D **11**, and as this rises to ♯E 5, ♭A 19 should fall to G 3, and not rise to ♭B 7. (*h*) Upon the supposition that the first chord is the common chord to the basis ♭B, and that ♯F is written for ♭G 19, or ♭G 7, this example is good; but as this is not the case, the progression of the parts is bad, and such as cannot be defended. (*i*) This grotesque progression is made to take place to avoid the consecutive 5ths, $\begin{matrix}\flat E\ 7 \text{—} D\ 3\\ \flat A\ 19 \text{—} G\ 1\end{matrix}$, between the first and third parts. It is to be observed, however, that the interval of these two 5ths is not identical, although it is so according to the notation and upon keyed instruments. But, be this as it may, the means by which they (consecutive 5ths) are avoided is, as in the present instance, frequently more offensive to the ear than the consecutive 5ths themselves. Ex. 224. (*k*) Bad, because ♭E should be ♯D **11**, which ought to resolve upon E 5, and not jump up to G 3. (*l*) Consecutive octaves evaded, but the effect not thereby removed. (*m*) It would surely be easier here for the singer to pass at once from ♯F **17** to its resolvent G 1, without the intervention of the note C. (*n*) In passing from the 10th to the 11th bar, in ex. 241, the first and 2nd parts proceed thus: $\begin{matrix}\flat E\ 7 \text{—} B\ 5\\ \flat A\ 19 \text{—} G\ 1\end{matrix}$. In this instance, the harmonic **7** to the sub-basis is not allowed to

descend to D 3, because consecutive 5ths would thereby be produced with the part below. Nevertheless, as it is thought that D 3 must be brought in by some means or other, the note C 3 is made to ascend to D 3, in one part, while in another part it proceeds to B 5. The licence thus taken to split one part into two, undoubtedly affords the means to escape from a difficulty, which is very convenient; but unfortunately the effect is not always agreeable. The unpleasant sensation produced by the ascent from C 3 to D 3 arises, in all probability, from its influence to divert the attention from the more pure, marked, and definite progression of C 3 to B 5. But if, after exhausting every possible manœuvre, with the view to get the note D 3 introduced as a legitimate and indispensable member of the last chord, it be found that the object sought—the avoidance of consecutive 5ths—is not really, but only speciously obtained, may I not ask why maintain this perpetual warfare against nature? Or, if we will have our own way, why not go to work boldly, and assume our right to introduce the consecutive 5ths, $\frac{7\text{—}3}{19\text{—}1}$, or $\frac{19\text{—}3}{19\text{—}1}$, as well as others, whenever we please. But, let us not condescend to bring them in by any sneaking process whatever, and at the same time inveigh against the only mode whereby they can legitimately make their appearance. But, as it is common to speak of consecutive 5ths which *really* exist, and of others which only *virtually* exist, how can such a distinction between the mere internal arrangement of the parts in respect to *one note only*, in a vocal and instrumental chorus, be of any consequence? Will any one undertake to prove his ability to detect the difference? And, if not, why have recourse to a contemptible quibble, with the view to impose upon ourselves? Lastly, if this be not the true statement of the case, how are we to avoid giving offence to the same delicate organization, by the introduction of the note D 3 after C 3, or after any of the other sounds of the chord ♭A 19, C 3, ♭E 7, ♯F 17? The conclusion to be drawn is, if we will have D 3 in the last chord, we certainly ought to make ♭E 7 occupy the same part in the first. But. since the effect is by no means improved by these two sounds, my advice is to *leave them both out*; for that which does not tend to render the course of the harmony more distinctly intelligible, must of necessity render it confused; that which does not enrich must impoverish, and that which does not strengthen must weaken, § 388, 483.

Ex. 244. (*o*) Here, by means of the strange and unexpected jump from ♭A 19 to E 5, this last note becomes suddenly reinforced, the effect of which is by no means good. (*p*) The effect of the consecutive 4ths at this place is quite as offensive as consecutive 5ths—and no good reason can be assigned for their introduction. (*q*) Another strange and very inconsistent jump, and unexpected reinforcement of the note F 1. (*r*) The jump from ♭A 19 to F 3 is introduced to escape from the consecutive 5ths which the natural resolution of ♭D 19, ♭A 19 or 7, would produce in the 6th bar, in passing to C 1, G 3, in the 7th bar —and the unpleasant descent from F 3, to C 1, at (*s*) is done with the view to set matters right again. (*t*) Inconsistent with any rational interpretation, and any key to which it can possibly be referred. (*u*) Here ♭E 7 falls upon ♯B 5, instead of D 3, to avoid consecutive 5ths with the base—and C 3, instead of resolving upon B 5 ascends to D 3. (*v*) Here the ascending chromatic harmonic ♯D 11, under the false signature ♭E, instead of resolving upon E 5, jumps up to G 3, the effect of which, with the part above, is exceedingly disagreeable. (*w*) The effect here is not agreeable, the jump from E 5 to ♭A 19 being abrupt, unexpected, and unnatural. (*x*) Here the insertion of ♭B 1, between F 3 and ♯E 5, spoils the effect of the simple and more natural progression from F 3 to E 5, and sounds bad with the part above. (*y*) Here we have the bad effects, described at (*m*), viz., the progression of ♯F 17 to its resolvent G 1, intercepted by the note C 3, in connexion with the objectionable progression in which C 3 proceeds to D 3, instead of B 5.

510. Though the sounds may not be chromatic, if, from the peculiar nature of the progression, the tendency of any particular part is decidedly in one direction, that is either to ascend to the note above, or to descend to the note below, the effect of doubling that part, and proceeding thence to a higher and lower sound simultaneously, is not good—the attention being thereby diverted from the only true progression to another which is less satisfactory, because less decidedly marked, or because, as in the present instance, the mediant of the last chord is thereby either omitted, or barbarously introduced.

511. In ex. 241, bar 2, the basis C to the two first chords is wanting. This will apppear from the examination of the 8 parts, as copied from the score, and also in ex. 242, wherein the 8 parts are all grouped together.

In the last 4 bars, the key of C is clearly proclaimed by the

base part alone—and if so, the harmony, in order to agree therewith, ought to be such as I propose, ex. 245, 246.

It may here be asked, can the combination ♭B 19 or 7, D 3, F 1 (ex. 242, bar 9), be formally introduced as an effective chord? And will it not be received by the ear merely as the common chord to the basis ♭B *out of tune*, and in some undefined key? If the ♭B 19 be united to F 7, their resolution in *practice* will produce consecutive 5ths, and *real* consecutive 5ths if ♭B be 7 and F 7.

512. In putting forth these strictures upon the structure of the harmony of so sublime a genius as Handel, let it not for a moment be supposed that I am insensible to his great, and in some respects unrivalled powers. Carried along, as we frequently are, by the loftiness and magnificence of his conceptions, we are apt to overlook the confusion and disorder which is now and then to be detected in the internal arrangements, when put to the test of a close and severe scrutiny,—and which, in fact, are chargeable more to the school to which he belonged, than to himself. They, nevertheless, are faults, and such as are well adapted to remind us of the great distinction ever to be found between the works of art and nature, into whose recesses the deeper we penetrate, the more we find to excite wonder and admiration.

513. The two plans for the arrangement of a vocal score, ex. 245, 246, are offered as suggestions for a ready and effective method of operation; and the part for the organ or piano, ex. 246, is added that the general effect may be produced upon these instruments.

514. Ex. 248, being obviously a mere simplification of ex. 247, is, for convenience, made to stand in its place. In this example (and there are many such), as the 2nd and 3rd chords are *in appearance* the same as the 1st and 2nd, transposed to the wholetone above, it may be supposed that the key is changed from D to E. But the 3rd and 4th chords may with more propriety be referred to A, because A is more nearly related to D than E,—and with still greater propriety to D, because in this case the key remains the same. Upon the supposition that the key be E, the symbols answering to these two chords are:—

E 1———E 3
D 7 or 19—♯C 5
B 3———} A 1
♯G 5———}

If A be the key-note, the symbols are:—

E 1———E 3
D 1 or 7—♯C 5
B 3 ——
♯G 5 —— } A 1

But if D be the key-note, they are:—

E 3—— E 3
D 3——♯C 5
B 5——
♯G 17—— } A 1

Furthermore, *in appearance*, the 5th and 6th chords are in B minor; but whether the two preceding chords be in the key of E, A, or D, there is no reason why, in passing to those which follow, the mode should become minor. For though this change in the mode is not inconsistent with the idea that the 1st, 2nd, 3rd, and 4th chords are all in D; this affords no sufficient reason for coming to the conclusion that the 5th and 6th chords are *really* in the key of B minor, since these chords also not only admit of being referred to D, but, in so doing, the unity of the whole is preserved better than by shifting the key, for one moment, merely for the sake of having to get back again the next. By refering the 5th and 6th chords to the key of B, the symbols by which they are represented are:—

♯F 1———♯F 3
E 1 or 7— D 19
♯C 3———
♯A 5——— } B 1

But if D be the key, they are:—

♯F 5——♯F 5
E 3—— D 1
♯C 5——
♯A 17 — } B 5

Of course the final test in all such matters should be a direct appeal to the ear, by means of suitable experiments.

515. In ex. 249, the key is evidently C from first to last, and, if so, the note ♯F, in bars 3, 4, 5, and 8, is the harmonic 17 to the basis ♮F. In bar 6, ♭E 7 in the base, may, with good effect

be united with ♭E 19 in the treble—and since nice points like these cannot be expressed by the notation, we must either be guided by experience, and a kind of intuition, or resort to experiment—and the same may be said in respect to the first chord in bar 8, and numerous other instances. If the note F in bar 7 be understood to represent the sub-basis, it is then the basis to the ♭A 19 above; but, if it be understood to represent the harmonic 7 to the super-basis G, it is thereby referred to the same basis as the notes ♯B 5, and D 3, with which it is also united.

516. Ex. 250, by S. Bach, is well worthy of attention, on account of the purity of the melody, richness of the harmony, and high chromatic ratios. That the note ♯D 11 requires to be much flatter, and ♯B 41 much sharper, than the sounds by which they are represented on the piano, many will intuitively perceive. For the farther elucidation of this example I have added the chords representing the harmony upon which it is based, ex. 251. It will be found upon examination that the nominal extreme ♭3rd, and practical wholetone, ♯B 41, ♯D 11, exceeds the diatonic semitone 15, 16, only as 165, 164.

517. In ex. 252, let the attention be directed first to the base, and to the reiteration of the notes ♭E, F, G, ♭A, and it will be evident that the key throughout is C. But if the key be C, the triplet in bar 2, with G in the base, and ♭B, ♭E, ♭D, in the treble, does not represent the chord of the ♭7th upon ♭E, nor the following triplet with ♭A in the base, and ♭A, ♭E, C, in the treble; the common chord of ♭A, as mere appearances might lead us to suppose, but which, by resorting to the spelling principle, we find to be fallacious, and know how to correct.

518. As far as the notation is concerned, the first and second portions of ex 253, bar 2, might be in C major; but in this case the general effect would be very different, and the peculiar beauty of the note ♯C 19, in the base, totally destroyed. Bar 3, and part of bar 4, considered *per se*, might be referred to D minor, in which case ♭B would be 19, and not 11 But there appears no sufficient reason for supposing the key to be D, and the more so as the effect of ♭B 11 is very beautiful, and quite consistent with the unity of the entire passage.

In regard to the fugue, ex. 254, the portion referred to D might also be referred to A—but in this instance I prefer the intonation answering to the key of D, were it only for the sake of the brilliant effect thereby imparted to the note B 3 at the

resumption of the original key A, at bar 4. Furthermore, if this last portion be also referred to D, instead of A, the difference in the intonation will only affect the notes B and ♯G, for B **5** to B 3 = 80, 81, and ♯ G **17** to ♯G 5 = 136, 135.* And it is to the dubious impression created in the mind in respect to the key that we owe the sublime, and somewhat mysterious effect which this passage is calculated to produce.

519. In ex. 255 the high ratio 41, in bars 2, 3, is evidently required that the note B may be chromatic, in agreement with the notes over which the chromatic harmonics 17, 13, 11, are marked, but more especially that this note may correspond in its general effect with the last note but one in bars 1, 3, and with ♯F 17 in bar 4.

520. The critics have been greatly puzzled with ex. 256. They have hesitated to speak well of it, because it is evidently compounded of $5 \times 2 = 10$ bars, and they have not had the courage to condemn it. Some of them have tried to represent each of the 5 bars as being compounded of 3 + 2, in defiance of the effect produced by the melody, and, apparently, without perceiving the incongruity of an alternate series of bars in triple and common time, § 440. That it is a genuine specimen of a phrase, consisting of twice 5 measure notes, is obvious.

521. In ex. 257 the central basis to the great system is D, to which the pitch of the key notes B, E, ♯F, must be rendered subservient; that is, the ratio of B to D should be as 16 to 19; D to E, as 8 to 9; and D to ♯F, as 4 to 5.

In this case, the ratio of B to ♯F will *not* be as 2 to 3, but as 64 to 95, and then the pitch of the central basis D will remain fixed.

The peculiar beauty of the introduction of the original subject, in the original key D, is entirely owing to the fact that the chord D **19**, A 19, ♯F △, in the preceding bar is *not* the chord of D major in the key of D; but the *apparent* major common chord of D pertaining to the key of ♯F minor.

522. Ex. 258 is as beautiful and effective as it is remarkable. It is moreover admirably adapted to withdraw the mind from the barbarous notions which the mere contemplation of sounds, by reference to the crude intervals of the semitonic scale, is apt to beget. For does it not almost speak for itself, that the

* The interval answering to the ratio 80, 81, is 0·2150628; and to the ratio 135, 136, it is 0·1277669.

ascending chromatic ♯C 17 requires for its accompaniment the descending chromatic 31, at each repetition; and the ascending chromatic ♯D 13, the accompaniment of the ascending chromatic B 41?

But, if ♮C 31 be the relative descending chromatic to C 1 and B 41, the relative ascending chromatic to B 5, it will be found that the interval between this acute B 41, and grave ♮C 31, is only as 41×3 is to $31 \times 4 = 123, 124$; that is, instead of a semitone, only about one-eighth of a semitone. Let this example be well studied in regard to its true ratios, and let the relative position of the harmonics be carefully examined by reference to the Ratiometer, and, if possible, let it also be put to the test of experiment.

523. The chief objects of attention in ex. 259 are the beautiful retrograde tonic transition from C to ♭A, and the thrice repeated combination ♮B 13 and G 41. That since in each case the harmonic 13 impinges as it were upon its resolvent C 5, it hence becomes requisite to make the note G, 41 and not 5, that this note may also impinge upon its resolvent ♭A 1 will, I am pursuaded, be intuitively perceived by every one capable of appreciating the merits of the entire passage.

524. The third bar in ex. 260 presents a beautiful and perspicuous instance of the chord of the extreme sharp 6th, ♭B 19, D 3, ♯G 17, in combination with the descending series G, F, E, in the upper part, wherein the note G must be ♮, because it descends to F, notwithstanding its being ♯ in the lower part, because therein it is chromatic, and ascends to A. This is invariably the case in all similar examples, and proves beyond contradiction the truth of my theory in regard to this chord, § 118.

525. The powerful notes ♭D 19, or 7, and ♮B 17, in ex. 261, are simultaneously resolved by the note C 5 in the last bar. Such examples address themselves to the feelings as well as to the understanding, and thereby enlighten the mind in regard to what is true and what is false concerning them.

526. Can anything be more obviously true than the analysis here given of ex. 262, bar 2? How else can we reconcile the semibreve ♯D in the treble with ♮D in the base? And who will dispute the propriety of making ♯F 5 in the 1st bar, ascend to 41 in the next? This, indeed, is impracticable upon the piano, but where the key is changed from D to G, and the first chord is G 1, ♭B 19, and the next ♯G 17, ♮B 5, the under part is

really made to ascend in the manner here required for the notes ♯F 5, 41, in bars 1, 2. The remainder of this example the analysis will explain—nevertheless, the peculiarly happy basal transition from G to ♭B ought not to be passed over without notice.

527. Strange and erratic as the melody may appear in ex. 263, the analysis will show that the course of the harmony is perfectly orderly. In passages of this kind it will be found that the general rules relating to the progression of chromatic harmonics are occasionally set aside for the express purpose of creating a wild and mysterious effect. The course which the ear anticipates is always that which leads, by the shortest course, to the repose produced by the common chord of the central basis. But it frequently happens, and especially in dramatic compositions, that this tendency must be made to give place to the higher objects relating to the progressive development of the piece, or the all-absorbing interest of the moment—and hence it is, that though the major common chord, considered in itself, is unquestionably more sweet and harmonious than the apparent major common chord ♭B 19, ♭E 11, G 1, in bar 5, it would be poor and insipid here, because quite inconsistent with the general effect, § 95, No. 6 of the Table.

528. In ex. 264 the tonic transition from C to ♭E, and thence to F, would lead to the ratio 513, instead of $2^9 = 512$ for the note F, unless, by the influence of the great system, the ratio of the note F be taken from C, and not from ♭E.

The explication of this example will, in every instance be found perfectly consistent with the analysis therein given.

529. In ex. 265, it might be supposed that the tonic transition from C to ♭D should be to the ♭D 11 of G—but the following analysis will show that though in this case the pitch of the note ♭D would be the same in both keys, the note F, in the key of C would be nearly a semitone sharper than in ♭D—and the note ♭A in the key of C, nearly half a semitone sharper than in ♭D. Whereas, by taking the key-note ♭D as the harmonic 17 to C, the note F in C will be sharper than in ♭D, in the ratio 256, 255; the note ♭A in C, flatter than in ♭D in the ratio 152, 153; and the note ♭D in C flatter than in ♭D in the ratio 33, 34. That is, the discrepancies in this case are considerably less than in that in which the ratio of the key-notes are as 2^5 to 33. Moreover, in the retrograde tonic transition from ♭D to C, in bar 8, the ratio 17 to 2^4 is again to be preferred to 33 to 2^5—and

experiment will prove that the true ratio of the key notes C, ♭D, is as 2^4 to 17, and not as 2^5 to 33.

$$33\left\{\begin{matrix}\frac{5}{1}\\ \frac{3}{19}\\ \frac{1}{11}\end{matrix}\right. = \begin{matrix}\flat D\\ C\end{matrix}\left\{\begin{matrix}\frac{f}{495}\\ \frac{\flat a}{512}\\ \frac{\flat d}{297}\\ \frac{}{304}\\ \frac{}{1}\\ \frac{}{1}\end{matrix}\right. \qquad 17\left\{\begin{matrix}\frac{5}{1}\\ \frac{3}{19}\\ \frac{1}{11}\end{matrix}\right. = \begin{matrix}\flat D\\ C\end{matrix}\left\{\begin{matrix}\frac{f}{255}\\ \frac{\flat a}{256}\\ \frac{\flat d}{153}\\ \frac{}{152}\\ \frac{}{34}\\ \frac{}{33}\end{matrix}\right.$$

530. In ex. 266, ♯C **17** in the base passes to ♮C, owing to the abrupt and unlooked-for transition from the key of G to C.

The transition from G to C is more simple than from G to F, and therefore to be preferred; the ratio of G to C is also less equivocal, and therefore more distinct and agreeable.

531. The retrograde tonic transition from F to ♭D, in ex. 267, bar 4, and the direct tonic transition from ♭D to F, in bar 7, are in both instances very beautiful. And the chord of the extreme ♯6th, ♭D **19**, ♮B **17**, F**3**, in bar 8, is a verification of the truth of what has been stated concerning this chord in § 118, since it is clear that here the key is F, notwithstanding the introduction of the note ♮B—and consequently that this note represents the harmonic **17** to the basis ♭B, and not the harmonic 5 to the basis G.

532. In ex. 268, the direct tonic transition from 𝟙 to **17**, bar 5, and then the retrograde tonic transition from **17** to 𝟙, bar 7, is very remarkable—too distinct and obvious to admit of dispute, and too beautiful and sublime for any one not destitute of an ear for music, to condemn.

533. In ex. 269 the tonic transition is from C to ♭A, but not from C to ♭A and thence to F and ♭A—for the chords in the 2nd phrase are not what they appear to be. Moreover, while the basis to the chord in bars 1, 2, 4, of the 1st phrase, is ♭A only, and to the chord in bars 1, 2, 4, of the 3rd phrase, is ♭D only; in the corresponding bars of the second phrase the bases are both ♭A and ♭D. In like manner, while in bar 3 of the 1st phrase the bases to the two compound chords are ♭A, ♭E, and in bar 3 of the 3rd phrase, ♭D, ♭A—in the corresponding bar of the 2nd phrase it is ♭D, ♭A, ♭E. Furthermore, the chord in bars 1, 2, 4, of the 2nd phrase, in addition to the conjunction of the bases appertaining to bars 1, 2, 4, of the 1st and 3rd phrases, generate also the medial basis F. And in like manner bar 3, of the 2nd phrase, in addition to the conjunction of the bases appertaining to bar 3 of the 1st and 3rd phrase, generates also the medial bases F and C. Such is the mode in which

nature unites together, and fortifies the members of an edifice like the present.

For, be it observed, herein the bases of the 2nd or middle phrase, link the present with the past by the retention of the bases of the 1st phrase, and with what is forthcoming by the anticipation of the bases of the 3rd phrase.

The following is a representation of the relative position, &c. of the two bases generated by the chords in the 3rd bar of the 1st and 3rd phrase, and of the five bases generated by the chords in the 3rd bar of the 2nd phrase.

The two bases generated by the chords in bar 3 of the 1st phrase.	♭E	♭E	The three primary and two medial bases generated by the chords in bar 3, of the 2nd phrase.
		c	
	♭A	♭A	
Do. do. 3rd phrase		f	
	♭D	♭D	

534. In the original of ex. 270, bars 4, 5, 6, the note ♯A **11** is ♮A, and the note B 5 is ♭B. This is done to distinguish *practically* B 5 from B 41. But it is a clumsy expedient to commence with the falsification of two notes, that a third may *seem* to be what it is not—nevertheless, this is frequently done, because in such instances the defects of the semitonic scale are calculated to perplex and embarrass the composer.

535. Ex. 271. Such an example as this is admirably adapted to substantiate the fact that the existence of a great system of sounds is not a chimera. Where the existence of the great system is denied, such rapid flights from key to key would, in almost every instance, lead us from the original or central key, never more to return.

536. Ex. 272. This splendid recitative deserves to be carefully studied by reference to the analysis here given, not forgetting to notice the necessity that exists for the admission of the great system, in order to preserve the pitch of the first and final key note C. To whom we are indebted for the words I cannot say.

537. In ex. 273, the key, at the commencement of the allegro movement is F. It is also F at the first double bar before the repeat; but after the repetition, instead of the chord F 1, A 5, C 3, ♭E 7, we have ♯F **5**, A **3**, ♯B 13 (incorrectly written ♮C), ♯D **17**, the effect of which is as beautiful as it is unexpected. By this means a direct tonic transition is effected

from F to A, that is, from the central basis to its harmonic 5 above. The ratio of the vibrations of the above two chords is as follows:—

5	5	3	13	17
	1	5	3	7

	♯F	A	♯B	♯D
Keys	A 25	1	65	85
	F 24	1	64	84
	F	A	C	♭E

An attentive observer will perceive that the large ratio $\frac{25}{24}$ is that which relates to the only note in which there is practically (that is upon keyed instruments) any difference at all; also, that the only sound in the chord which is really identical to both keys is A. That this must be the case is moreover evident, since the harmonic 5 to the former key is the very sound assumed as the new key note. That the ratio of this sound becomes $\frac{1}{1}$ is also evident, the numerator 3 being multiplied by 5 = 15, for one purpose, and the denominator 5 by 3 = 15, for another, and $\frac{1}{1}\frac{5}{5}$ being equivalent to $\frac{1}{1}$. Lastly, it is to be noted that ♯B is above C, and ♯D above ♭E, and not *vice versa*, according to the doctrine heretofore laid down in regard to such sounds. The first of these two chords is the ♭7th upon the tonic basis, and the second a chromatic modification of the added 6th.

538. In ex. 274 will be found the chord ♭D **11**, C 1, F **7**, B **41**, C 1, alternating with the chord ♯D **3**, C 1, F **7**, B **5**, C 1. The reason why B is first **41** and then **5** is, that this note may be chromatic and diatonic, in conjunction with ♭D **11** and ♯D **3**; so that while the depressed sound ♭D **11** impinges upon the note C below, the ascending chromatic may press upwards to the C above. The chord ♭D, C, F, B, C, has been deemed inexplicable in a certain theoretical work: but to those who can spell by reference to the scale, it is as easy to decipher one chord as another.

539. Ex. 275, if studied in detail, as it should be, will be found to present a variety of extraordinary combinations or chords, which nothing but the spelling principle can decipher, and at the same time render intelligible, and which the analysis will fully explain.

540. In ex. 276, bars 2, 5, we have a repetition of the harmonics **37**, **41**, and then a tonic transition. The note ♯D **13**, in the lower part in bar 2, is by Beethoven, incorrectly written

♭E; and in bar 5, the note ♯C $\frac{13}{}$ is incorrectly written ♭D. The ratio of the key notes G, D, F, C, is, of course, dependent upon the great system of G. The exquisite beauty of the entire movement is well adapted to call forth our admiration.

541. The consecutive 5th in ex. 277, bar 2, symbolically represented, is $\frac{5—3}{5—1}$, and is of frequent occurence, and faultless. Of course the interval represented by the medial harmonics $\frac{5}{5}$ is the same as by $\frac{3}{1}$.

542. In passing from the 2nd to the 3rd bar, in ex. 278, we have B 5, C 1, in the upper part, set against C 1, B 5 in the lower, whereby is involved the basal transition, Class 7, No 2. In passing from the 3rd to the 4th bar, we have C 1, B 5 in the upper part set against E 5, F 1 in the lower, whereby is involved the basal transition Class 3, No. 1. And in passing from the 4th to the 5th bar, we have E 5, F 1 in the upper part, set against F 1, E 5, in the lower, whereby is involved the basal transition, Class 7, No. 1, § 346. But, as Beethoven was probably unable to account for the union of such conflicting elements, he has, apparently with the view to evade their occurrence, written B 5, instead of C 1, in the 2nd and 3rd bars, and E 5, instead of F 1 in the 4th bar. But, whatever he might have thought concerning such matters, I am convinced had he trusted to his ear, as to the general effect, he would have written the passage as here given. Besides, if it be allowed to vitiate the melody in the base at bars 2 and 4, why not accomplish the same object by changing B 5 in the upper part of bar 2 to C 1, and E 5 to F 1?

543. Should it be said that in ex. 279, bar 6, the key is ♭E major, and that it is changed in bar 13, to C minor, the unity of the melody will be destroyed, for in this case the note G 1 or 3 will become G 5, that is, too flat in the ratio 95, 96, which is quite opposed to the intended effect whenever it occurs, but especially at the *ff* in bar 7. The notes ♭E and ♭A are the same in either key; but the note F differs in the ratio 512, 513, the higher sound being in the key of ♭E.

544. In ex. 280, bar 2, a good effect is produced by the introduction of the harmonic ♭B 7 in an ascending progression in the base. Now, though 7 is not here formally resolved as

one of the members of the subordinate basal transitions, it is nevertheless resolved in a sort of general way, as one of the members of the principal basal transition, wherein the following chord contains the resolvent **A 5**.

545. In ex. 281, the sounds represented in bar 2 by the notes D, B, F, in the key of A, differ from the same notes in the key of C by the ratios 512, 513—96, 95—64, 63, respectively; and the note ♯G from ♮G by the ratio 20, 19—and hence the peculiar effect resulting from this tonic transition.

546. In ex. 282, in the upper base part, we have the following series of relative diatonic and chromatic harmonics, C1, ♯C 17, —D 3, ♯D 13—E 5, E 41—F 1, ♯F 17. And though E 5 and E 41 are the same in their notation and upon keyed instruments, who will dispute the propriety of the distinction here made between them, or of the analogy as to position between these and the other diatonic and chromatic relatives? Here, then, we have the prime 41 introduced in a manner so marked and perspicuous as almost to enforce our assent to its claims upon our attention as a musical ratio, and, if so, to all the lower primes also. In the semiquavers below we shall find a perfect consistency between the groups severally connected with the diatonic and chromatic harmonics above, and also in respect to the individual notes of which each group is composed; and, here again, consistency as to position, &c., demands the use of the prime 41 for the note ♮B in bar 2.

547. In ex. 283, we have a bold and splendid specimen of a retrograde tonic transition to the major 3rd below, or, in other words, *from* the harmonic 5—and in ex. 284, a direct tonic transition *to* the harmonic 19. In ex. 284, the common chord of G major proceeds to the common chord of ♭E major; but this affords no reason why the ratio of the bases of these two chords should not be rendered subservient to the ratio of their respective key notes, and not upon themselves as isolated individuals in neither case referable to any key or triune system.

548. In ex. 285, the key of B, after ♭E, is to be understood as being substituted for ♭C, the major 3rd below ♭E; the notation of the key of B with five sharps being simpler than that of ♭C with seven flats.

The ratio of the key notes ♭E, B, being taken as 5 to $2^2 = 4$, and of the key notes ♭E, F, as $2^3 = 8$ to $3^2 = 9$—the difference

between the chord ♯C, E, G, ♯A, in bars 11, 12, and the chord ♮C, E, G, ♭B, in bar 13, is as follows:—

$$5 \times 9 = 45 \left\{ \begin{array}{c|c|c|c} 1 & 5 & 3 & 7 \\ \hline 3 & 1 & 19 & 5 \end{array} \right.$$

Keys	C	E	G	♭B
F	15	2025	1215	63
B	16	2048	1216	64
	♯C	E	G	♯A

Again, the difference between bars 23, 24, the key being F, and the same notes, the key being C, is as follows:—

$$3 \left\{ \begin{array}{c|c|c|c|c} 11 & 1 & 19 & 3 & 5 \\ \hline 3 & 11 & 19 & 5 & 17 \end{array} \right.$$

Keys	C	F	♭A	D	B
C	1	1	1	81	135
F	1	1	1	80	136
	C	F	♭A	D	B

549. Can anything be more beautiful, clear, and unequivocal than the *retrograde* tonic transition, ex. 286, from the harmonic 17? (C to B), and the subsequent *direct* tonic transition to the harmonic 17? (B to C). And what is the value of any special pleading against a fact which experiment can at any time confirm?

The reader is also requested to pay attention to the harmonics 11, 13, and 41, towards the end of this example.

550. If ex. 287 be studied by reference to the chords to which it is reduced, ex. 288, and the chords ♭A, C, ♭E—♭B, D, F, ♭A, in bar 4, and the chord ♭E, G, ♭B in bar 5, be understood as being in reality what they appear to be, the unity of the passage and general effect would be injured. And although bars 9 and 10 would naturally be referred to the key of G, had the mind been previously impressed with the idea of that key—but, since the attractive central force is evidently C, and not G, the harmonics by which these two bars are symbolized, and thereby analysed, are marked accordingly.

Examples like these should be studied with minute attention to every circumstance relating to them, and with the view to the intended effect, without which the propriety of many of the particulars relating to the analysis will sometimes appear questionable.

551. In ex. 289, bar 6, the ascending chromatic harmonic ♯F **17**, instead of ascending to its resolvent G 1, is here made to descend to ♮F 7, to which G 1 is the basis. But instead of

♯F 17 we may substitute ♭G 15, that is the mean between the preceding note G 1 and the following note F 7. The ratio of the sounds ♯F 17, ♭G 15, is 136, 135. Furthermore, instead of ♯F 17, we may substitute ♭G 111. The ratio of the sounds ♯F 17, ♭G 111, is 34, 33. As this combination is of frequent occurence it is desirable to pay great attention to what is here said concerning it. The consecutive 5th which takes place in passing from bar 6 to 7, is not real, but only apparent, and consequently faultless and the effect good.

552. The prevailing sensations pertaining to ex. 290, are mystery, and consternation—and the means employed to call forth these effects are a series of strange and eccentric movements in respect to the introduction and resolution of the chromatic harmonics, and the harmony to which they belong. If, for instance, ♭A 7, in the 8th and 9th bars, be understood as ♯G 13, the chord in bar 10 is thereby fully and clearly anticipated; but, by so doing, the progression is rendered tame and common-place. The same may be said of the unexpected introduction of ♭E 19 instead of F 1, A 5, C 3, or F 1, ♭A 19, C 3, in the key of F, and the change of key thereby effected, (bar 13)—of the unexpected introduction of the chord F 1, A 5, D 3, instead of D 3, G 1, B 5, D 3, or G 1, ♭B 19, D 3, in the key of G (bar 15)—and of the unexpected transition from ♭E 7 (bar 17), to E 5 (bar 18), instead of passing from ♭E 7, to D 3, or changing ♭E 7, to ♯D 13. Now, if it be admitted that the above is a true representation of the authors feelings and intention, the fact that the anticipated progression of any particular harmonic is in such instances disregarded, is obviously in perfect keeping with the state of indecision and hesitation which it is intended to depict. In ex. 292, the chord which precedes the slur is taken from ex. 290, the bars being marked to correspond. The chords under the slur will show how the harmony might, in every instance, be smoothly brought to a close.

553. If all the sharps and flats in ex. 291 were erased, the melody and harmony would still be fundamentally the same, and I am disposed to think that this will be intuitively perceived upon its being put to the test, and listened to with attention. But if it be admitted that the diatonic harmonics are merely the initial elements of the triune system, and that this system is rendered complete by the addition of the chromatic harmonics, this example is itself sufficient to prove that the principles upon which this work depend are true. That the key is C there can be no doubt, and this is moreover proved by

the fact that in every group of 4 semiquavers the last note is uniformly diatonic, and therefore all naturals, and the first in every group an ascending chromatic, to which may be added the elegance of the passage, and the utter impossibility of altering a single note without spoiling the effect.

554. In ex. 293, bar 6, we have an ascending progression, wherein 11 proceeds to 5, first in the lower, and then in the higher octave. As a diatonic note appertaining to the 3rd mode, this is quite consistent, but not so as a chromatic note.

At bar 9, we have the notes ♭E, G, ♭B, in the key of ♭E, and at bar 13, the same practical notes in the key of D. In bar 9, the note ♭E is 1̇, and in bar 13, it is 11. It would be useless to attempt to convince those who cannot perceive the difference between them that these sounds are not really the same.

The harmonics and key being marked, I leave the student to make his own observations upon the portions extracted from var. 9 and 12, but trust he will not overlook the beauty of the tonic transition from E to ♯C, var. 12, bar 14, and thence to G, bar 17, or forget what has been said concerning the great system, and its application to practice, in reference to the tonic transitions in this example.

555. In ex. 294, the reality of the direct tonic transition to 17, (♭E to ♯E) is as perspicuous as it is beautiful, as is also the retrograde tonic transition from 17, (♯E to ♭E) which takes place afterwards. If, at the resumption of the three flats, the key be understood as being C for two bars, and afterwards ♭E, and if the ratio of the key notes E, C, be as 5 to 4, and of the key notes C, ♭E, as 16 to 19, the pitch of the original key note ♭E will not be restored.

556. The combinations or compound chords, ex. 295, marked thus *, though harsh in themselves, and especially if the attention be diverted from the general effect, by playing them slow, are, nevertheless, quite correct, and consistent with the unity of the entire passage.

557. In the analysis here given of ex. 296, the tonic transitions are from C to ♭B, and thence to C and F. But if the notes under the first slur be referred to the key of F, the analysis will be :—

F 1̇ —	♭G 11 —	— —	F 1̇
C 3 —	♭D 19 —	— —	C 3
A 5 —	♭B 1 —	— —	A 5

And if the notes under the second slur be referred to the key of G, the analysis will be:—

C 1	— C 1	C 7 —
F 7	— G 3	A 3 —
C 1	—♭E 19	D 1 —
♭A 11	— G 3	♯F 5 —

And since by so doing we introduce the harmonic 11 (the first of the elements of the 3rd mode), the effect will be rendered more intense and profound.

558. Ex. 297, 298, in addition to many others, are all admirable specimens of the 3rd mode, which not only merit the closest attention to their effects, but also as to every fact observable therein.

559. Erratic and mysterious as is the harmony of the variation, ex. 299, to the preceding waltz, ex. 188, it is, nevertheless, referable to the same changes of key and general outline. To chord-mongers it must be a regular poser.

560. In the original, the first portion of ex. 300 commences with bar 144, the second portion with bar 554, and the third with bar 612. The chief points of attention are, first, the unusual but perfectly consistent combination ♭B 1, ♭E 1, ♭G 19, A 5, and the subsequent basal transitions, which the harmonic analysis will explain. In the second portion, the important point to determine is, whether, after the *pianissimo* close upon the keynote (♭E), in the sudden burst upon the *fortissimo* chord, ♭D, F, ♭A, the best effect would result from the key C or ♭D? If the former, the analysis of the chord would of course be ♭D 11, F 5, ♭A 3—if the latter, ♭D 11, F 1, ♭A 19, in which case the key coincides with what follows, wherein a second *fortissimo* crash takes place with the chord of C major, after the mysterious effect produced by the unexpected introduction and dying away of the chord ♭D, F, ♭A. The passage commencing at the third portion is introduced merely to show what singular harmonic combinations are sometimes to be found in works of this kind, and against which no valid objection can be made, and the analysis of which is, nevertheless, as easy, clear, and satisfactory, by the adoption of the spelling principle, as it is unattainable by any other means.

561. Ex. 301, 302, from Beethoven's grand sonata pathetique, Op. 13, is a proof that authors have come to no settled deter-

mination in respect to the chord in bar 3—and that they are not only at variance one with another, but with themselves also. For though, with the exception of the key, these two examples are evidently meant to be the same—nevertheless, in ex. 301, the harmonic **5** in the base is accompanied by **3** and **1** only, but in ex. 302, by **8, 7, 1**. In ex. 301, **5** proceeds to **19**, but in ex. 302, the same sound is repeated. In ex. 301, in passing from the 3rd to the 4th bar, we have the disagreeable progression $\frac{3-3}{19-1}$, which is as bad as the apparent consecutive 5ths $\frac{7-3}{19-1}$, or the real consecutive 5ths $\frac{19-3}{19-1}$. In passing from the 3rd to the 4th bar, in ex. 302, we have the disagreeable progression $\frac{3-3}{5-1}$, which is a bad as $\frac{7-3}{5-1}$ or $\frac{19-3}{5-1}$, and each of these as bad as $\frac{7-3}{19-1}$, or $\frac{19-3}{19-1}$. As a substitute for the above, I recommend the progressions pointed out in ex. 303, 304, or 305, and in the 2nd mode by ex. 306, 307, but not those in ex. 308, 309, and 310.

562. Ex. 311 presents an effective basal transition from ♭B to ♭D, wherein the note ♭D **19** is changed to ♭D **1**—♭E **1** to E **3**—♭G **19** to ♭G **7**, the ratio of their vibrations being in all cases dependent upon the ratios of their respective key-notes.

563. In ex. 312, the chord occupying the first four bars, harsh as it may seem, is not out of place in the symphony, and would be still more effective if heard in its true ratios, and especially the note F **11**. A scientific knowledge of music will, no doubt, at some future period, lead to the study and realisation of the effect intended by the composer.

564. In ex. 313, the tonic transition from C to ♭G is unequivocal, and very effective, and experiment will prove that the ratio of the two key-notes should be C 1, ♭G 11. Experiment will also prove that the ratio of the two key notes C, ♭D, in ex. 314, should be C 1, ♭D 17—and that the ratio of the key notes C, ♭B, in ex. 315, should be C 1, ♭B 7, and the ratio of the key notes C, A, in the same example, C 19, A 1.

565. Ex. 316, in the original, is in the key of ♭A minor, with 7 flats, which gives birth to strange appearances in respect to the tonic transitions which follow; but when transposed to the semitone above, as in the example before us, these extraordinary appearances vanish, and if examined by reference to the

great system, everything, even in the tonic transition from ♭E to A, is seen to be consistent and orderly. The last 11 bars are obviously in the 3rd mode.

566. In reference to ex. 317, the editor of the *Musical Times* (No. 129) remarks:—"In this beautiful Sanctus, Orlando Gibbons has overlooked two-fifths between the treble and tenor parts, but as they do not produce a bad effect in performance (on the contrary, the ♭B has a charming effect on the D in the base), they have been left just as they were written by the composer."

Now though this is by no means a bad specimen of musical composition for the period in which it was written, I cannot assent to its claims as being "beautiful," since, upon the whole, it is a very common-place affair. That the consecutive 5th should have been "overlooked" is not at all surprising; for why, and especially in music, should that which has proved harmless in respect to the ear, be suspected of shedding a malignant influence in the direction of the eyes? According to the statement of the critic of the *Musical Times*, these "overlooked" 5ths "do not produce a bad effect in performance," and, if not in performance, it is difficult to know when, unless the learned critic is prepared to prove that the bad effect flows from their *non*-performance. Lastly, I suppose we are to understand that by virtue of the "charming" progression $\frac{C—♭B}{C—D}$ a charm is somehow diffused through the harmony of the whole, the consecutive 5th not excepted. The note ♯B **17** is worthy the attention of those who fancy music of this kind to be purely diatonic, and are prone to despise the chromatic under the notion that it is artificial. The transition from D minor to F major might be made to take place with the word "glory be," by means of the common chord of F major instead of D minor, rather than at the word "to" which follows.

567. In ex. 317, bars 1, 2, ♯D **17** passes to ♯E **13**—in bars 2, 3, ♯E **13** passes to ♯G **17**—in bars 3, 4, ♯G **17** passes to ♯F **5**, and in so doing the resolution of the first of the two chromatic harmonics is, in every instance, evaded.

But in all such cases, the resolution is, or ought always to be, sufficiently obvious to render the introduction of the resolvent unnecessary. Some of the combinations which take place at the last quaver in the bar must appear strange, if considered in the light of formal chords—but not so when it is perceived that the

treble note merely serves to postpone the introduction of that which follows.

That the change in the notation from ♭D to ♯C, in bar 20, is resorted to as a mere matter of convenience, because the notation of ♯C minor with four sharps is simpler than ♭D with eight flats, is evident—as is also the fact that the license to make such changes in the notation affords no ground for the nonsense and quackery that has so long been maintained concerning what are called enharmonic modulations.

568. Ex. 319 represents, in the key of C, the combination to be found in var 1, bar 13—viz. 𝄩, **1**, **19**, 3, **1**, 5, 𝄩—and ex. 320, the combination to be found in var 4, bar 14—viz., 1, **19**, **3**, **17**, 1, which, by the adoption of the spelling principle, is as easy to decipher as the simplest chord in existence.

569. Palestrina, the author of ex. 220, was born in 1524; 129 years before Corelli. In speaking of Palestrina I am quite at a loss to discover wherein the merit of such so-called music consists. It is, however, but fair to confess that I know nothing of his compositions except what could be obtained by a hasty survey of a large volume. And as the general aspect of all the pieces therein contained was so very much alike, it was impossible to form any correct judgment upon the matter, so as to choose one of the best, as an example. Should what is here given be an unfortunate specimen, I shall be glad to find it is so, as the works of Palestrina are by some persons still held in high estimation. But until I can be assured that he really has written something very superior to this I cannot but entertain a very mean opinion of his genius. The all-important question to which I am anxious to draw attention, is—does the world contain a single example of really good music, four or even three hundred years old—and, if not, what is the secret influence that has given rise to its existence at the present time? Let those who can, answer this question.

The tonic transitions and harmonics marked in connexion with this example are only to be understood as one out of several other interpretations that might be given. For, in this sort of music the harmony is scarcely ever pure and definite. The parts being either engaged in a sort of mimicry, or in elbowing each other, the effect of which is a continual snarl from beginning to end. It is stated that Pope Marcellus was so enraptured with a mass dedicated to him by Palestrina, that he compared it to the heavenly melodies which the apostle John

heard in his ecstatic trance. This may or may not be the case, for anything I know to the contrary—or it may so happen that Pope Marcellus' judgment in regard to music was not infallible. It is due, however, to Palestrina to state that he has the reputation of being a prodigiously scientific composer—though judging from the example before us I cannot comprehend upon what ground—or under what sense the word science is to be understood. At all events, it must be in a sense widely different from that by which the feelings and thoughts of such men as Handel, Mozart, and Beethoven, were influenced.

It may be questioned whether such compositions, and especially such wretched harmony, would at any time have been tolerated, were it not for the pompous mode in which it is represented, by means of a variety of clefs and a distinct stave for each part. It is true an experienced performer may, by means of the base, with or without "a thorough base," and an occasional glance at the treble, form a general idea of what it is about—but, as to entering into the particulars, the thing is impossible. It must be confessed, therefore, that the tenor clef, occupying two or three positions upon the staff, in addition to the treble and base clefs, is admirably adapted to confound the distinction between good and bad harmony, so as to render their appearance much the same. At all events it is impossible to devise a more absurd scheme than the employment of a multiplicity of clefs.

570. The analysis of ex. 322, is given symbolically, and separate from the notes, that the harmonics of this peculiar and perpetually descending series of sounds, may be more distinctly perceived. The note written ♯F in the original is here changed to G 29,—B to ♭C 29,—♯C to ♭D 11,—and A to ♭♭B 13.

571. In ex. 323, bar 8, 9, the notes ♭A 1, ♭C 19, in the key of ♭E, pass to ♯G 5, B 3, in the key of ♯E.

But though these sounds are the same upon keyed instruments, they are not so in respect to the ratios of their vibrations. The effect of this direct tonic transition to 17 is exquisite—the return to the original key, bar 25, being, as it were, welcomed with rapture, which is hightened by the previous hesitation and indecision produced by the sounds represented by the notes E 1, ♯D 5, ×C 11, in bars 21—24.

572. The analysis of ex. 324 will show that the harmony of the sounds in bars 1, 2, differs from bars 3, 4, and these again from bar 5, notwithstanding their being all the same in respect

to their practical intervals. That bars 9—13, are essentially the same as bars 1—5, is obvious. It is also evident that the notes C, C, in bar 14, really represent two distinct sounds, in the the ratio 1 to 31.

In this example I have changed the notation in a few instances, where I considered it to be incorrect, but not so as to affect its performance upon keyed instruments.

573. In ex. 325 the ratio of the tonic transitions from A to F, and thence to E, both of which are exceedingly beautiful, must be estimated by reference to the great system of E, and not the one from the other, that the unity of the whole may be be preserved.

574. In ex. 326 the tonic transition $\begin{matrix} C\ 3 & \text{——} & A\ 3 \\ A\ 5 & \text{——} & \sharp F\ 5 \\ F\ 1 & \text{——} & D\ 1 \end{matrix}$ gives birth to consecutive fifths between the two extreme parts, but as the notes are heard in succession, and the tonic transition peculiar and tasteful, the effect is by no means bad. In bars 5, 6, the key is obscure, being neither in D minor, nor F major. What it ought to be is shown in ex. 327.

575. In ex. 328 the chord A **1**, C **19**, F **11**, in bars 3, 4, is obviously not the major common chord of F. The sensations it is adapted to excite, whether contemplated by reference to its notes individually, or collectively as a chord, are utterly opposed to such an idea. Neither can the chord which follows ♮C **19**, ♮F **11**, A **1**, ♯D **17**, be mistaken for the chord of the added 6th, F **1**, A **5**, C **3**, ♯D 13, in the key of C, by any one whose perception is in the smallest degree elevated—and what is more, neither is it possible that any mistake of this sort can ever occur in the course of analization while the spelling principle is adopted.

576. In ex. 329 we find the note ♭A **19** united with ♯G 17, the ratio of whose vibrations is, as $19 \times 2^3 = 152$ to $17 \times 3^2 = 153$ —and who can show why two sounds vibrating in this ratio may not come together.

577. Can his taste be pure who can find fault with the consecutive 5ths in ex. 330? And what is that but ignorance and pedantry, which, after confounding their several distinct varieties, proceeds to pass a sentence of condemnation against them all, because *some* are bad—not simply because they are 5ths, but because, in such cases, the rules of harmony are violated in respect to other particulars.

578. In var 1 of ex. 192, Beethoven has been led astray

by the imperfections appertaining to the semitonic system and the notation. He felt that the second note for the left hand should be lower than the first, and in making it to descend, as it is impossible on the piano to represent the transition from 𝟙 to 𝟛𝟙, instead of holding on to the same note and sound—the best expedient that can be adopted in such cases—he passed on to the semitone below, and, in so doing, not only displaced the *six* following notes in the same part, but, as a necessary consequence, threw the upper parts into disorder also.

In the corrected example underneath, dotted lines are placed to draw attention to the simplicity and regularity of the *diatonic* progressions throughout. These progressions are lost sight of, in the original, four times in succession. In var. 8, the several parts of the harmony are not represented by Beethoven with his usual perspicuity. The defects in the original will be seen by comparison with the present example. Var. 12 presents a splendid series of tonic transitions, which, unless we admit the existence of the great system, must render a return to the exact pitch of the original key impossible.

579. In ex. 119, we have a splendid example of the 3rd mode, for, surely no one can be so devoid of feeling and reason as to imagine this to be the chord of ♭D major, or in any other key than C.

In this example, the lowest part is the harmonic **19**, and not **1**, as is almost invariably the case with this chord. But, in no instance, that I have ever met with, does **11** occupy the lowest part, and yet, if this be really the common chord, because according to the notation, it appears as such, who can account for the basis never being seen in the lowest part?

580. Ex. 113, is a good specimen of consecutive 5ths, which are really what they appear to be; and also of the union of ♭B 𝟟, with ♮B 5. I may here ask who can improve the harmony of this passage?

581. In the two sonatas, by Beethoven, which follow, I have introduced what I conceive to be improvements in the notation. Every one competent to form a correct opinion upon this subject must admit that our present system of musical notation is, in many respects, ingenious and admirably adapted for the purposes for which it is intended; but, it does not follow from hence that it is entirely free from defects, or that these defects cannot be removed. Let it be understood then, that nothing can be farther from my views than the idea of substituting

another system instead of the one now in existence, and thereby upsetting that system. My object is to render it more clear and distinct, where at present it is complicated and obscure. And that the advantages of the proposed improvements may be distinctly perceived, and fairly put to the test, I have chosen two pieces not only well calculated for the purpose, but which are already in the hands of an immense number of persons. Wherefore, let the corresponding passages of the present copy be carefully compared with other copies, and while engaged in the attempt to form a just estimate of the value of the innovations proposed, do not forget to take into consideration the importance of knowing the key at all times, and the advantages, both mental and practical, to be derived therefrom; and also from the analysis of the harmony generally, by means of an occasional small numeral to the right, above, or below any note. The point of importance next to the correct analysis of the composition, is, how best to express the divisions and sub-divisions arising from the prime 3, and its products $3 \times 2 = 6$; $3 \times 4 = 12$; $3 \times 3 = 9$, &c. Now, though a dot after a note is well enough adapted to represent the time *one and a half* of some real measure note (*i.e.* the minim or crotchet in common time, and the minim, crotchet, or quaver in triple time) it is an abuse of this sign, (the dot) to employ it so as to represent a measure note in connexion with the quantity *one and a half*, in order that $\frac{1}{2}$ of some real measure note may also represent $\frac{1}{3}$ of this fictitious measure note, and thereby fit it to become a clumsy substitute for a sign representing $\frac{1}{3}$ of a real measure note. The evil consequences of this awkward expedient is seen by the substitution of $\frac{12}{8}$, instead of simple common time, in Beethoven's Sonata Appassionata, and the numerous complications thence arising in the course of the first movement. Now, my copy will show that there is no occasion for all this complexity and confusion, provided it be understood that 3 quavers, when united in groups, go to one crotchet &c., without the figure 3 being placed over them. Should circumstances imperatively demand the representation of a note equal in time to $\frac{2}{3}$ the measure crotchet, this can be easily and effectually done by placing the stem on the contrary side of the head, and in the case of the quaver by turning the hook to the left. To represent corresponding portions of time, rests may be turned upside down, in which case the inverted crotchet rest will represent the $\frac{2}{3}$ crotchet rest, and the inverted quaver rest the $\frac{2}{3}$ quaver rest.

Very high and very low notes can be rendered perspicuous, at first sight, by the *temporary* adoption of a supplementary treble or base staff, see ex. 331—333 in which case those in the supplementary treble staff are two octaves above, and those in the supplementary base staff, two octaves below, the corresponding notes in the ordinary staff. As the sharps and flats at the beginning of every staff occupy space—are seldom noticed by the performer—are far more frequently wrong than right, in respect to the key, and therefore worse than useless, it will be a real boon to society to get rid of them. Concerning the clefs the rule which ought to be adopted is, to mark them only when necessary—that is at the commencement of every movement—at every change—and at the commencement of the page after turning the leaf, especially in music for the piano, when the upper staff has the base clef, or the lower the treble clef.

Instead of the usual symbol to represent the value of one semibreve in the bar, and of fractions for other bar measures, it would surely be better to mark the common or bi-equal measure by *two* minims, or *two* crotchets, and the triple measure by *three* minims, *three* crotchets, or *three* quavers, as in these two sonatas. The compound measures of 6, 9, 12, &c. in the bar, are, in my opinion, objectionable; but, if not rejected, 6 and 12 in the bar should be represented by *two dotted* minims, crotchets or quavers, 9 in the bar by *three dotted* crotchets or quavers. The rule for children to count the time by, would then be to count two in the bar, when the value of the bar is represented by 2 notes, and 3 when represented by 3 notes. Or, instead of this, to count 2 or 4 for each note when not dotted, and 3 or 6 for each note when dotted. Lastly, it will be seen that in these examples, where the time of any note is made up by uniting two or more into one, by means of the bind, I have represented such additional portions of the time by stems only, without heads, with the view to render it conspicuous that they are to be held on, but not struck a second time.

As I think it unnecesary, after the numerous examples already introduced, to expatiate upon the particulars relating to these two sonatas, I refrain from so doing; I therefore leave them for the student to make his own observations, more especially as the tonic transitions are all distinctly portrayed, and the 3rd mode symbolically represented wherever it occurs.

PART 24.

FURTHER REMARKS UPON THE CHORD OF THE DIMINISHED OR EXTREME ♭7TH. VERY LOW AND VERY HIGH SOUNDS. THE NUMBER OF VIBRATIONS IN A SECOND, ACCORDING TO THE EQUAL TEMPERAMENT. CHANGE OF KEY. A SUPPOSITIOUS SYSTEM OF FIVE BASES. AN IDEAL LAW OF PROGRESSION.

582. The subjects which occupy the present part are intended to confirm the truth of the principles already advanced, sometimes by the introduction of new matter, and sometimes by representing what has already been stated under a new point of view.

The chord of the diminished or extreme ♭7th, § 114, 117, *and* 493.

583. By attending to the coincident vibrations of the sounds ♯c, e, g, ♭b (fig. 36), resulting from the ratios of the old theory, it will be seen that during the period of the entire long cycle (the ♭D cycle), the two harmonic sounds:—

g, ♭b,	in the ratio 5 to 6	generate a vibration of the primary basis	...	♭E,	36	times.
e, g	do.	do.	...	...	...	do.	...	C,	30	do.
♯c, e	do.	do.	...	...	...	do.	...	A,	25	do.
e, ♭b,	do.	5^2 to 6^2	...	...	...	secondary basis	...	♭A	6	do.
♯c, g,	do.	do.	...	...	...	do.	...	F	5	do.
♯c, ♭b,	do.	5^3 to 6^3	...	...	...	tertiary basis	...	♭D	once.	

Furthermore that the two primary bases—

C, ♭E,	in the ratio 5 to 6,	also generate a vibration of the secondary basis	♭A,	6	times.
A, C,	do.	do.	...	...	...	do.	...	F,	5	do.
A, ♭E,	do.	5² to 6²	...	...	...	tertiary basis	...	♭D,	once.	

And lastly that the two secondary bases—

F, ♭A, in the ratio 5 to 6, generate a vibration of the tertiary basis ... ♭D, once.

It will, moreover, be seen, that the two primary bases A, C—C, ♭E, and the two secondary bases F, ♭A, are respectively minor 3rds in the ratio 5 to 6, the same as the given notes above—and that the tertiary basis ♭D, differs from a true octave

to the harmonic ♯C above, as 5^3 to $2^8 = 125$ to 128. Let us now turn to fig. 37, wherein, as the ratio of the vibrations of the two sounds, e, g, is the same as in fig. 36, they are represented by marks at exatly the same distance apart. But as the ratio of the vibrations of g, ♭b, is here 6 to 7, instead of 5 to 6—and ♯c, e, 17 to 20, instead of 5 to 6, the spaces between the marks representing these vibrations will be found to differ in agreement therewith—the value of which is pointed out in fig. 36 by the dotted lines in connexion with the small numerals $\frac{7}{6}$; $\frac{14}{12}$; $\frac{21}{18}$; $\frac{28}{24}$; $\frac{35-36}{30}$; $\frac{40}{34}$; and $\frac{60}{50-51}$.

584. In ex. 331, 332, the generated bases to the chords above are correctly represented in agreement with their ratios, and in the octave in which they are actually generated. The ledger lines are carried out as recommended in § 581, every supplementary staff being two octaves below the one above.

In ex. 332, for instance, the duet combination G, ♭B in the ratio 5, 6, generates ♭E,—E, G, in the ratio 5, 6, generates C—and ♯C, E, in the ratio 5, 6, generates A. The duet combination E, ♭B, in the ratio 5^2, $6^2 = 25$, 36 generates ♭A—and ♯C, G, in the ratio 5^2, $6^2 = 25$, 36, generates F. Lastly, the duet combination ♯C, ♭B, in the ratio 5^3, $6^3 = 125$, 216, generates ♭D. In regard to the chord ♯C 17, E 5, G 3, ♭B 7, it is important to observe that as the vibrations of bases are coincident with the extreme terms of the cycle to which they belong, and with these only, the basis to the harmonic 17 can never, by possibility, introduce a vibration extraneous to the harmony of the given sounds or chord.

585. If *A* represent a true sound, and *x*, *y*, *z*, three false sounds, it is evident the six duet combinations *Ax*, *Ay*, *Az*, *xy*, *xz*, *yz*, must all be false. If *A*, *B*, represent two true, and *y*, *z*, two false sounds, the duet combination *AB*, only will be true, and the remaining five, *Ay*, *Az*—*By*, *Bz*—*yz*, all false. If *A*, *B*, *C*, represent three true, and *z* one false sound, then the three duet combinations *AB*, *AC*, *BC*, will all be true, and *Az*, *Bz*, *Cz*, all false—and it is constantly to be borne in mind that false intonation is in all cases the fruitful parent of horribly discordant generated bases.

586. If it be said that no one before ever thought of putting the truth of the bases of chords to the test of the ear—that is no reason why it should not be done—and at all events it

remains for other theorists to explain how it happens that my theory, and mine only, can abide the severity of such a test. Besides, since it is universally admitted that the ear ought to constitute the ultimate test of truth in all other instances, in whatever relates to music, why not in this case also?

587. Coincident vibrations ought not, however, to be considered as the sole cause of the superiority of the more simple ratios in respect to their harmonious effects, when compared with others which are less simple. For, if so, what becomes of this principle in respect to the same sounds, when heard in succession? The perception of order, relation, proportion, or ratio must therefore be looked upon as the physical cause of the effects of both melody and harmony, to which coincident vibrations undoubtedly serve to add brilliancy and force, by marking the outlines and boundaries of such relations, and thereby distinguishing the various members the one from the other.

Very low and very high sounds.

588. Practically speaking, very *low* sounds have no existence, because vibrations which do not result from the continuous motion of the particles of the air do not produce the sensation commonly understood by a musical sound. For, the effects produced by vibrations which proceed from the alternating states of motion and rest, are distinguished by the term beats or knocks—and though such beats or knocks only affect the ear periodically, that is during the state of motion, the effect produced when the intonation is perfect, and these vibrations in harmony with those that are quicker, is very sublime and powerful—and, on the contrary, when these conditions are violated, the result is more or less discordant and offensive in proportion to the multitude and magnitude of the errors existing at any given time, during the sequence, which constitutes the unity of the whole. Without descending into the abyss of the region relating to beats, the point to which musical sound is limited as to depth, is therefore dependent upon the limit to which the individual particles of the atmosphere can be severally forced aside from their central or quiescent position, without being violently thrust out of their own spheres, and thereby made to exchange places one with another—or without becoming quiescent between their several vibrations. Beyond this point the air ceases to operate as the medium for the production of sounds denominated musical, and to suppose the ear adapted

to a scale of sounds at variance with that of the medium in which we are placed, and which constitutes the instrumental cause, is contrary to reason and universal experience. But, with very high sounds, since it is impossible to set any limit either to the movements of the particles of the air, or mechanism of the ear in respect to very small animals and insects, the case is entirely different. It is stated that "The human ear has been made to perceive sounds whose vibrations are 24,000 in a second." In musical notes this sound will be found to lie between ♯F on the 18th space, and G on the 18th line above the treble staff, as represented in ex. 333. To produce this sound by means of a steel string, of the very best quality, the length, at the utmost, must not exceed $\frac{3}{8}$ of an inch. A pipe or slip of glass is perhaps better adapted for the production of such sounds.

TABLE,

Showing the absolute and relative number of vibrations of the sounds of the semitonic scale, according to the equal temperament, from C with 16, to C, the eighth octave above, with 4096 vibrations in a second.

C	*	D	*	E	F	*	G	*	A	*	B	C
$2^{11}=2048$	2169.78	2298.80	2435.49	2580.32	2733.73	2896.30	3068.54	3250.99	3444.32	3649.12	3866.10	$4096=2^{12}$
$2^{10}=1024$	1084.89	1149.40	1217.74	1290.16	1366.86	1448.15	1534.27	1625.49	1722 16	1824.56	1933.05	$2048=2^{11}$
$2^{9}=512$	542.44	574.70	608.87	645.08	683.43	724.07	767.13	812.74	861.08	912.28	966.52	$1024=2^{10}$
$2^{8}=256$	271.22	287.35	304.43	322.54	341.71	362.03	383.56	406.37	430.54	456.14	483.26	$512=2^{9}$
$2^{7}=128$	135.61	143.67	152.21	161.27	170.85	181.01	191.78	203.18	215.27	228.07	241.63	$256=2^{8}$
$2^{6}=64$	67.80	71.83	76.10	80.63	85.42	90.50	95.89	101 59	107.63	114.03	120.81	$128=2^{7}$
$2^{5}=32$	33.90	35.91	38.05	40.31	42.71	45.25	47.94	50.79	53.81	57.01	60.40	$64=2^{6}$
$2^{4}=16$	16.95	17.95	19.02	20.15	21.35	22.62	23.97	25.39	26.90	28.50	30.20	$32=2^{5}$
$2^{3}=8$ $2^{2}=4$ 2 1 } Beats.												

589. The vibrations $2^9 = 512$ in a second produce the sound C upon the 3rd space in the treble, in reference to which the ratios below severally answer to the octaves below, and the ratios above to the octaves above.

As the vibrations 1, 2, 4, 8, in a second are sufficiently slow to be counted, and individually perceived as so many distinct knocks or blows, they are called *beats* (§ 63); but as the effect of continuity is essential to the idea of musical sounds, beats, correctly speaking, are not musical sounds, although the junctures at which they take place are continually coincident with the prominent features in melody, and the crash of chords in harmony—and hence it follows that since the low sounds called beats are, as it were, spontaneously generated by means of these higher sounds, the attempt to re-enforce these natural concussions, at a vast expense, by artificial means, is a lamentable proof of the utter want of scientific knowledge in regard to such matters. Nevertheless, the attempt to do this is evinced every time it is proposed to build an organ with monster pipes, 64 feet long, like that recommended to the proprietors of the Crystal Palace, Sydenham. At the time this was in agitation, I took the liberty to forewarn the proprietors through their agents, that if put into execution it would for certain turn out a failure.

590. Although the vibrations of the note C upon the 3rd space are here set down as being 512 in a second, it is not to be understood that this, or any other number, has been unanimously agreed upon by professors, composers, or theorists. A few years ago the French, by command of the present Emperor, undertook to investigate this matter, the result of which was that they came to the resolution that, in France, the vibrations of the note A upon the first ledger line in the treble should be 870 in a second, which would give 517·303, instead of 512 for C upon the third space—whereupon it became the law of the land that 870 vibrations in a second should be the standard pitch for this note A, and consequently 435, $217\frac{1}{2}$, $108\frac{3}{4}$, $54\frac{3}{8}$, and $27\frac{3}{16}$, for the octaves below.

Shortly after this, in imitation of the French, the English took up the subject, but after going through the formality of a few ceremonious meetings in London, the matter was quietly dropped without coming to any decision whatever—thereby leaving every free-born Englishman, while in England, in perfect freedom to let down or screw up the pitch, just as he may think proper.

The only reasons I have to give for the pitch I have chosen is —(1) Because it is about midway between the two extremes adopted in different countries. (2) Because it presents throughout the series, whole numbers for the notes denominated C— and :—(3) Because it thereby becomes, as it were, linked with the grand operations of nature, in respect to the diurnal revolution of the earth upon its centre.

591. In § 315 it is shown that 2^{49} vibrations would require 34,865 years to complete the series, at the rate of 512 in a second. But if, instead of vibrations we understand the product 2^{49} to represent miles, and then enquire how long it would take for light to traverse that distance, the velocity of which is stated as being 192,000 miles in a second *—the answer is, $\frac{562,949,953,421,312}{192,000}$ seconds=93 years very nearly. Now, though 93 years is but a short period compared with 34,865—if we reflect upon it in connexion with the velocity of 192,000 miles in a second, the idea of its immensity is equally overpowering and inconceivable. But if, in reference to sound, we are struck with the very great length of time required for the completion of 2^{49} vibrations, at the rate of 512 in a second, we shall perhaps be still more so at the shortness of time in which the same number is completed, when the vibrations relate to color.

In order to show this, I here subjoin an extract from the Bridgewater Treatise on Astronomy and General Physics (p. 137), by the Rev. Wm. Whewell, M.A.

"It is only within certain limits that the effect" (*i.e.* of color) "is produced, and these limits are comparatively narrower here than in the case of sound. The whole scale of color, from violet to crimson, lies between vibrations which are 458 million millions, and 727 million millions in a second; a proportion much smaller than the corresponding ratio for perceivable sounds." According to this statement, and taking only the three first figures (562), relating to the vibrations of the note C 2^{49}, the time required for the same number of vibrations in the production of the violet colour would be $\frac{562}{458}$ = about $1\frac{1}{4}$ seconds—and for the production of the crimson color only $\frac{562}{727}$ = about $\frac{4}{5}$ of a second.

* This now contradicted by M. Foucault, who states that a reduction of about 3 or 4 per cent must be made from this amount.

Observations on change of key.

592. Should every common chord involve the idea of a different key? If so, then the key to every note in melody is changed with every basal transition—and then in harmony we must be sometimes in two and even three keys at the same time. Again, if in the common cadence of the chords C, F, G, C, the key be C throughout, what difference can it make in passing from the chord of C to F, whether the ♭7th be added to the chord of C or not? Are we in the key of F in one case and not in the other? And, in like manner, in passing from the chord of F to G, if ♯F be introduced between F and G, are we in G in the one case and not in the other? Has not a centre of attraction been felt and tacitly acknowledged under the idea and name of key-note—and now, because, for the first time it can be clearly shown to have a real existence, is this to be made the ground for raising a doubt as to the fact? If there be no centre of attraction, then ought we always to feel in a quiescent position, or, as it were, at home with any and every new basis, and common chord to that basis. But as this is not the case, then is the great fact of a centre of attraction fully proved—and if so, the key is not changed by the introduction of the ♭7th upon any given basis—the change effected being merely that of passing from the centre, outwards to the circumference whereby the influence of the centre becomes palpably felt from the effort it makes to draw us back again—an effort that increases in power and intensity as we approach, by means of chromatic sounds, to the central point itself.

As to the term *partial modulation*, or *partial change of key*, as introduced by some theorists, as I confess myself entirely ignorant as to whether these expressions are to be understood as implying the condition of being between two keys, and consequently in no key at all, or in two keys at once, of course I am unable to enter in a discussion upon the subject.

593. If it be asserted that a tonic transition can be anticipated by the ear, I would ask what species of tonic transition it is? For, if any one in particular, then are all others necessarily excluded—and if none in particular, this is equivalent to none at all. And, again, if one in particular, whence the cause of this distinction? If it be said to the 5th above, on account of its simplicity, then why not to the octave above or below, which is still more simple?—or rather, why not to the 5th

below, which is equally simple? And, again, if any tonic transition in particular be anticipated, then it follows that this natural desire must be thwarted whenever any other species of tonic transition takes place. If it be said that in a composition already known a tonic transition is anticipated, this proves nothing more than that the composition or passage in question is impressed on the memory. In short *the mind never anticipates a change of key*—for, in the first place, every chromatic harmonic finds its resolvent *in* the key, and consequently every chromatic chord must also find its resolvent *in* the key. Furthermore, if the mind could by anticipation forestall a new key, it must necessarily be under the dominion of the forthcoming as well as the present key—consequently under the dominion of at least two keys at the same time. But, admitting the principle that the attraction of the keys which may be said to surround any given central key is in all directions, and equal—a perfect equilibrium is the necessary consequence—whence arises perfect freedom—that is, the freedom to pass to the right or the left, either to the nearest, or some of the more or less remote keys.

594. Should any one still feel disposed to withhold his assent to the existence of a great system, simply because it does not force itself upon the conviction of those who will not be at the trouble to examine the evidence in its favour—let them, in justice to themselves as well as to others, tell us how it is that all authors, in all ages, and of all nations, have been induced in more than 99 cases out of 100, to end in the key in which they began.

Upon a suppositious system of 5 bases.

595. Should any one imagine that a system of sounds might be formed of 5 simple systems, that is of 1 central in conjunction with 2 sub and 2 super simple systems—such, for instance, as that of C for the central basis, ♭A in the ratio $\frac{5}{1}$, F in the ratio $\frac{3}{1}$, for the two sub-bases, and E in the ratio $\frac{1}{5}$, G in the ratio $\frac{1}{3}$, for the two super-bases—let him reflect for a moment upon the consequences resulting from the admission of such a principle, and he will assuredly reject it. In such a scale C is not the immediate central basis to either of the two extreme bases ♭A, E—the central basis to ♭A as a sub-basis being ♭E—and the central-basis to E as a super-basis being A. But a scale consisting of 4 minor centres, in conjunction with 1 common centre, and an heterogeneous mass of particulars, is no scale at all. In

the triune system the basis C is the central-basis in respect to the sub-basis F, and the super-basis G—and the simple system of C a central-system in respect to the sub-system of F, and the super-system of G. But in the above suppositious system of 4 minor centres and 1 common centre, C is neither the basis nor an harmonic to the two fictitious centres ♭E, A, that is, if we take into consideration the true ratio of these sounds. For, in this case, the basis ♭E would not be in unison with the harmonic 19 to C, neither would the basis C be in unison with the note C, the harmonic 19 to the basis A.

An ideal law of progression.

596. There can be no such thing as a law of progression—and the yearning that is felt for such a monitory guide arises from the want of creative power. The thoughts originating in the feelings which give birth to this idea, do not therefore express anything corresponding to the emotions and pleasurable sensations which we experience from the operation of the laws of nature, but, on the contrary, a somewhat allied to vacuity, hunger, or thirst. From hence it follows that the law of progression, if anything, must be subject to the idiosyncracy peculiar to every individual, and consequently not of universal application. But, be this as it may, at what point is the law of progression to begin to take effect? Is it to do everything, from first to last, while we sit still and look on? or is it to commence. after the introduction of a given portion? and, if so, how is this introductory portion to be accomplished without the aid of this law?

PART 25.

NEW TABLES OF LOGARITHMS, AND THEIR ADAPTATION TO MUSICAL PURPOSES—SCALE OF 53 SOUNDS IN THE OCTAVE—CONCLUSION.

597. Having frequently experienced the inconvenience arising from the unfitness of the common logarithms to represent musical intervals, in consequence of the octave being practically divided into 12 and not into 10 equal parts, I resolved upon

the removal of this evil by the construction of a table of logarithms, in which the semitones are represented by whole numbers, and the fractional parts of the semitone by decimals.

598. Though the mathematician will see at once the object of the following logarithmic tables, and wherein they differ from the ordinary logarithms, in which 10 is represented by 1,—100 by 2,—1000 by 3, &c., it may be necessary to observe that by making the whole numbers 1—11 represent the 11 semitones of the chromatic scale, between the octave 0—12, and by the use of decimals in connexion with these whole numbers to represent intermediate sounds, I adopt a system of logarithms especially adapted to musical purposes, and by means of which the pitch of any sound is accurately represented to the twenty-millionth part of a mean semitone, and consequently to the two hundred and forty-millionth part of the octave.

In other words, though the common logarithms, as well as these, in respect to their natural numbers, stand in the relation of intervals to vibrations—the intervals represented by the common logarithms refer to quantities indirectly and awkwardly connected with the interval of the octave, and the 12 mean semitones into which this interval is practically divided—and moreover convertible into these quantities only by means of a troublesome and laborious process—whereas by the new set of logarithms this is seen at once, freed from these obscurities and the necessity of any additional labour to render them intelligible.

599. In these tables I have throughout given the pedigree of all the numbers, for example, $3 \times 5 = 15$; $7 \times 3^2 = 63$. The primes being distinguished by thicker figures and numbered.

The first 4 pages contain the logarithms of *all* numbers from 1 to 512. Page 230, of the *primes only* between 512 and 1024. And with the view to facilitate the practical application of these logarithms for ordinary purposes, I have appropriated page 231, to the primes 1 to 41, only, with their octaves.

TABLE OF LOGARITHMS

Of all numbers from 1 *to* 512 *; the octaves to the basis being represented by the number of their mean semitones,* 12, 24, 36, *&c. To this is added the indices of the primes whence the powers and products are derived.*

	Harmonic Ratios.							Logarithms.
{1 The 1st prime and absolute Basis for which 2, 2^2, 2^3, &c., is allowed to pass as its practical representative.}								
2nd Prime	2—	4—	8—	16	32	64	128	256		·0000000
56th Prime	...	...	...	...	...	...	...	**257**		·0674946
3×43=	...	...	...	...	...	...	129	258		·1347271
7×37=	...	...	...	...	...	...	...	259		·2016995
5×13=	...	...	...	...	65	130	260		·2684138
3^2×29=	...	...	...	...	...	...	261		·3348720
33rd Prime	...	...	...	...	...	**131**	262		·4010760
57th Prime	...	...	...	...	...	...	**263**		·4670279
3×11=	...	...	...	33	66	132	264		·5327294
5×53=	...	...	...	...	...	...	265		·5981826
7×19=	...	...	...	...	...	133	266		·6633892
3×89=	...	...	...	...	...	...	267		·7283511
20th Prime	...	...	...	...	67	134	268		·7930703
58th Prime	...	...	...	...	...	...	**269**		·8575484
3^3×5=	...	...	...	...	...	135	270		·9217872
59th Prime	...	...	...	...	...	...	**271**		·9857885
								Semitone 0, and its Octaves.	
8th Prime	...	17	34	68	136	272		·0495541
3×7×13=	...	...	...	...	...	...	273		·1130857
34th Prime	...	...	...	...	...	**137**	274		·1763850
5^2×11=	...	...	...	...	...	...	275		·2394537
3×23=	...	...	...	69	138	276		·3022935
60th Prime	...	...	...	...	...	...	**277**		·3649060
35th Prime	...	...	...	...	...	**139**	278		·4272929
3^2×31=	...	...	...	...	...	...	279		·4894558
5×7=	...	...	35	70	140	280		·5513962
61st Prime	...	...	...	...	...	...	**281**		·6131159
3×47=	...	...	...	...	...	141	282		·6746162
62nd Prime	...	...	...	...	...	...	**283**		·7358989
21st Prime	...	...	...	71	142	284		·7969654
3×5×19=	...	...	...	...	...	...	285		·8578173
11×13=	...	...	...	...	...	143	286		·9184560
7×41=	...	...	...	...	...	...	287		·9788831
								Semitone 1, and its Octaves.	

	Harmonic Ratios.							Logarithms.
3^2=	...	9	18	36	72	144	288		·0391000
17^2=	...	...	...	...	...	...	289		·0991082
5×29=	...	...	...	...	...	145	290		·1589091
3×97=	...	...	...	...	...	...	291		·2185041
22nd Prime	...	...	...	...	**73**	146	292		·2778947
63rd Prime	...	...	...	...	...	...	**293**		·3370823
3×7^2=	...	...	...	...	...	147	294		·3960682
5×59=	...	...	...	...	...	...	295		·4548537
13th Prime	...	...	...	**37**	74	148	296		·5134404
3^3×11=	...	...	...	...	...	...	297		·5718295
36th Prime	...	...	...	...	...	**149**	298		·6300223
13×23=	...	...	...	...	...	...	299		·6880201
3×5^2=	...	...	...	...	75	150	300		·7458243
7×43=	...	...	...	...	...	...	301		·8034361
37th Prime	...	...	...	...	...	151	302		·8608569
3×101=	...	...	...	...	...	...	303		·9180878
9th Prime	...	**19**	38	76	152		304		·9751302
								Semitone 2, and its Octaves.	
5×61=	...	...	...	...	...	...	305		·0319852
3^2×17=	...	...	...	...	...	153	306		·0886541
64th Prime	...	...	...	...	...	...	**307**		·1451382
7×11=	...	...	...	...	77	154	308		·2014385
3×103=	...	...	...	...	...	...	309		·2575564
5×31=	...	...	...	...	...	155	310		·3134929
65th Prime	...	...	...	...	...	...	**311**		·3692493
3×13=	...	...	...	39	78	156	312		·4248266
66th Prime	...	...	...	...	...	...	**313**		·4802262
38th Prime	...	...	...	...	...	**157**	314		·5354490
3^2×5×7=	...	...	...	...	...	...	315		·5904962
23rd Prime	...	...	...	...	**79**	158	316		·6453690
67th Prime	...	...	...	...	...	...	**317**		·7000684
3×53=	...	...	...	...	...	159	318		·7545955
11×29=	...	...	...	...	...	...	319		·8089514
4th Prime	**5**—	10—	20	40	80	160	320		·8631372
3×107=	...	...	...	...	...	...	321		·9171539
7×23=	...	...	...	...	...	161	322		·9710026
								Semitone 3, and its Octaves.	

	Harmonic Ratios.							Logarithms.
$17\times19=$	...	...	...	...	...	323	Semitone 4, and its Octaves.	·0246843
$3^4=$ ...	...	...	...	81	162	324		·0782000
$5^2\times13=$	...	...	...	...	...	325		·1315509
39th Prime	...	...	...	...	**163**	326		·1847379
$3\times109=$	...	...	...	...	...	327		·2377619
14th Prime	...	...	**41**	82	164	328		·2906241
$7\times47=$	...	...	...	...	...	329		·3433253
$3\times5\times11=$	...	...	...	...	165	330		·3958666
68th Prime	...	...	...	...	...	**331**		·4482489
24th Prime	...	...	...	**83**	166	332		·5004732
$3^2\times37=$	...	...	...	...	...	333		·5525404
40th Prime	...	...	...	...	**167**	334		·6044515
$5\times67=$	...	...	...	...	...	335		·6562074
$3\times7=$	...	21	42	84	168	336		·7078091
69th Prime	...	...	...	...	...	**337**		·7592574
$13^2=$ ...	...	...	...	...	169	338		·8105533
$3\times113=$	...	...	...	...	...	339		·8616976
$5\times17=$	...	...	...	85	170	340		·9126912
$11\times31=$	...	...	...	...	...	341		·9635352
$3^2\times19=$	...	...	...	...	171	342	Semitone 5, and its Octaves.	·0142302
$7^3=$ ...	...	...	...	...	...	343		·0647772
15th Prime	...	...	**43**	86	172	344		·1151771
$3\times5\times23=$	...	...	...	...	...	345		·1654306
41st Prime	...	...	...	...	**173**	346		·2155387
70th Prime	...	...	...	...	...	**347**		·2655022
$3\times29=$	...	...	...	87	174	348		·3153220
71st Prime	...	...	...	...	...	**349**		·3649987
$5^2\times7=$	...	...	...	...	175	350		·4145334
$3^3\times13=$	...	...	...	...	...	351		·4639267
6th Prime	**11**	22	44	88	176	352		·5131794

	Harmonic Ratios.							Logarithms
72nd Prime	...	...	...	...	...	**353**	Semitone 5 continued.	·5622925
$3\times59=$	...	...	...	...	177	354		·6112666
$5\times71=$	...	...	...	...	...	355		·6601026
25th Prime	...	...	...	**89**	178	356		·7088011
$3\times7\times17=$	...	...	...	...	...	357		·7573632
42nd Prime	...	...	...	...	**179**	358		·8057893
73rd Prime	...	...	...	...	...	**359**		·8540804
$3^2\times5=$	...	...	45	90	180	360		·9022372
$19^2=$	...	...	...	...	...	361		·9502603
43rd Prime	...	...	...	...	**181**	362		·9981507
$3\times11^2=$	...	...	...	...	...	363	Semitone 6, and its Octaves.	·0459089
$7\times13=$	...	...	...	91	182	364		·0935357
$5\times73=$	...	...	...	...	...	365		·1410319
$3\times61=$	...	...	...	...	183	366		·1883981
74th Prime	...	...	...	...	...	**367**		·2356351
10th Prime	...	**23**	46	92	184	368		·2827435
$3^2\times41=$	...	...	...	...	...	369		·3297241
$5\times37=$	...	...	...	...	185	370		·3765775
$7\times53=$	...	...	...	...	...	371		·4233045
$3\times31=$	...	...	...	93	186	372		·4699057
75th Prime	...	...	...	...	...	**373**		·5163819
$11\times17=$	...	...	...	...	187	374		·5627335
$3\times5^3=$	...	...	...	...	...	375		.6089614
16th Prime	...	...	**47**	94	188	376		.6550662
$13\times29=$	...	...	...	...	...	377		·7010486
$3^3\times7=$	...	...	...	...	189	378		·7469091
76th Prime	...	...	...	...	...	**379**		·7926485
$5\times19=$	...	...	...	95	190	380		·8382673
$3\times127=$	...	...	...	...	...	381		·8837663
44th Prime	...	...	...	...	**191**	382		·9291460
77th Prime	...	...	...	...	...	**383**		·9744070

Harmonic Ratios.						Logarithms.		
3rd Prime **3**	6	12	24	48	96	192	384	·0195500
5×7×11=	...	...	...	...	...	...	385	·0645756
45th Prime	...	...	...	...	...	**193**	386	·1094845
3^2×43=	...	...	...	...	...	...	387	·1542771
26th Prime	...	...	...	...	**97**	194	388	·1989541
78th Prime	...	...	...	...	...	...	**389**	·2435162
3×5×13=	...	...	...	...	...	195	390	·2879638
17×23=	...	...	...	...	...	...	391	·3322976
7^2= ...	...	...	...	49	98	196	392	·3765181
3×131=	...	...	...	...	...	...	393	·4206260
46th Prime	...	...	...	...	...	**197**	394	·4646218
5×79=	...	...	...	...	...	...	395	·5085061
3^2×11=	...	...	...	...	99	198	396	·5522794
79th Prime	...	...	...	...	...	...	**397**	·5959424
47th Prime	...	...	...	...	...	**199**	398	·6394955
3×7×19=	...	...	...	...	...	...	399	·6829393
5^2= ...	...	...	25	50	100	200	400	·7262743
80th Prime	...	...	...	...	...	...	**401**	·7695011
3×67=	...	...	...	...	...	201	402	·8126203
13×31=	...	...	...	...	...	...	403	·8556323
27th Prime	...	...	...	...	**101**	202	404	·8985378
3^4×5=	...	...	...	...	...	...	405	·9413371
7×29=	...	...	...	...	...	203	406	·9840310
11×37=	...	...	...	...	...	...	407	·0266198
3×17=	...	...	...	51	102	204	408	·0691041
81st Prime	...	...	...	...	...	...	**409**	·1114844
5×41=	...	...	...	...	...	205	410	·1537612
3×137=	...	...	...	...	...	...	411	·1959350
28th Prime	...	...	...	...	**103**	206	412	·2380063
7×59=	...	...	...	...	...	...	413	·2799757
3^2×23=	...	...	...	...	...	207	414	·3218435
5×83=	...	...	...	...	...	...	415	·3636103

Semitone **7**, and its Octaves.

Semit. **8**, and its Octaves.

Harmonic Ratios.						Logarithms.		
7th Prime	**13**	26	52	104	208	416		·4052766
3×139=	...	...	...	...	...	417		·4468429
11×19=	...	...	...	...	209	418		·4883096
82nd Prime	...	...	...	...	...	**419**		·5296772
3×5×7=	...	...	...	105	210	420		·5709462
83rd Prime	...	...	...	...	...	**421**		·6121171
48th Prime	...	...	...	...	**211**	422		·6531903
3^2×47=	...	...	...	...	...	423		·6941662
17th Prime	...	...	**53**	106	212	424		·7350455
5^2×17=	...	...	...	...	...	425		·7758284
3×71=	...	...	...	...	213	426		·8165155
7×61=	...	...	...	...	...	427		·8571072
29th Prime	...	...	...	**107**	214	428		·8976039
3×11×13=	...	...	...	...	...	429		·9380060
5×43=	...	...	...	...	215	430		·9783142
84th Prime	...	...	...	...	...	**431**		·0185287
3^3= ...	...	27	54	108	216	432		·0586500
85th Prime	...	...	...	...	...	**433**		·0986786
7×31=	...	...	...	...	217	434		·1386148
5×87=	...	...	...	...	...	435		·1784590
30th Prime	...	...	...	**109**	218	436		·2182119
19×23=	...	...	...	...	...	437		·2578737
3×73=	...	...	...	...	219	438		·2974447
86th Prime	...	...	...	...	...	**439**		·3369256
5×11=	...	...	55	110	220	440		·3763166
3^2×7^2=	...	...	...	...	...	441		·4156181
13×17=	...	...	...	...	221	442		·4548307
87th Prime	...	...	...	...	...	**443**		·4939547
3×37=	...	...	...	111	222	444		·5329904
5×89=	...	...	...	...	...	445		·5719382
49th Prime	...	...	...	...	**223**	476		·6107988
3×149=	...	...	...	...	...	447		·6495723

Semitone **8** continued.

Semitone **9**, and its Octaves.

	Harmonic Ratios.							Logarithms.
5th Prime	7	14	28	56	112	224	448	·6882591
88th Prime	...	...	...	...	...	...	**449**	·7268596
$3^2\times5^2=$	...	...	...	...	...	225	450	·7653743
$11\times41=$	...	...	...	...	...	...	451	·8038035
31st Prime	...	...	...	...	**113**	226	452	·8421476
$3\times151=$	...	...	...	...	...	...	453	·8804069
50th Prime	...	...	...	...	...	**227**	454	·9185819
$5\times91=$	...	...	...	...	...	...	455	·9566728
$3\times19=$	...	...	...	57	114	228	456	·9946802
89th Prime	...	...	...	...	...	...	**457**	·0326043
51st Prime	...	...	...	...	...	**229**	458	·0704455
$3^3\times17=$	...	...	...	...	...	...	459	·1082041
$5\times23=$	...	...	...	...	115	230	460	·1458806
90th Prime	...	...	...	...	...	...	**461**	·1834753
$3\times7\times11=$	...	...	...	...	...	231	462	·2209885
91st Prime	...	...	...	...	...	...	**463**	·2584206
11th Prime	...	**29**	58	116	232	...	464	·2957719
$3\times5\times31=$	...	...	...	...	...	...	465	·3330428
52nd Prime	...	...	...	...	...	**233**	466	·3702338
92nd Prime	...	...	...	...	...	...	**467**	·4073449
$3^2\times13=$	...	...	...	...	117	234	468	·4443766
$7\times67=$	...	...	...	...	...	...	469	·4813294
$5\times47=$	...	...	...	...	...	235	470	·5182034
$3\times157=$	...	...	...	...	...	...	471	·5549990
18th Prime	...	...	**59**	118	236	...	472	·5917166
$11\times43=$	...	...	...	...	...	...	473	·6283565
$3\times79=$	...	...	...	...	...	237	474	·6649190
$5^2\times19=$	...	...	...	...	...	...	475	·7014045
$7\times17=$	...	...	...	...	119	238	476	·7378132
$3^2\times53=$	...	...	...	...	...	...	477	·7741455
53rd Prime	...	...	...	...	...	**239**	478	·8104017
93rd Prime	...	...	...	...	...	...	**479**	·8465822
$3\times5=$ ...	15	30	60	120	240	...	480	·8826872
$13\times37=$	...	...	...	...	...	...	481	·9187170
54th Prime	...	...	...	...	...	**241**	482	·9546721
$3\times7\times23=$	...	...	...	...	...	...	483	·9905526

Semitone 9 continued.

Semitone 10, and its Octaves.

	Harmonic Ratios.							Logarithms.
$11^2=$ ...	...	...	...	...	121	242	484	·0263589
$5\times97=$	...	...	...	...	...	...	485	·0620912
$3^5=$ ...	...	...	...	...	...	243	486	·0977501
94th Prime	...	...	...	...	...	...	**487**	·1333356
19th Prime	...	...	**61**	122	244	...	488	·1688481
$3\times163=$	...	...	...	...	...	...	489	·2042879
$5\times7^2=$	...	...	...	...	...	245	490	·2396553
95th Prime	...	...	...	...	...	...	**491**	·2749506
$3\times41=$	...	...	...	...	123	246	492	·3101741
$17\times29=$	...	...	...	...	...	..	493	·3453260
$13\times19=$	...	...	...	...	...	247	494	·3804068
$3^2\times5\times11=$	...	...	...	...	...	...	495	·4154165
12th Prime	...	**31**	62	124	248	...	496	·4503557
$7\times71=$	...	...	...	...	...	...	497	·4852245
$3\times83=$	...	...	...	...	...	249	498	·5200232
96th Prime	...	...	...	...	...	...	**499**	·5547521
$5^3=$ ...	...	...	...	...	125	250	500	·5894114
$3\times167=$	...	...	...	...	...	...	501	·6240015
55th Prime	...	...	...	...	...	**251**	502	·6585227
97th Prime	...	...	...	...	...	...	**503**	·6929751
$3^2\times7=$	...	...	...	63	126	252	504	·7273591
$5\times101=$	...	...	...	...	...	...	505	·7616749
$11\times23=$	...	...	...	...	...	253	506	·7959229
$3\times13^2=$	...	...	...	...	...	...	507	·8301032
32nd Prime	...	...	...	**127**	254	...	508	·8642163
98th Prime	...	...	...	...	...	...	**509**	·8982622
$3\times5\times17=$	...	...	...	...	...	255	510	·9322413
$7\times73=$	...	...	...	...	...	...	511	·9661538
2nd Prime	**2**-4-8-16-32			64	128	256	512	·0000000

Semitone 11, and its Octaves.

Semitone 12

TABLE OF LOGARITHMS

Of all the intermediate primes between 512 *and* 1024.

Number of the Prime.	Primes.		Logarithms.
99	521	Semitone 0, and its Octaves.	·3016748
100	523		·3680057
101	541		·9538174
102	547	Semitone 1, and its Octaves.	·1447643
103	557		·4584022
104	563		·6438934
105	569		·8274181
106	571		·8881632
107	577	Semitone 2, and its Octaves.	·0691301
108	587		·3666003
109	593		·5426596
110	599		·7169463
111	601		·7746542
112	607		·9466325
113	613	Semitone 3, and its Octaves.	·1169192
114	617		·2295202
115	619		·2855472
116	631		·6179544
117	641		·8901666
118	643		·9440992
119	647	Semitone 4, and its Octaves.	·0514328
120	653		·2112702
121	659		·3696159
122	661		·4220776
123	673		·7335524
124	677		·8361443
125	683		·9889012
126	691	Semitone 5, and its Octaves.	·1905028
127	701		·4892476
128	709		·6357018
129	719		·8781755
130	727	Semitone 6, and its Octaves.	·0697387
131	733		·2120327
132	739		·3531667
133	743		·4466208
134	751		·6320292
135	757		·7697939
136	761		·8610317
137	769	Semitone 7, and its Octaves.	·0420775
138	773		·1318953
139	787		·4426379
140	797		·6612310
141	809		·9199507
142	811		·9626973
143	821	Semitone 8, and its Octaves.	·1748610
144	823		·2169835
145	827		·3009223
146	829		·3427895
147	839		·5503240
148	853		·8368232
149	857		·9178168
150	859		·9581719
151	863	Semitone 9, and its Octaves.	·0386010
152	877		·3171964
153	881		·3959785
154	883		·4352356
155	887		·5134836
156	907		·8995049
157	911		·9756869
158	919	Semitone 10, and its Octaves.	·1270526
159	929		·3144175
160	937		·4628629
161	941		·5366110
162	947		·6466474
163	953		·7559889
164	967	Semitone 11, and its Octaves.	·0084656
165	971		·0799298
166	977		·1865770
167	983		·2925713
168	991		·4328950
169	997		·5373964
170	1009		·7481132
171	1013		·8130215
172	1019		·9152601
173	1021		·9492058

LOGARITHMIC TABLE

Of the Primes from 1 *to* 41, *in connexion with several of their Octaves, and in which the octaves to the Basis are represented by the number of their mean semitones* 12, 24, 36, *&c.*

	Hms.	Logs.	Hms.	Logs.	Hms.	Logs.	Hms.	Logs.	Hms.	Logs.	Hms.	Logs.	Hms.	Logs.	Hms.	Logs.	Hms.	Logs.	Hms.	Logs.	Hms.	Logs.
2^8 =	256	96.	384	103.	320	99.	448	105.	352	101.	416	104.	272	97.	304	98.	368	102.	464	106	496	107.	296	98.	328	100.
2^7 =	128	84.	192	91.	160	87.	224	93.	176	89.	208	92.	136	85.	152	86.	184	90.	232	94.	248	95.	148	86.	164	88.
2^6 =	64	72.	96	79.	80	75.	112	81.	88	77.	104	80.	68	73.	76	74.	92	78.	116	82.	124	83.	74	74.	82	76.
2^5 =	32	60.	48	67.	40	63.	56	69.	44	65.	52	68.	34	61.	38	62.	46	66.	58	70.	62	71.	37	62.	41	64.
2^4 =	16	48.	24	55.	20	51.	28	57.	22	53.	26	56.	17	49.	19	50.	23	54.	29	58.	31	59.		50.		52.
2^3 =	8	36.	12	43.	10	39.	14	45.	11	41.	13	44.		37.		38.		42.		46.		47.		38.		40.
2^2 =	4	24.	6	31.	5	27.	7	33.		29.		32.		25.		26.		30.		34.		35.		26.		28.
2 =	2	12.	3	19.		15.		21.		17.		20.		13.		14.		18.		22.		23.		14.		16.
1 = The absolute Basis for which $2, 2^2, 2^3$, &c., is allowed to pass as its practical representative.		0 .0000000		7. .0195500		3. .8631372		9. .6882591		5. .5131794		8. .4052766		1. .0495541		2. .9751302		6 .2827435		10. .2957719		11. .4503557		2. .5134404		4. .2906241
0	12		7		4		10		5—6		8—9		1—2		3		7—6		10		12		3		4	

The decimals placed vertically are referable to every octave in the series, and also to every octave above and below. The numerals in the lowest line represent the practical intervals of the harmonics above, and severally stand for the logarithms to the mean intervals of the semitonic scale. The semitonic intervals represented as practical, are those only which lie within the compass of the octave.

600. The following calculations show the difference between the logarithmic ratios of the same practical intervals, and thereby prove the accuracy of those ratios.

To find the log. to the harmonics of the sub-basis, subtract the log. of the harmonic 3 (7·0195500) from the corresponding harmonic of the central-basis. For example, the log. of the sub-basis is 12·0000000—7·0195500=4·9804500.

OCTAVES.

Log. of the harmonic 2, the basal 8th or octave 12.0000000

				Practical Intervals.
Log. of the harmonic 3		7·0195500 12		7th semitone
,, ,, ,,	3^{12}	84·2346000	7×12=84th	,,
,, ,, ,,	5	3·8631372 3	4th	,,
,, ,, ,,	5^3	11·5894116	4×3=12th	,,
,, ,, ,,	7	9·6882591 6	10th	,,
,, ,, ,,	7^6	58·1295546	10×6=60th	,,
,, ,, ,,	11	5·5131794 2	6th	,,
,, ,, ,,	11^2	11·0263588	6×2=12th	,,
,, ,, ,,	13	8·4052766 3	8th	,,
,, ,, ,,	13^3	25·2158298	8×3=24th	,,
,, ,, ,,	17	49·0495541 12	49th	,,
,, ,, ,,	17^{12}	588·5946492	49×12=588th	,,
,, ,, ,,	19	2·9751302 4	3rd	,,
,, ,, ,,	19^4	11·9005208	3×4=12th	,,

FIFTHS.

Log. of the harmonic 3, the basal 5th .	7·0195500*
Log. of the harmonic 5 . . .	15·8631372
" " " 3^3	9·0586500
" " intermediate 5th . . .	6·8044872

For example, D3^2 minus A5, in the key of C.
Hence the log. to the ratio 80, 81, is 7·0195500—6·8044872 = 0·2150628.

MAJOR THIRDS.

Log. of the harmonic 5, the basal major 3rd	3·8631372
Log. of the harmonic 3 . . .	7·0195500
" " " 19	2.9751302
" " intermediate major 3rd . .	4·0444198

For example, ♭E 19, G 3, in the key of C.

Log. of the harmonic 3^2 . . .	14·0391000
" " " 7 . . .	9·6882591
" " intermediate major 3rd . .	4·3508409

For example, ♭B 7, D 3^2, in the key of C.

MINOR THIRDS.

Log. of the harmonic 19, the basal minor 3rd	2·9751302
Log. of the harmonic 3 . . .	7·0195500
" " " 5 . . .	3·8631372
" " intermediate minor 3rd . .	3·1564128

For example, E 5, G 3, in the key of C.

Log. of the harmonic 7 . . .	9·6882591
" " " 3 . . .	7·0195500
" " intermediate minor 3rd . .	2·6687091

For example, G 3, ♭B 7, in the key of C.

* Let it be remembered that the log. of the *tempered* interval of the 5th, major 3rd, and minor 3rd, are respectively 7, 4, 3.

MAJOR SECONDS OR WHOLETONES.

Log. of the harmonic 2	12·0000000
„ „ „ 7	9·6882591
„ „ intermediate wholetone	2·3117409

For example, ♭B7, C 1, and F 7, G 1, in the key of C.

Log. of the harmonic 3^2	14·0391000
„ „ „ 2	12·0000000
„ „ basal wholetone	2·0391000

For example, C 1, D 3, in the key of C.

Log. of the harmonic 5	3·8631372
„ „ „ 3^2	2·0319000
„ „ intermediate wholetone	1·8240372

For example, D 3, E 5, in the key of C.

MINOR SECONDS OR SEMITONES.

Log. of the harmonic 15	10·8826872
„ „ „ 14	9·6882591
„ „ intermediate semitone	1·1944281

For example, ♭B 7, ♯B 15, in the key of C.

Log. of the harmonic 16	12·0000000
„ „ „ 15	10·8826872
„ „ intermediate semitone	1·1173128

For example, B 5, C 1, in the key of C.

Log. of the harmonic 17	13·0495541
„ „ „ 16	12·0000000
„ „ basal semitone	1·0495541

For example, C 1, ♯C 17, in the key of C.

Log. of the harmonic 18	2·0391000
„ „ „ 17	1·0495541
„ „ intermediate semitone	0·9895459

For example, ♯C 17, D 3^2, in the key of C.

Log. of the harmonic 19 . . .	2·9751302
„ „ „ 18 . . .	2·0391000
„ „ intermediate semitone . .	0·9360302

For example, D 3^2, ♭E 19, in the key of C.

Log. of the harmonic 20 . . .	3.8631372
„ „ „ 19 . . .	2·9751302
„ „ intermediate semitone . .	0·8880070

For example, ♭E 19, ♯E 5, in the key of C.

Log. of the harmonic 21 . . .	4·7078091
„ „ „ 20 . . .	3·8631372
„ „ intermediate semitone . .	0·8446719

For example, E 5, F 7, in the key of C.

601. Having given a logarithmic table for all numbers from 1 to 512, and for all the primes between 512 and 1024, it will now be seen that the logarithms in the following table represent, by reference to the octave and the 12 mean semitones into which it is divided, the real and practical intervals severally answering to the vibrations of the sounds represented in the table, pages 112, 113. And as the value of the vibrations and intervals set forth in these tables is, in every instance, rendered an object of occular demonstration by fig. 38, which will be explained presently, it is desirable that they should be studied conjunctively, that the whole may be rendered clear and distinct.

TABLE

Representing the true and substituted Intervals of the Great System—in agreement with the Tables, pp. 112, 113.

1st semitone above and below the Central System.		2nd semitone above and below the Central System.		3rd semitone above and below the Central System.	
Nos.	Logarithms.	Nos.	Logarithms.	Nos.	Logarithms.
...	...	..	...	19×231	2.9751302
17×239	1.0495541	$17^2\times231$	2.0991082	$17^3\times223$	3.1486623
15^{11}	11.7095587	15^{10}	0.8268715	15^9	1.9441844
...	...	...	...	...	...
...	...	...	...	...	...
...	...	9×236	2.0391000	...	...
...	...	$7^5\times225$	0.4412954	...	...
...	...	...	...	...	...
37×232	11.1368501	33×236	2.0391000	39×221	3.1759501
...	...	...	...	...	...
Equal Temperament	1.0000000	Equal Temperament	2.0000000	Equal Temperament	3.0000000

4th semitone above and below the Central System.		5th semitone above and below the Central System.		6th semitone above and below the Central System.	
Nos.	Logarithms.	Nos.	Logarithms.	Nos.	Logarithms.
...	...	...	...	192×216	5.9502603
174×215	4.1982164	175×27	5.2477705	176	6.2973246
15^8	3.0614972	15^7	4.1788101	$15^6\times2$	5.2961229
$13^2\times224$	4.8105533	...	...	...	...
...	...	...	...	11×221	5.5131794
92×225	4.0782001	...	...	93×215	6.1173001
74×220	2.7530363	...	...	73×216	5.0647772
5×239	3.8631371	...	...	...	...
34×225	4.0782001	311×210	5.2150502	36×215	6.1173001
...	...	...	...	...	...
Equal Temperament	4.0000000	Equal Temperament	5.0000000	Equal Temperament	6.0000000

7th semitone above and below the Central System.			8th semitone above and below the Central System.			9th semitone above and below the Central System,		
Nos.		Logarithms	Nos.		Logarithms.	Nos.		Logarithms.
...	...	...	...	...	...	$19^3 \times 2^{24}$	...	8.9253905
17^7	...	7.3468787	17^8	...	8.3964828	17^9	...	9.4459869
$15^5 \times 2^9$	...	6.4134358	$15^4 \times 2^{17}$	...	7.5307486	$15^3 \times 2^{27}$	...	8.6480615
...	..	...	13×2^{29}	...	8.4052766	...	...	...
...	...	...	...	...	...	...	...	...
...	..	...	$9^4 \times 2^{20}$	...	8.1564001	...	...	...
...	..	...	$7^2 \times 2^{27}$	...	7.3765181	...	...	...
...	...	...	$5^2 \times 2^{28}$	...	7.7262743	...	...	...
3×2^{27}	...	7.0195500	$3^8 \times 2^{20}$	...	8.1564001	$3^3 \times 2^{32}$	...	9.0586500
...	...	...	...	...	...	...	...	...
Equal Temperament.	...	7.0000000	Equal Temperament	...	8.0000000	Equal Temperament	...	9.0000000

10th semitone above and below the Central System.			11th semitone above and below the Central System.			12th semitone above and below the Central System.		
Nos.		Logarithms.	Nos.		Logarithms.	Nos.		Logarithms.
...	...	...	...	...	...	$19^4 \times 2^{32}$	...	11.9005207
17^{10}	...	10.4955410	17^{11}	...	11.5450951	17^{12}	...	12.5946492
$15^2 \times 2^{33}$	...	9.7653743	15×2^{41}	...	10.8826872	$15^{12} \times 2^2$	...	10.5922458
...	...	...	...	...	...	$13^3 \times 2^{38}$	...	13.2158299
...	...	...	...	..	...	$11^2 \times 2^{42}$	...	11.0263589
$9^5 \times 2^{25}$	...	10.1955002	...	.	...	$9^6 \times 2^{30}$	...	12 2346002
7×2^{28}	...	9.6882591	...	...	...	$7^6 \times 2^{32}$	...	10.1295544
...	...	...	...	...	...	$5^3 \times 2^{42}$	...	11.5894114
$3^{10} \times 2^{25}$	...	10.1955002	$3^5 \times 2^{37}$	...	11.0977501	$3^{12} \times 2^{30}$	...	12.2346002
...	...	...	...	...	...	2^{49}	...	12.0000000
Equal Temperament	...	10.0000000	Equal Temperament	...	11.0000000	Equal Temperament	...	12.0000000

602. In fig. 38, at the top, the octave is represented by the notation, and by the diatonic and chromatic intervals answering to the key of C—and at the bottom, by the chromatic intervals counted from the higher C downwards. In the 2nd line from the bottom, by the figure 2 at its two extremities, is represented the *perfect* octave divided into 12 mean semitones, by lines carried from top to bottom.

By counting from the lower C upwards the numerals enclosed in a circle severally represent the position of each particular sound in respect to the series in which it is placed, and by the small figures are represented various powers of these numbers. The white figures on a black ground severally represent the position of each particular sound in respect to the series in which it is placed, by counting from the higher C downwards —whence it follows that the numerals encircled by a plain ring answer to the central bases of the super-systems, and those on a black ground to the central-bases of the sub-systems of the great system.

In this diagram C is made the central-basis—but this of course can be transposed to any other key. These particulars being arranged, and the position of the ratio 3^{12} (third line from the bottom) in respect to 2^{19} being ascertained, marked out and divided into 12 equal parts—one of these divisions on the right of the tempered sound answering to the note G will represent the true position of G, the perfect 5th to C,—the space equal to two of these divisions on the right of the tempered sound D will represent the true position of the perfect 5th to G,—and the space equal to three of these divisions on the right of the tempered sound A, will represent the true position of the perfect 5th to D, &c.

In the fourth line from the bottom the position of the ratio 5, 5^2 from the lower C, and of 5 downwards from the upper C, is obtained by the same means, and with great accuracy from the position of 5^{12}. But since, *practically*, the circuit of the octave is obtained by 5^3, and consequently four times over by 5^{12}, after finding the true position of 5^{12}, the space between this point and the note C, here answering to the ratio 2^7, has first to be divided into 4 equal parts. Having by this means obtained the true position of the ratios 5^9, and 5^6, by dividing the space between the lower C and the point 5^3 into three equal parts, we obtain the two intermediate points E 5 and ♯G 5^2,—and by inversion we also obtain the point answering to the note ♭A 5, the major 3rd below the higher C.

603. As 7, 11, and the powers of these numbers, in order to keep within the bounds of one octave, proceed from right to left—the difference between 2^{17} and 7^6, and between 2^{47} and 15^{12} must necessarily appear on the left—while the discrepancies between 2^{19} and 3^{12},—7^2 and 5^2,—2^{19} and 9^6,—2^7 and 11^2,—2^{11} and 13^3,—2^{49} and 17^{12},—2^{17} and 19^4, necessarily appear on the right.

The perpendicular lines numbered 1, 2, 3, 4, 5, above the higher C, on the right, and 1, 2, 3, 4, below the lower C, on the left, represent a continuation of mean semitones—whence it appears that 13^3 oversteps the the true octave more than one, and 13^{12} nearly five mean semitones—that 11^2 falls short of the true octave nearly one, and 11^{12} nearly six mean semitones,—that 15^{12} oversteps the true octave nearly one and a half, mean semitones,—that 7^6 oversteps the true octave nearly two, and 7^{12} nearly four mean semitones,—that 17^{12} oversteps the true octave more than half a mean semitone,—and that 5^{12} falls short of the true octave more than one and a half mean semitones.

All these, and many more particulars are rendered matters of occular demonstration, and thereby freed from the mental distortions arising from misapprehension—for hereby it may be seen that the true basal 5th, and the true basal minor 3rd, differ but slightly from the sounds presented by the scale of mean semitones—and also that they pull as it were in contrary directions—that in respect to diatonic intervals the sounds presented by the tempered scale of 12 mean semitones is most at variance with the basal major 3rd,—that the harmonic 3 in connexion with the powers of 3, *practically* work their way through all the intervals of the semitonic scale, and that this is not peculiar to this number and its powers, since 15 and its powers, and 17 and its powers do the same—and consequently, that if this circumstance affords a valid reason for rejecting the other ratios in one case, it ought to do so in other instances also,—upon this principle the claims of 15 and its powers, and of 17 and its powers, for universal dominion, might be set up in opposition to 3 and its powers.

As 9 is the same as 3^2,—9^2 the same as 3^4, &c., these points will, of course, be found to correspond.

Upon examination it will also be found that the discrepancy of 15^{12} with the octave is, as might be expected, exactly equal to that of 5^{12}, minus 3^{12}.

As a proof of the accuracy of this diagram, I herewith sub-

join the following calculations relating to several points which differ but little the one from the other.

Log. of	$17^2 = 1{\cdot}0991082$	Log. of	$3^{10} = 10{\cdot}1955000$
,,	$3^2 = 1{\cdot}0391000$	,,	$11^4 = 10{\cdot}0527176$
	$0{\cdot}0600082$		$0{\cdot}1427824$
,,	$15^4 = 7{\cdot}5307488$	,,	$15^6 = 5.2961232$
,,	$7^2 = 7{\cdot}3765180$	,,	$3^{11} = 5{\cdot}2150500$
	$0{\cdot}1542308$		$0{\cdot}0810732$
,,	$15^2 = 9{\cdot}7653744$	,,	$9 = 9{\cdot}9609000$†
,,	$7 = 9{\cdot}6882590$	,,	$15^2 = 9{\cdot}7653744$
	$0{\cdot}0771154$		$0{\cdot}1955256$
,,	$3 = 4{\cdot}9804500$*	,,	$15^7 = 4{\cdot}1788104$
,,	$13^2 = 4{\cdot}8105532$	,,	$3^4 = 4{\cdot}0782000$
	$0{\cdot}1698968$		$0{\cdot}1006104$
,,	$3^8 = 8{\cdot}1564000$	,,	$11^{12} = 6{\cdot}1581528$
,,	$11^8 = 8.1054352$	,,	$9^3 = 6{\cdot}1173000$
	$0{\cdot}0509648$		$0{\cdot}0408528$
,,	$11^6 = 9{\cdot}0790764$	,,	$5^3 = 11{\cdot}5894113$
,,	$3^3 = 9{\cdot}0586500$	,,	$17^{11} = 11{\cdot}5450951$
	$0{\cdot}0204264$		$0\ 0443162$

604. The notion, commonly entertained, that all intervals are derived from the octave, is suggested by keyed instruments; which, though in itself diametrically opposed to the truth, by holding the mind in subjection to visible and material objects, thereby clogs its ideas and thoughts, until they also become material, or at all events incapable of any degree of elevation. Thus, upon this supposition, the primes 1, 2, constitute the only bond of connexion existing between what to the affections is good, and to the understanding truth, since in respect to the subdivision of the octave into 12 mean semitones, numbers themselves are inadequate to afford the true expression. For example, were this the case, the prime 3 would not express

* Here we take the complement to the log. of 3, in order to compare the sub 3 point with the 13^2 point—and at † we take the complement to the log. of 9, in order to compare the sub 9 point with the 15^2 point.

the ratio of the 5th, but the ratio $\sqrt[12]{}524,288$, (*i.e.* 2^{19}) = 2·9966 + *ad infinitum*. Neither would the prime 5 express the ratio of the major 3rd, but the ratio $\sqrt[3]{}128$, (*i.e.* 2^7) = 5·03968 + *ad infinitum*. Nor the prime 7 the ratio of the ♭7th, but the ratio $\sqrt[6]{}131,072$, (*i.e.* 2^{17}) = 7·12719 + *ad infinitum*. Nor the prime 11 the ratio of the ♭5th, but the ratio $\sqrt[2]{}128$, (*i.e.* 2^7) = 11·3137 + *ad infinitum*. Nor the prime 13, the ratio of the ♯5th, but the ratio $\sqrt[3]{}2,048$, (*i.e.* 2^{11}) = 12·6992 + *ad infinitum*. Nor the prime 17 the ratio of the ♯1st, but the ratio $\sqrt[12]{}562, 949, 953, 421, 312$ (*i.e.* 2^{49}) = 16·9514 + *ad infinitum*. Nor the prime 19 the ratio of the minor 3rd, but the ratio $\sqrt[4]{}131,072$ (*i.e.* 2^{17}) = 19·0273 + *ad infinitum*.

And hence it follows that, according to this idea, all simple ratios, except that of the octave, being excluded, and the number of musical intervals limited to 12,—every one of these must necessarily involve a surd number—a doctrine as absurd as it is repulsive. Wherefore, turning aside from these *falsified harmonic ratios*, let the student reflect upon the irreconcileable differences existing between what is artificial and what is true in relation to music, and compare the one with the other by means of the Ratiometer.

605. The object in view for the introduction of the following scale of 53 sounds in the octave, is to show that no artificial system of equal intervals in the octave can, in any single instance, by possibility coincide with the true scale of nature, *i.e.* the triune system.

It may be necessary to assure the reader who may not perceive the truth of what is here affirmed, that nevertheless it is strictly true, and that every mathematician must know that it is so.

Wherefore upon examination it will be found that, the decimal parts in excess of the mean semitones, 0 . . . 12 on the left, and those placed under the same heading on the right, are in no single instance the same. Furthermore, by referring to the Ratiometer, and thereby scrutinizing all the particulars relating to the artificial scale of 53 sounds in the octave, in conjunction with those of the 12 mean semitones, and of the triune system, their relative positions will be found to correspond throughout with the utmost exactitude, and thereby afford an occular demonstration of all the facts relating to this subject. In every instance the mean semitones on the left of the decimal parts represent the whole number appertaining to those decimal parts.

Scale of 53 mean proportionals in the octave, in connection with the intervals of the semitonic scale—and also with the ratios of the vibrations appertaining to the triune-system.

Mean Semi-tones.	Decimal Parts in excess of the Mean Semitone.	No. of Sounds.	Bases and Harmonics of the Triune System. Sub-System.	Central-System.	Super-System.	Mean Semi-tones.	Decimal Parts in excess of the Mean Semitone.
12	.0000000	53	3	or 1	...	12	·0000000
11	·7735849	52	...	...	...	11	
	·5471698	51	...	31	... ...	...	·4503557
	·3207547	50	...	...	41 ...	...	·3101741
	·0943396	49	23	...	... ...	...	·2631935
10	·8679245	48	...	...	5 ...	10	·8 826871
	·6415094	47	11	...	... ...	...	·4936294
	·4150943	46	...	29	... ...	...	·2957719
	·1886792	45	...	...	19 ...	9	·9946892
9	·9622641	44					
	·7358490	43	...	7	... ...	...	·6882591
	·5094339	42	...	...	37 ...	...	·5329904
	·2830189	41	41	...	... ...	...	·2710741
	·0566038	40	5	...	... ...	8	·8435871
8	·8301887	39					
	·6037736	38	...	13	... ...	...	·4052766
	·3773585	37					
	·1509434	36	...	...	17 ...	...	·0691041
7	·9245283	35	19	...	... ...	7	·9555802
	·6981132	34	37	...	... ...	...	·4938904
	·4716981	33					
	·2452830	32	...	3	or 1 ...	...	·0195500

6	·7924528	30							
	·5660377	29	...	...	31	...	6	·4699057	
	·3396226	28	...	23	...	...	...	·2827435	
	·1132075	27	17	...	...	...	...	·0300041	
5	·8867924	26							
	·6603774	25	...	11	...	...	5	·5131794	
	·4339623	24	...	...	29	...	...	·3153219	
	·2075472	23							
4	·9811321	22							
	·7547170	21	1	...	...	...	4	·9804500	
	·5283019	20	...	...	7	...	...	·7078091	
	·3018868	19	31	...	...	...	...	·4308057	
	·0754717	18	...	41	...	...	...	·2906241	
			...	5	...	...	3	·8631371	
3	·8490566	17							
	·6226415	16	...	...	13	...	...	·4248866	
	·3962264	15	29	...	...	..	...	·2762219	
	·1698113	14	...	19	...	...	2	·9751302	
2	·9433962	13							
	·7169811	12	7	...	...	...	...	·6687091	
	·4905660	11	...	37	...	...	...	·5134404	
	·2641509	10	...	...	3	...	...	·0391000	
	·0377358	9							
1	·8113208	8							
	·5849057	7	13	...	...	...	1	·3857266	
	3584906	6	...	...	23	...	...	·3022935	
	.1320755	5	...	17	...	...	...	·0495541	
0	·9056604	4							
	·6792453	3	...	...	11	...	0	·5327294	
	·4528302	2							
	·2264151	1							
	·0000000	0	3	or 1	...	...	...	·0000000	

606. With the view to the farther elucidation of this subject, let it be observed:—(1) That the divisions in connexion with the numbers 192 to 384, relating to the scale of nature (*i.e.* the triune system), represent the nodal points of harmonic sounds, while those of the artificial systems of 12 to 53 in the octave represent the positions for stopped notes. (2) That hence it is that in the scale of nature, the series commences with 192, and terminates with 384, while in all artificial systems, the series commences with 0, and terminates with the highest number of the series. (3) That as equal musical intervals are the result of graduated spacial intervals, equal spacial intervals must necesrily give birth to unequal musical intervals. (4) That if the divisions of the scale of nature which represent harmonic sounds were all equal, the sound represented would, in every instance, be the sound actually produced by touching that node. (5) That since, in ascending from lower to higher sounds, the same musical interval gradually occupies a smaller spacial interval, to counteract this inconvenience, the arithmetical series of equal spacial divisions is converted into a corresponding geometrical series of equal musical intervals, that is in respect to their vibrations, equal ratios. (6) That hence it follows that the artificial scales of 12 and 53 sounds represent, in reality, only a repetition of one and the same musical interval and ratio each. (7) That in all such divisions, if the number be a prime, the interval produced will be the same, and not various, and still less prolific of variety, as is the case with the series 192 to 384. (8) That if the divisions of the artificial systems of 12 and 53 sounds in the octave be severally treated as the nodes of a harmonic sound, only the primary harmonics 2 and 3, and the super harmonic 12, can result from the harmonic 12, and only the primary harmonic 53 from the scale of 53 divisions. (9) That if the divisions of the graduated scale of 53 sounds in the octave be treated as representing a series of frets for stopped notes, only a repetition of one and the same musical interval and ratio can result therefrom. (10) That though the artificial scales of 12 and 53 equal musical intervals in the octave can only imperfectly represent the true scale of nature, they nevertheless possess the advantage of being transposable from any one to any other of the sounds appertaining to its series.

Discrepancies between the tempered systems of 12 *and* 53 *sounds in the octave.*

$$\frac{1}{12} - \frac{4}{53} = \frac{53}{636} - \frac{48}{636} = \frac{5}{636}$$

$$\frac{9}{53} - \frac{1}{6} = \frac{54}{318} - \frac{53}{318} = \frac{1}{318}$$

$$\frac{1}{4} - \frac{13}{53} = \frac{53}{212} - \frac{52}{212} = \frac{1}{212}$$

$$\frac{1}{3} - \frac{17}{53} = \frac{53}{159} - \frac{51}{159} = \frac{2}{159}$$

$$\frac{5}{12} - \frac{22}{53} = \frac{265}{636} - \frac{264}{636} = \frac{1}{636}$$

or

$$\frac{1}{2} - \frac{26}{53} = \frac{53}{106} - \frac{52}{106} = \frac{1}{106}$$

$$\frac{27}{53} - \frac{1}{2} = \frac{54}{106} - \frac{53}{106} = \frac{1}{106}$$

$$\frac{31}{53} - \frac{7}{12} = \frac{372}{636} - \frac{371}{636} = \frac{1}{636}$$

$$\frac{2}{3} - \frac{35}{53} = \frac{106}{159} - \frac{105}{159} = \frac{1}{159}$$

$$\frac{3}{4} - \frac{39}{53} = \frac{159}{212} - \frac{156}{212} = \frac{3}{212}$$

$$\frac{5}{6} - \frac{44}{53} = \frac{265}{318} - \frac{264}{318} = \frac{1}{318}$$

$$\frac{11}{12} - \frac{48}{53} = \frac{583}{636} - \frac{576}{636} = \frac{7}{636}$$

Discrepancies between the tempered systems of 12 *and* 41 *sounds in the octave.*

$$\frac{5}{12} - \frac{17}{41} = \frac{205}{492} - \frac{204}{492} = \frac{1}{492}$$

$$\frac{24}{41} - \frac{7}{12} = \frac{288}{492} - \frac{287}{492} = \frac{1}{492}$$

&c. &c.

607. Exclusive of all very high numbers, as utterly impracticable, the fitness of any number of equal intervals in the octave, to represent the triune-system, depends upon the following conditions:—(1) If the wholetone 𝟙, 1, be divided into any odd number of steps or degrees, the octave must also be divided into an odd number of degrees, that the intervals 𝟙, 𝟙, and 1, 𝟙, may each have the same number of degrees. (2) For the same reason, if the wholetone 𝟙, 1, be divided into an even number of degrees, so also must the octave. (3) The interval 𝟙, 3, must have one degree more than 3, 𝟝,—and the product of the degrees 𝟙, 3×6 must exceed that assigned to the octave by 1. (4) The product of the degrees 𝟙, 𝟝×3 must be less than than that assigned to the octave by 2,—and (5) the product of the degrees 𝟙, 𝟙𝟡×4 must be 1 less than that assigned to the octave. For example, in the scale of 53 degrees in the octave, the wholetone 𝟙, 1, has 9 degrees, and $9 \times 6 = 53 + 1 = 54$:—in the same scale, the major 3rd 𝟙, 𝟝, has 17 degrees, and $17 \times 3 = 53 - 2 = 51$,—and the minor 3rd 𝟙, 𝟙𝟡, has 13 degrees—and $13 \times 4 = 53 - 1 = 52$. Furthermore, in the scale of 41 degrees in the octave, the wholetone 𝟙, 1, has 7 degrees, and $7 \times 6 = 41 + 1 = 42$:—in the same scale the major 3rd 𝟙, 𝟝, has 13 degrees, and $13 \times 3 = 41 - 2 = 39$,—and the minor 3rd 𝟙, 𝟙𝟡, has 10 degrees—and $10 \times 4 = 41 - 1 = 40$. The Ratiometer will show that 53 degrees for the octave is better than 41—more especially in its application to the harmonics 5, 17, and 19.

608. Having, at length, as I presume, unravelled the mystery of the science of music, by tracing effects to their causes, and done all in my power to arrange the particulars relating thereto in their natural order, and to render all its details clear and distinct—I now leave it to its fate, without the slightest misgiving as to its final success, but not so as to the period when justice will be awarded to it. That it will live, I feel assured, because I know it contains a vital principle which cannot die, and also the source of that principle. But what may be the nature of the storms and perils it may have to encounter, or the time that may elapse in its progress through these more than probable events, I know not.

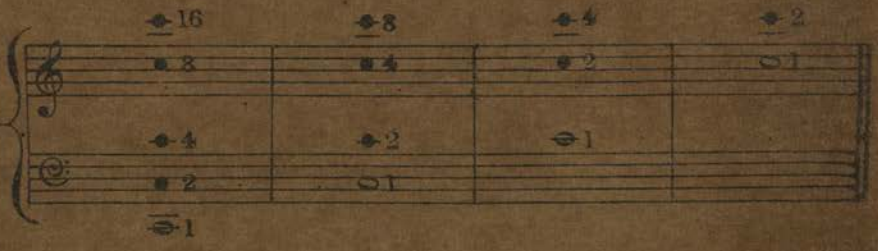
THE END.

ARLISS, PRINTER, GREAT QUEEN STREET, LINCOLN'S INN FIELDS.

Music Examples.

The sections (§) at the beginning of each example refer to the text where the subject is treated.

Ex. 1.
§ 11.



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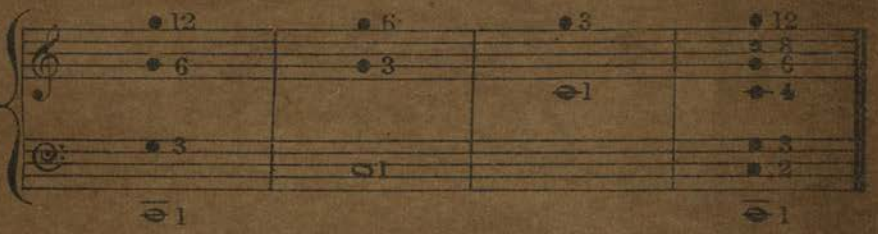
§ 13.



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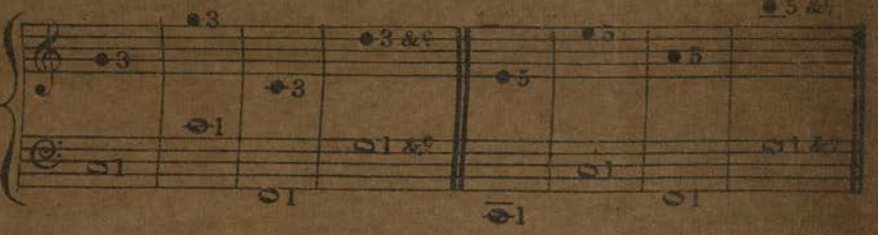

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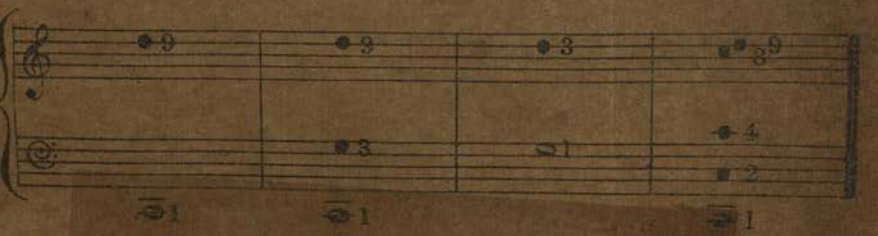

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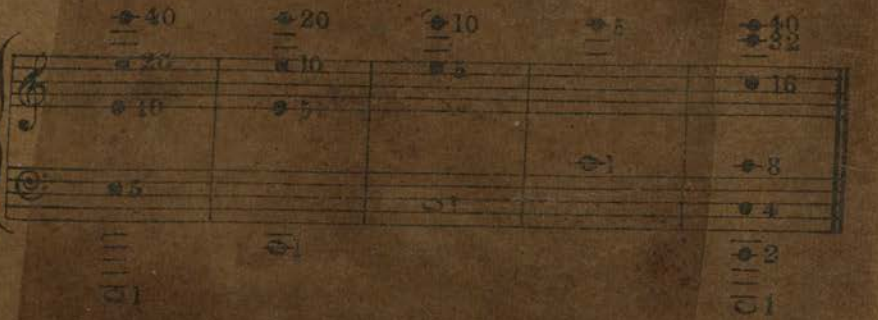

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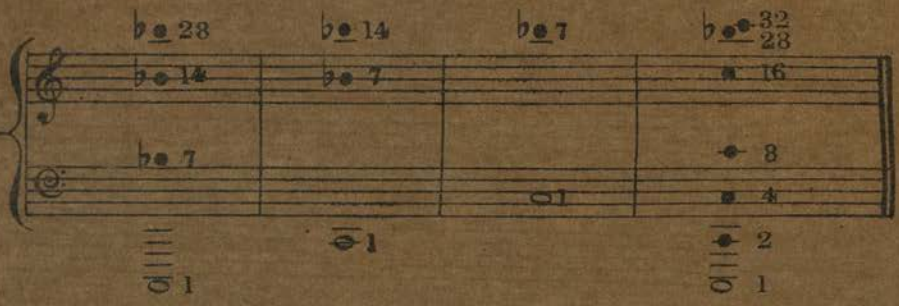

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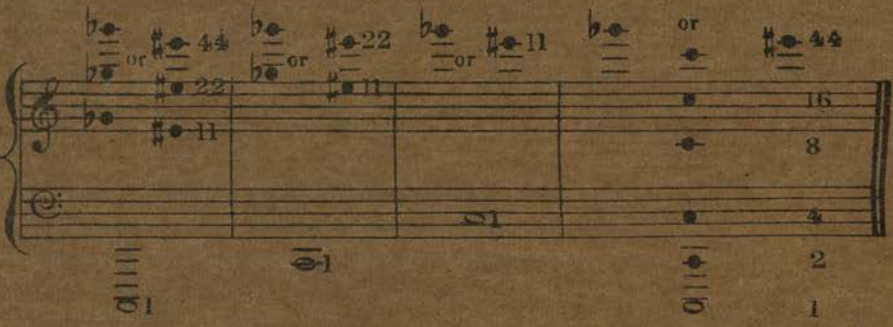

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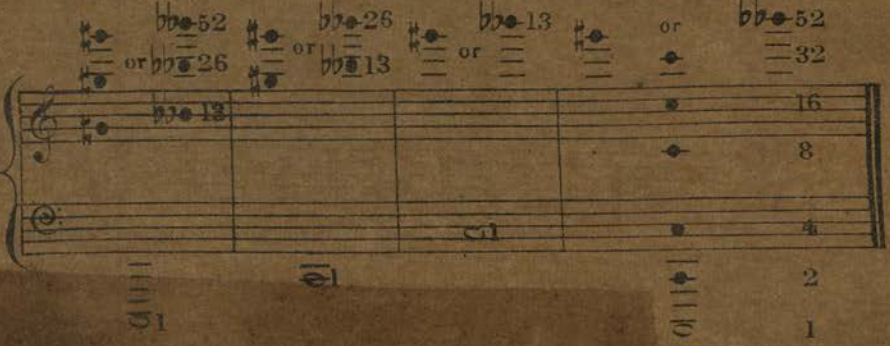
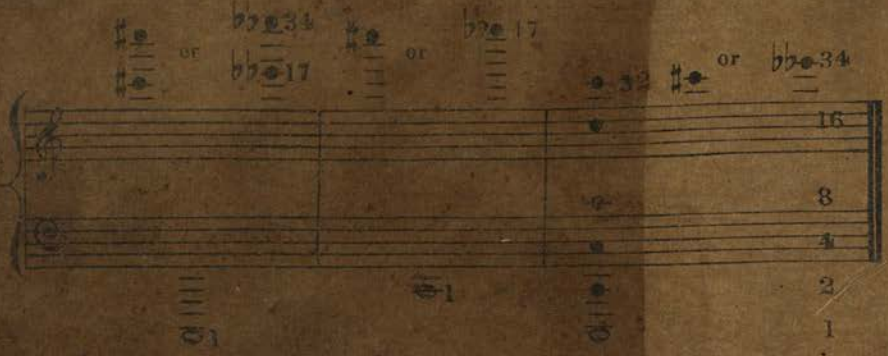


Ex. 9. § 20



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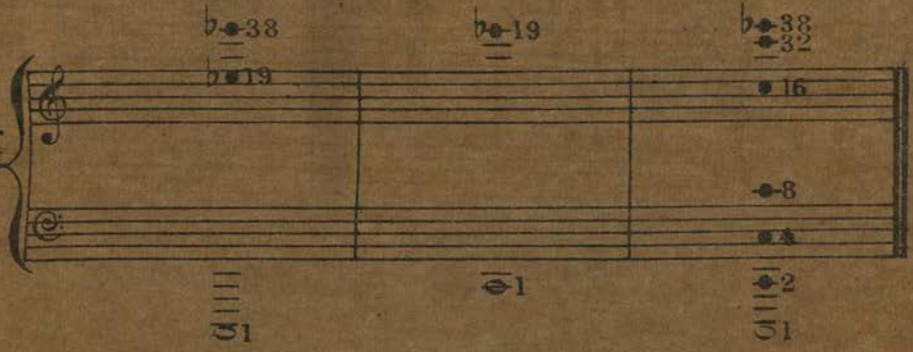
§ 21.



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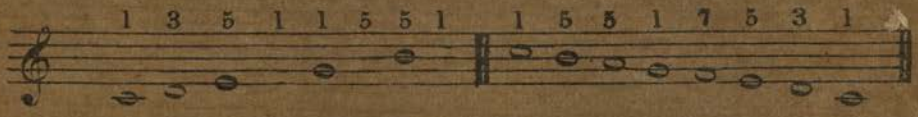

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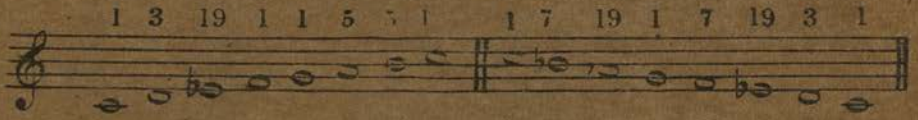

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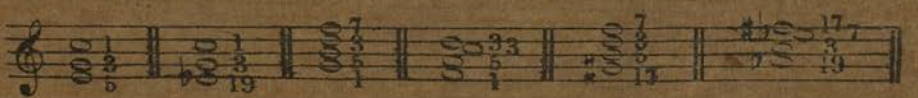

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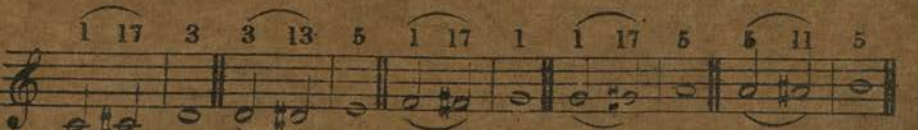

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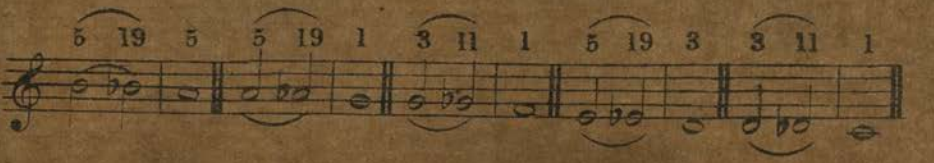

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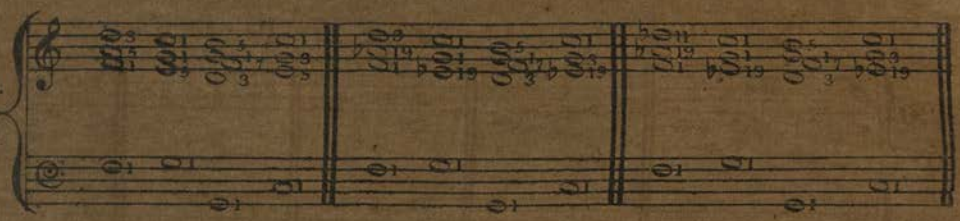

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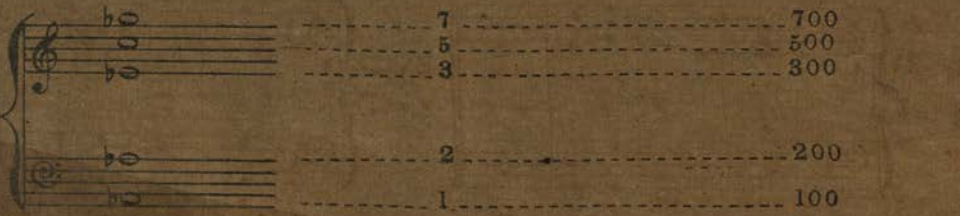

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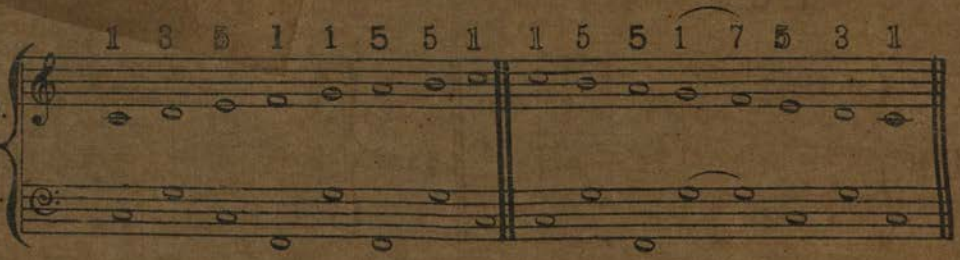
82–84.



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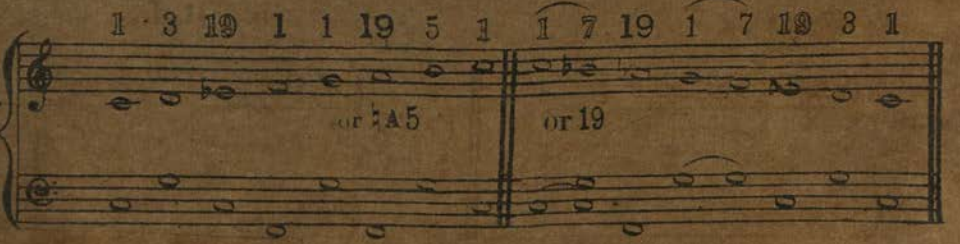

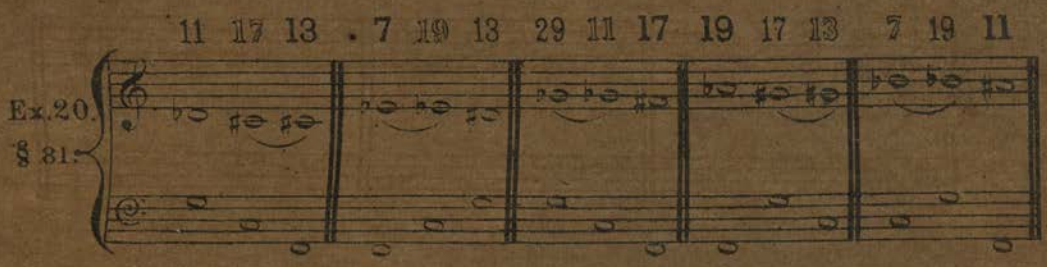
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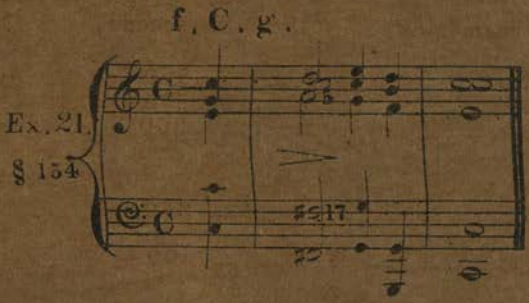
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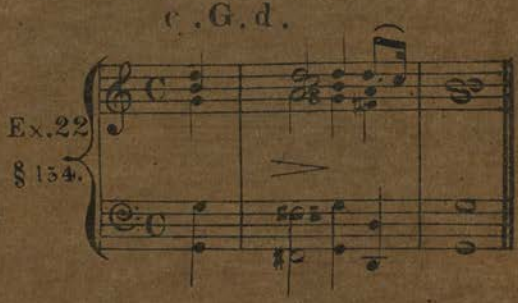
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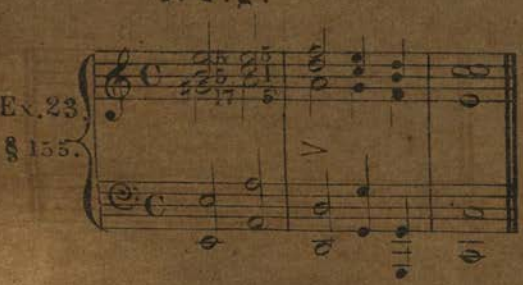
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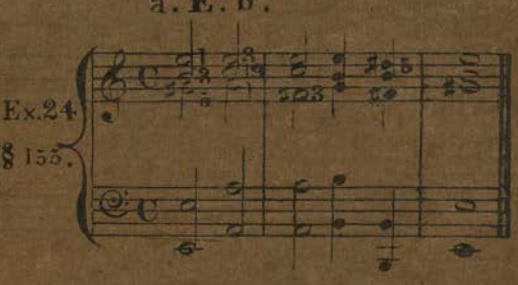
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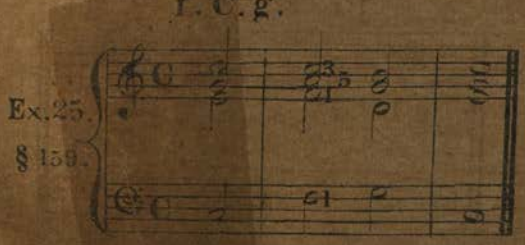
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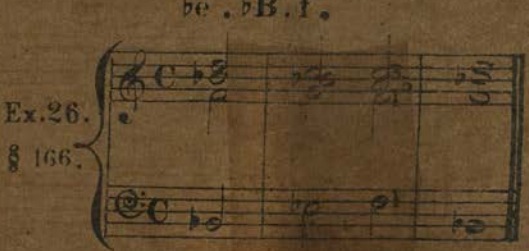
be . bB. f.
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§ 166.

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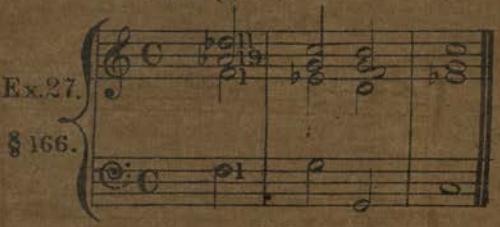
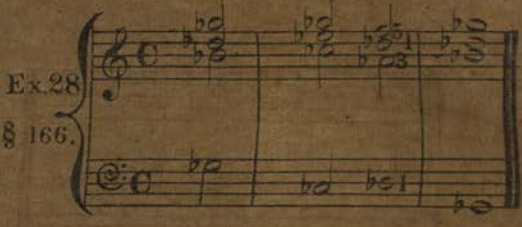


♭c. ♭G. ♭d.



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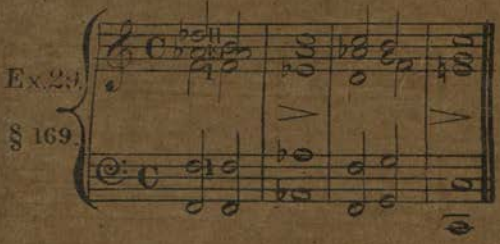
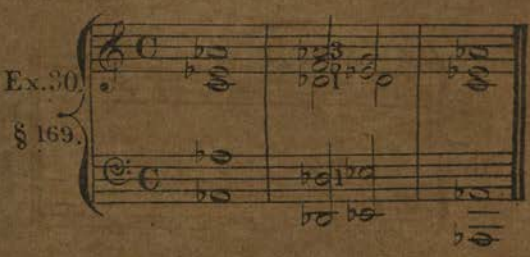


♭d. ♭A. ♭c.



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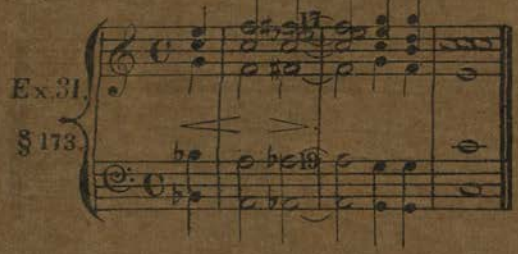
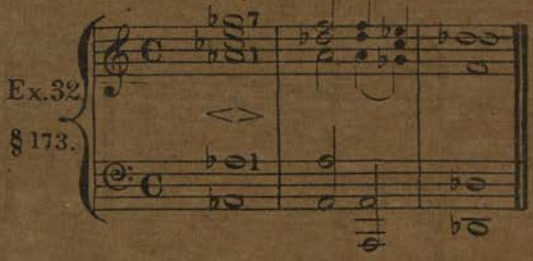


♭g. ♭D. ♭a.



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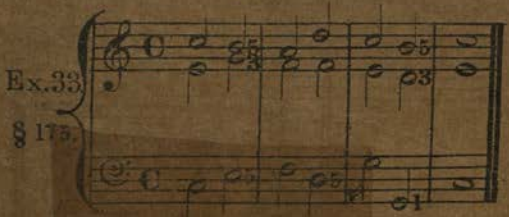
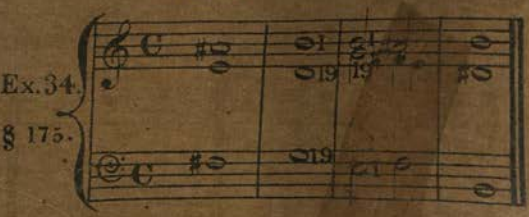


e. B. f.



f. C. g

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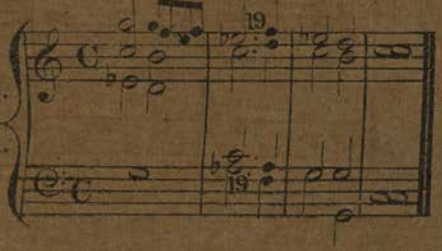

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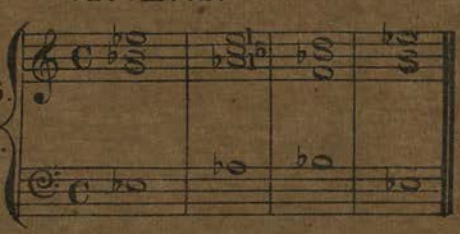
Ex. 36.

§ 176.



f. C. g.

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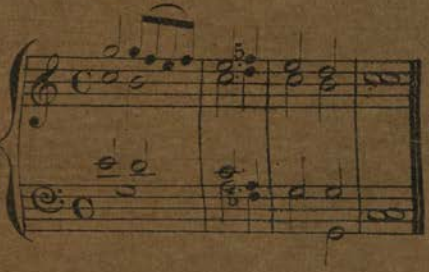
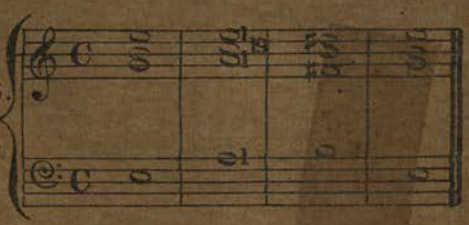


a. E. b.

Ex. 38.

§ 176.



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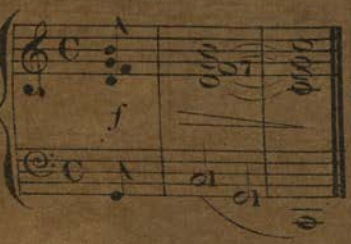
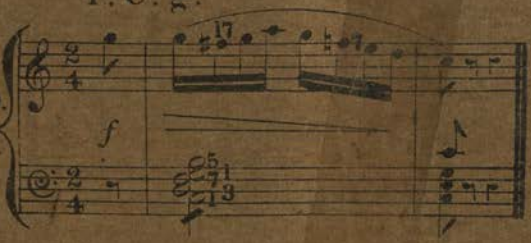


f. C. g.

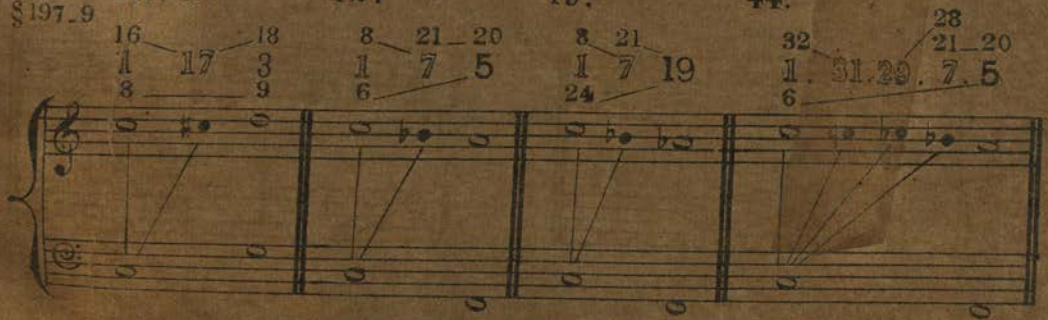
Ex. 40.

§ 184.

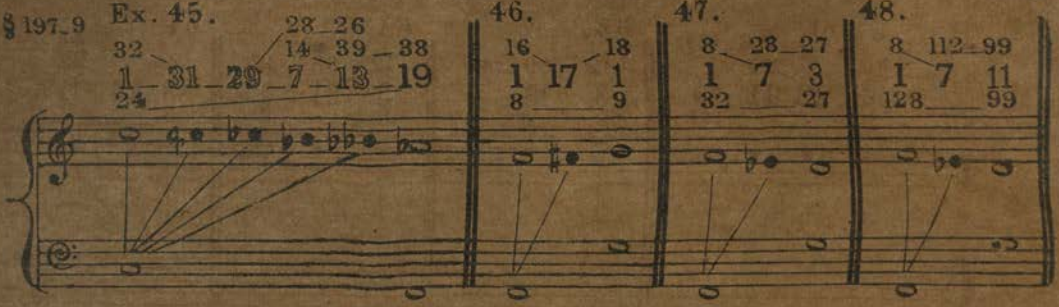


§ 197_9

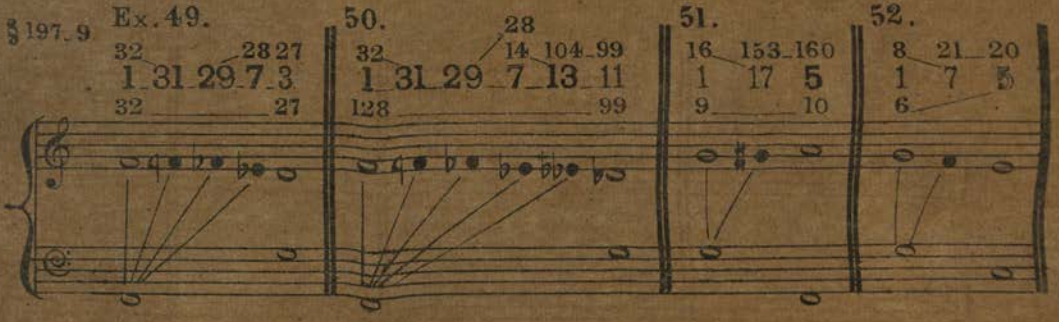
Ex. 41. 42. 43. 44.



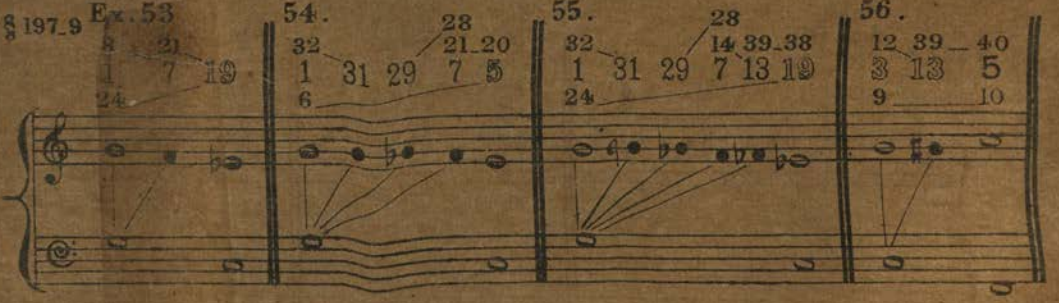
§ 197. 9 Ex. 45.



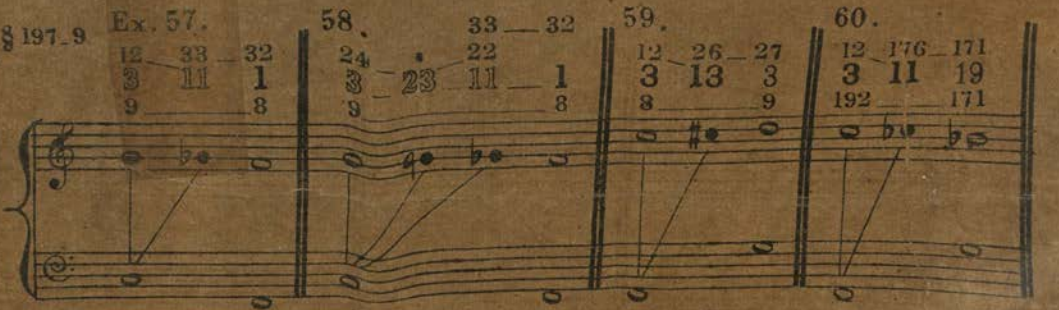
§ 197. 9 Ex. 49.

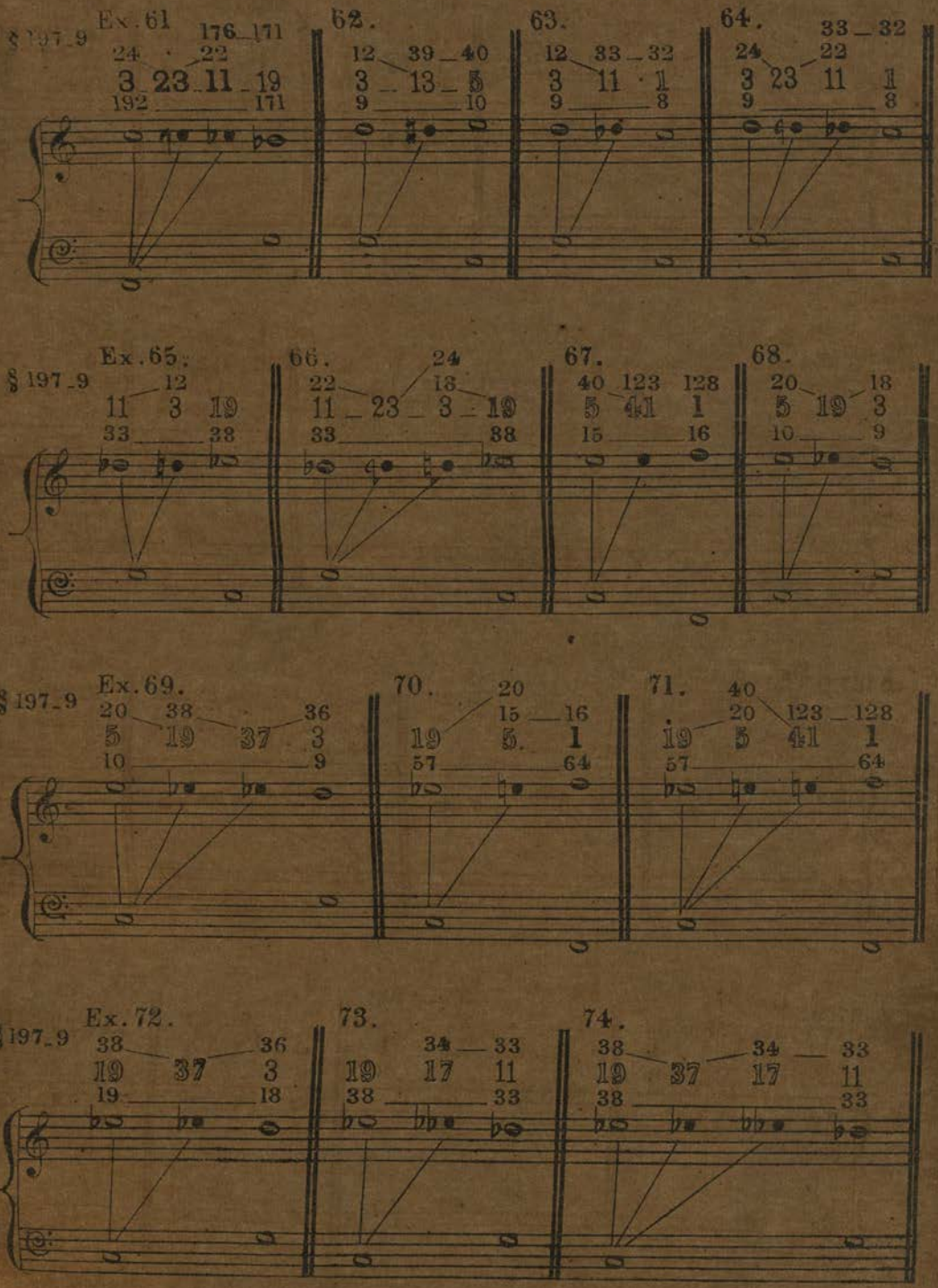


§ 197. 9 Ex. 53

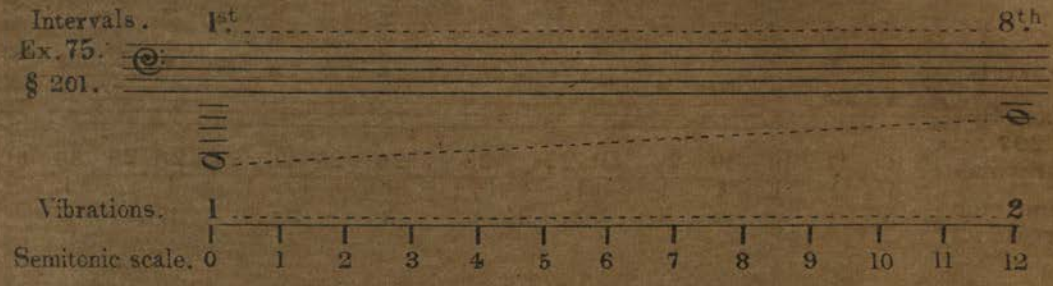


§ 197. 9 Ex. 57.

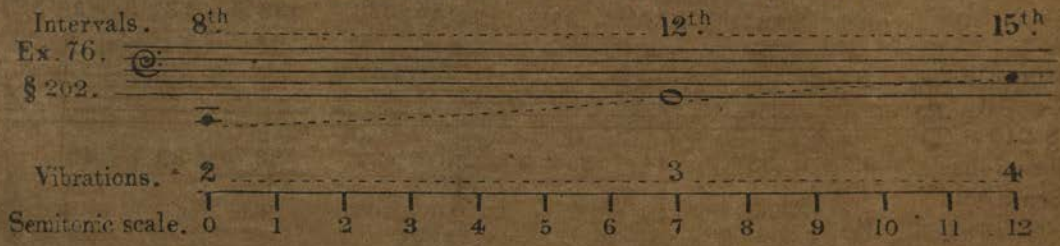



§ 197_9
Ex. 61
62.
63.
64.
Ex. 65.
66.
67.
68.
Ex. 69.
70.
71.
Ex. 72.
73.
74.

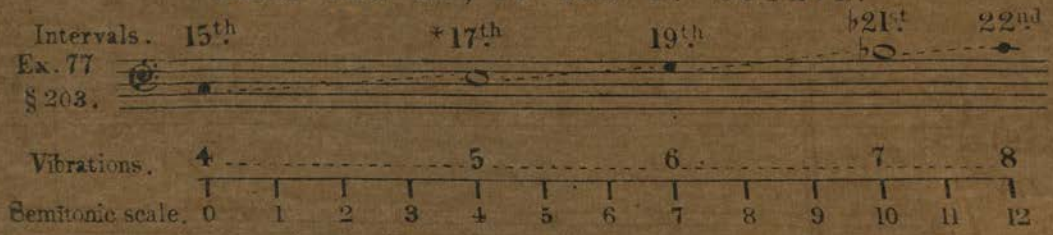
FROM 0, TO THE 1st OCTAVE.



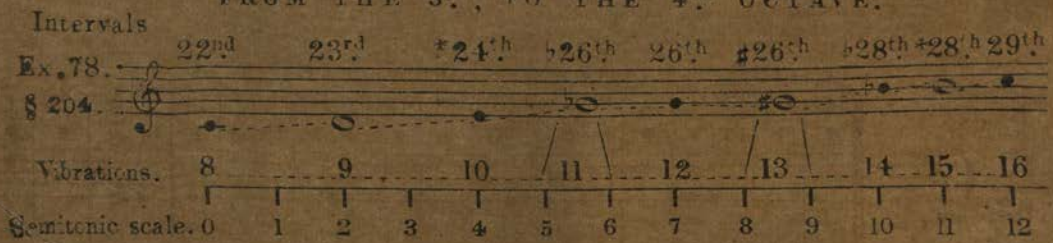
FROM THE 1st, TO THE 2nd OCTAVE.



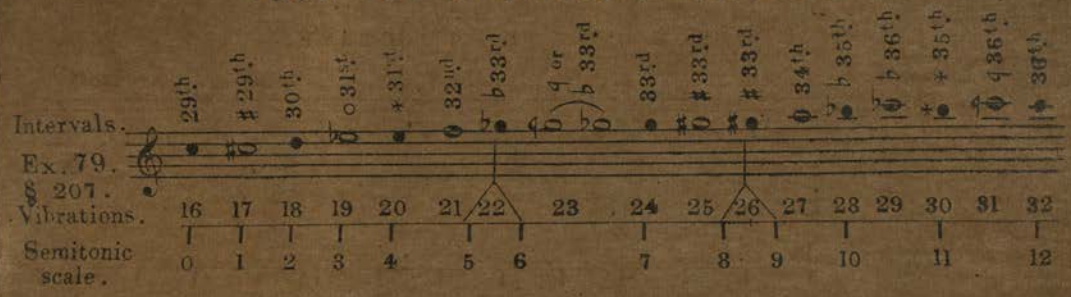
FROM THE 2nd, TO THE 3rd OCTAVE.



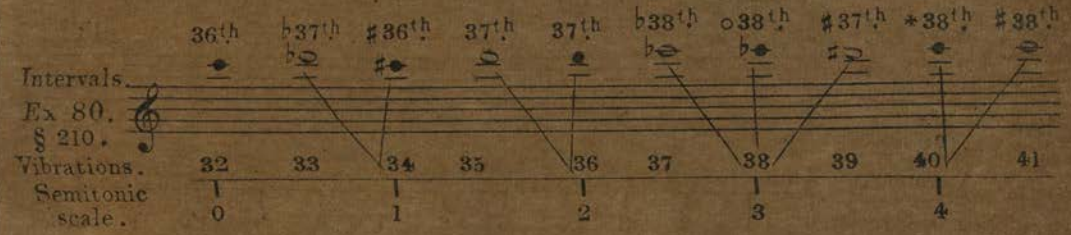
FROM THE 3rd, TO THE 4th OCTAVE.

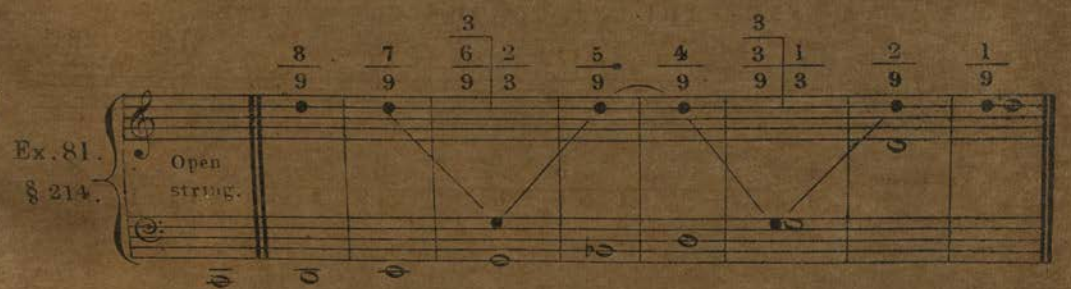


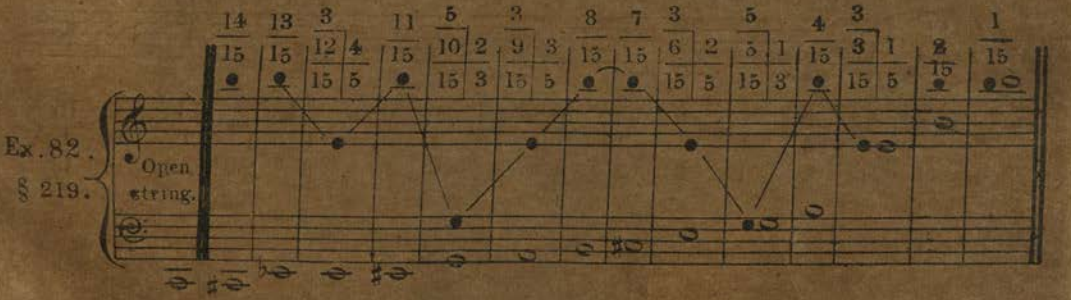
FROM THE 4th TO THE 5th OCTAVE.



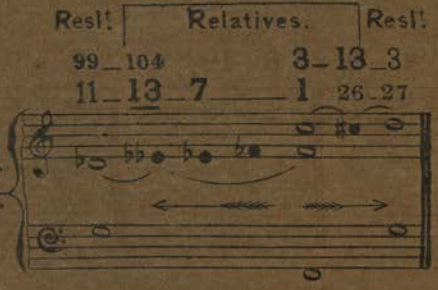
FROM THE 5th OCTAVE UPWARDS.



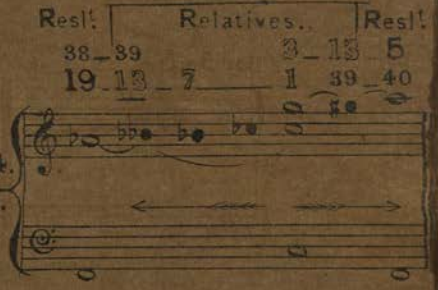




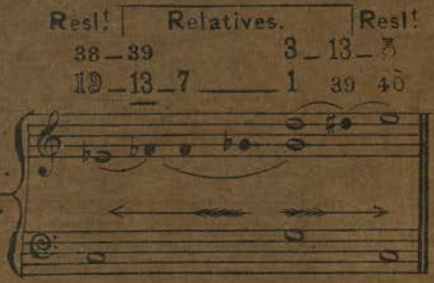
Ex. 83. § 249.



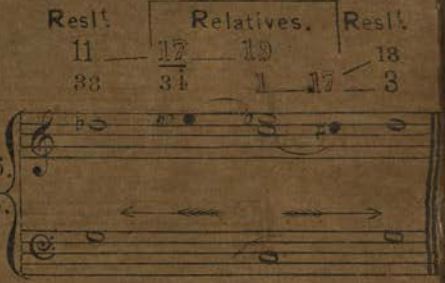
Ex. 84. § 249.



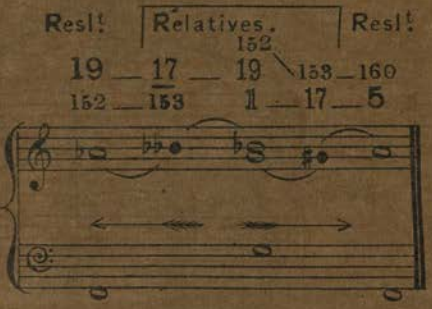
Ex. 85. § 249.



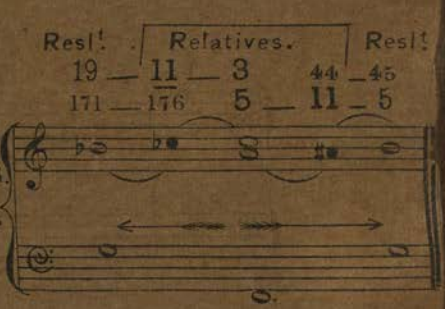
Ex. 86. § 249.



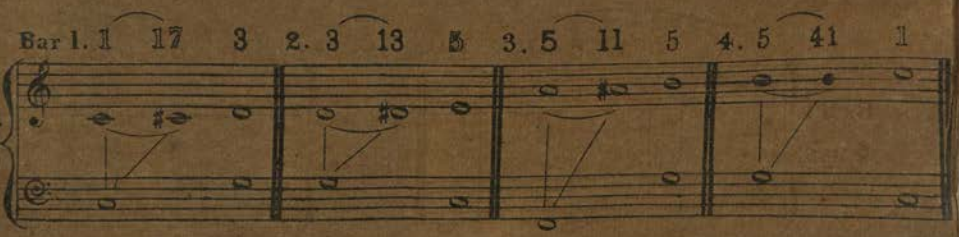
Ex. 87. § 249.

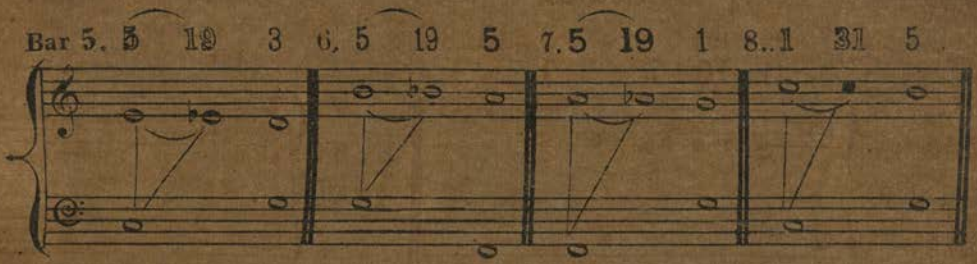


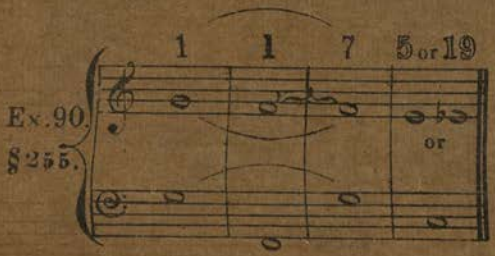
Ex. 88. § 249.

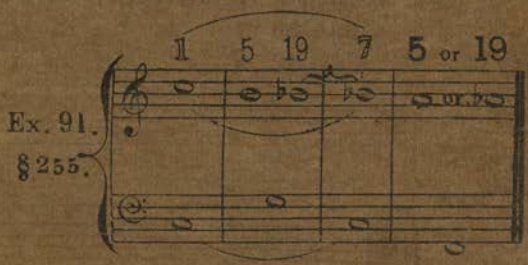


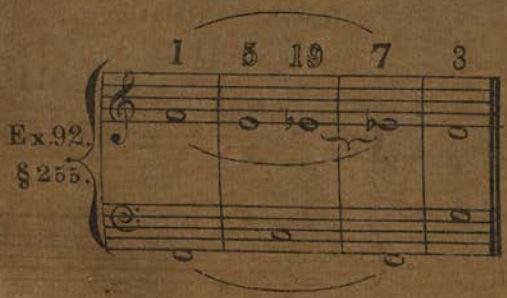
Ex. 89. § 252.

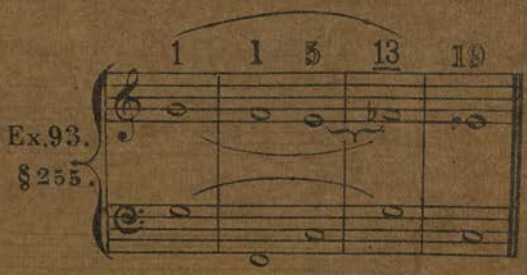


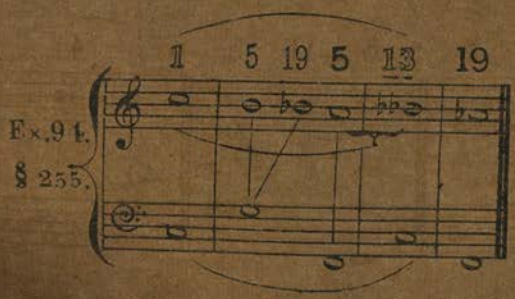

Bar 5. 5 19 3 6. 5 19 5 7. 5 19 1 8. 1 31 5

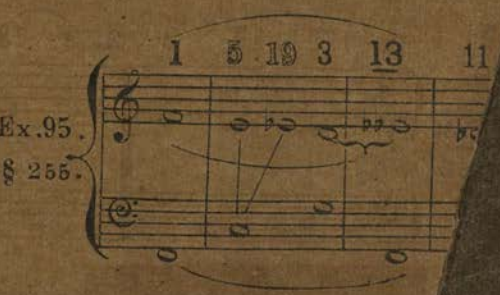

Ex. 90.
§ 255.
1 1 7 5 or 19
or


Ex. 91.
§ 255.
1 5 19 7 5 or 19
or

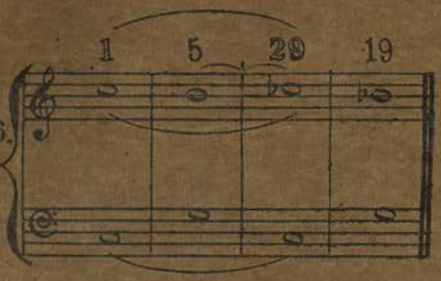

Ex. 92.
§ 255.
1 5 19 7 3


Ex. 93.
§ 255.
1 1 5 13 19

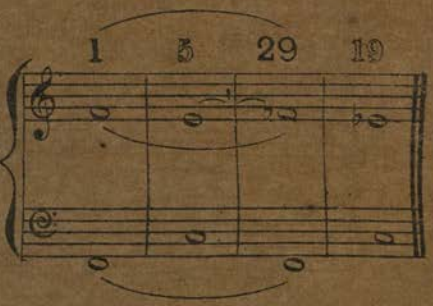

Ex. 94.
§ 255.
1 5 19 5 13 19


Ex. 95.
§ 255.
1 5 19 3 13 11

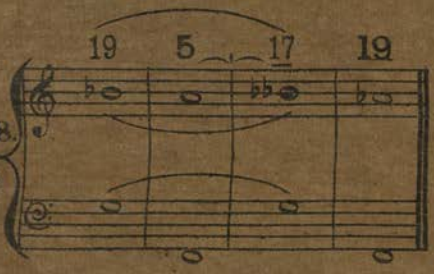
Ex. 96. § 255



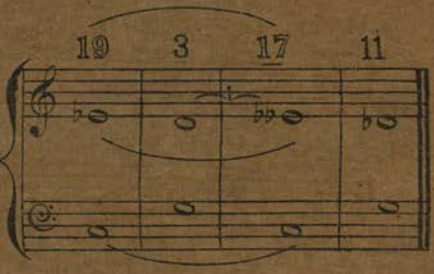
Ex. 97. § 255



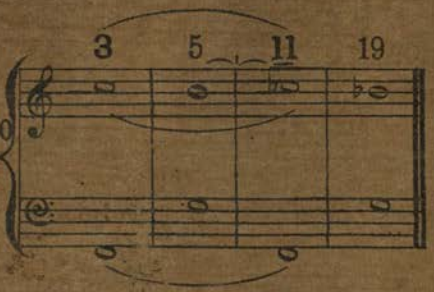
Ex. 98. § 255



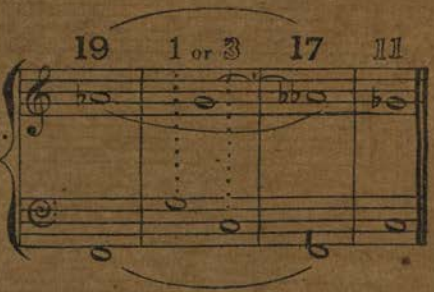
Ex. 99. § 255



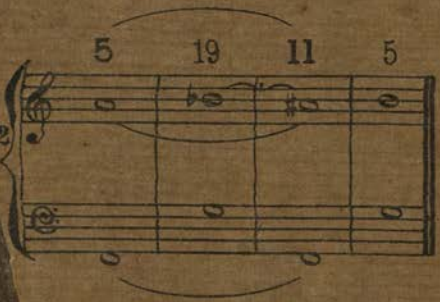
Ex 100 § 255



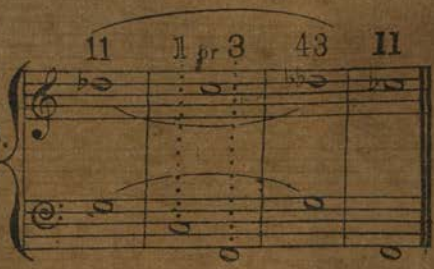
Ex. 101. § 255



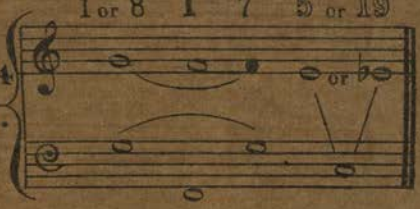
02



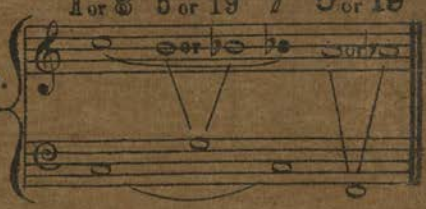
Ex. 103. § 255



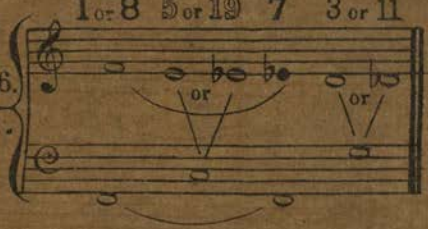
Ex.104 § 261.



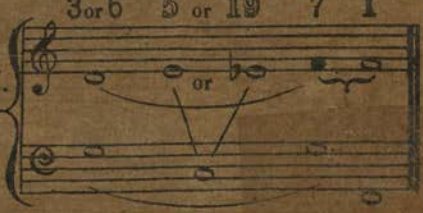
Ex.105. § 261.



Ex.106. § 261.

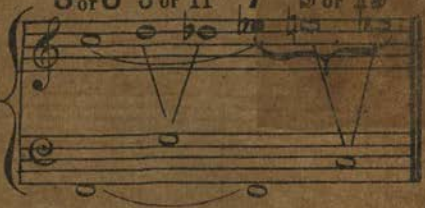


Ex.107. § 261.



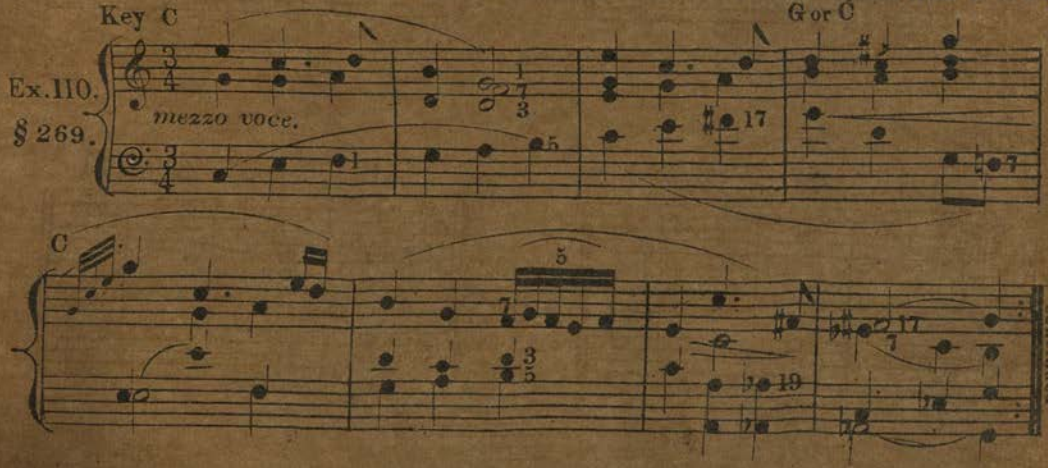
Ex.108. § 261.

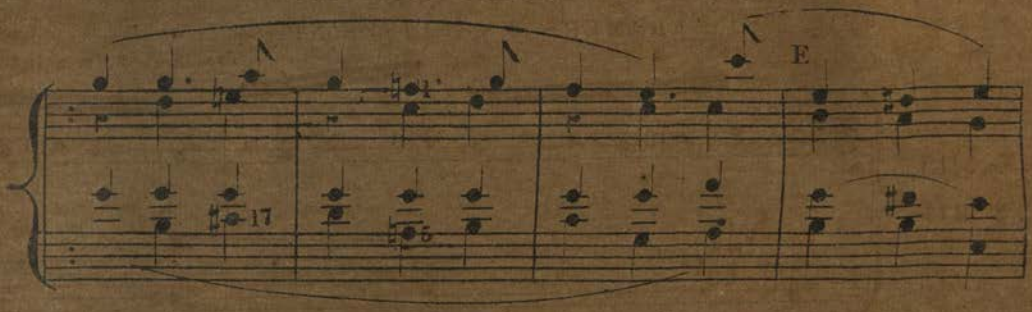
Ex.109. § 261.

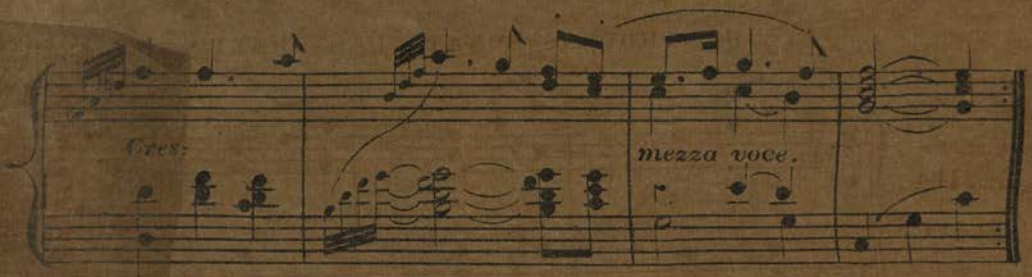


BEETHOVEN'S SONATA Op: 109. Transposed from E to C. G or C

Ex.110. § 269.



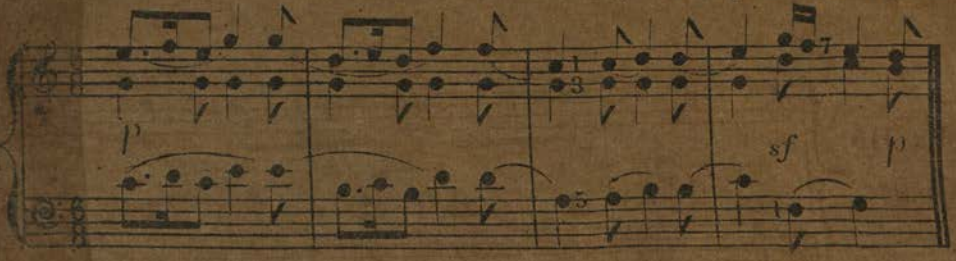




MOZARTS SONATA Op: 19.

Transposed from A to C.

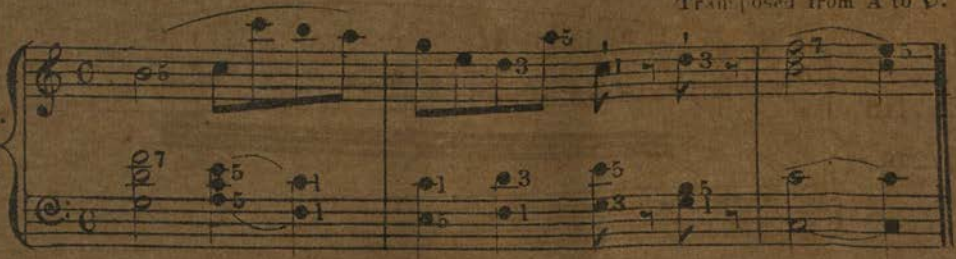
Ex.111. § 270.



MOZARTS SONATA Op: 15.

Transposed from A to C.

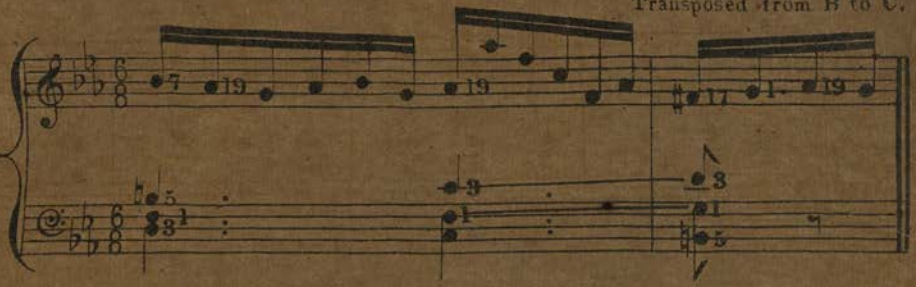
Ex.112. § 274.



CRAMERS EXERCISES Nº 10.

Transposed from B to C.

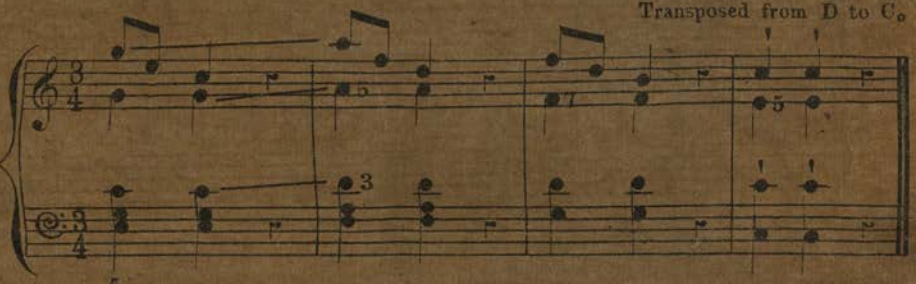
Ex. 113.
§ 274.



BEETHOVENS SONATA PASTORALE.

Transposed from D to C.

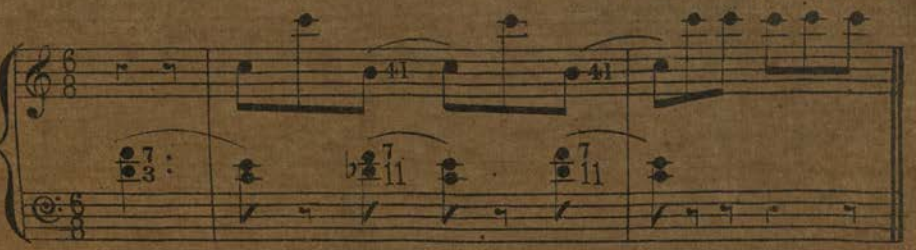
Ex. 114.
§ 274



MOZART'S SONATA Op: 19 Nº 2.

Transposed from D to C.

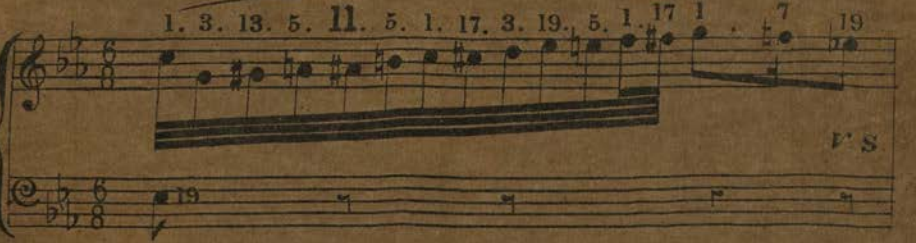
Ex. 115.
§ 276.

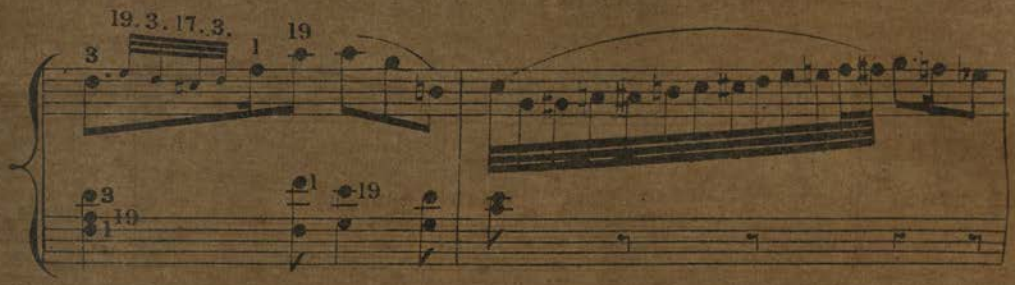


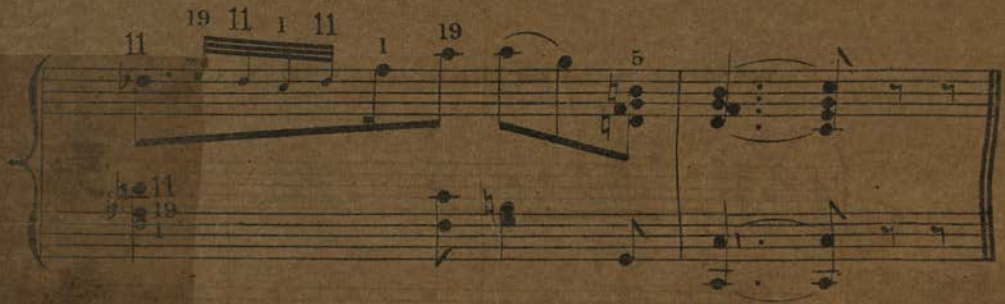
MOZART'S SONATA Op: 2. Nº 3.

Transposed from D to C.

Ex. 116.
§ 276.





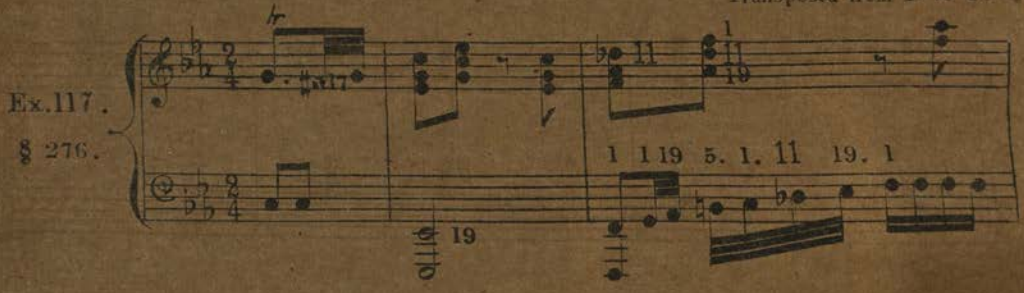


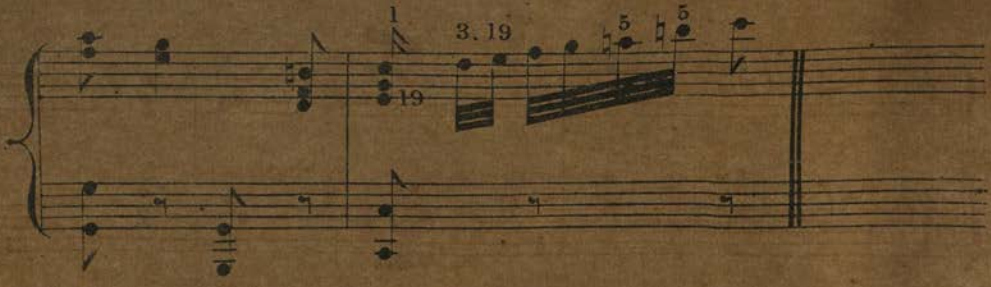
MOZART'S SONATA Op: 2. Nº 3.

Transposed from D to C.

Ex. 117.

§ 276.



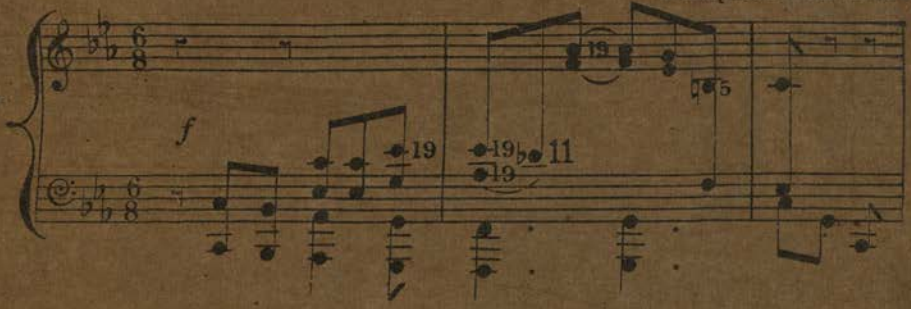


AIR WITH VARIATIONS BY MOZART Nº 12.

Transposed from ♭E to C.

Ex. 118.

§ 276.

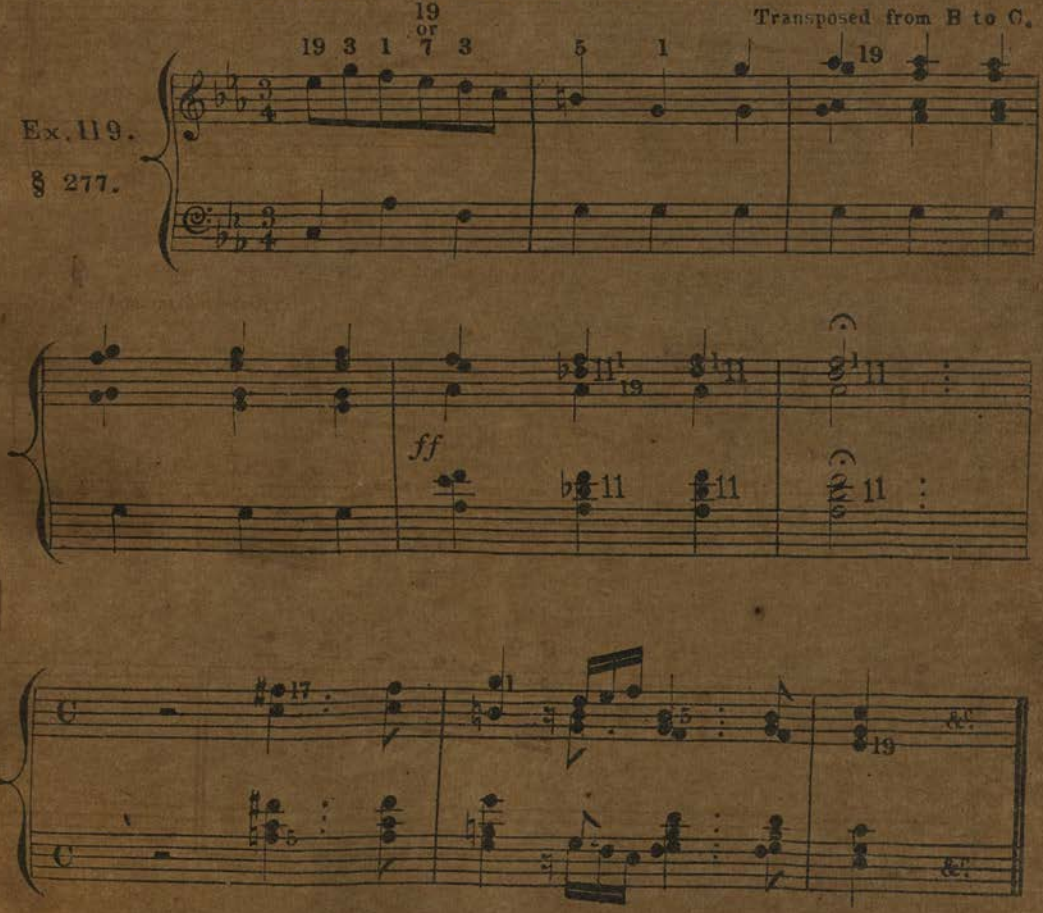


HANDEL'S OVERTURE TO ARMINIUS.

Transposed from B to C.

Ex. 119.

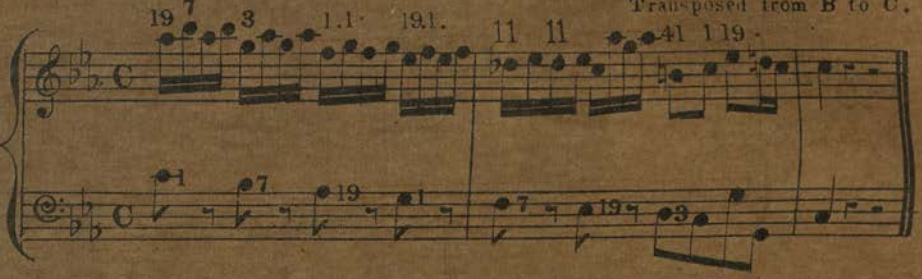
§ 277.



HANDEL'S CORONATION ANTHEM.

Transposed from B to C.

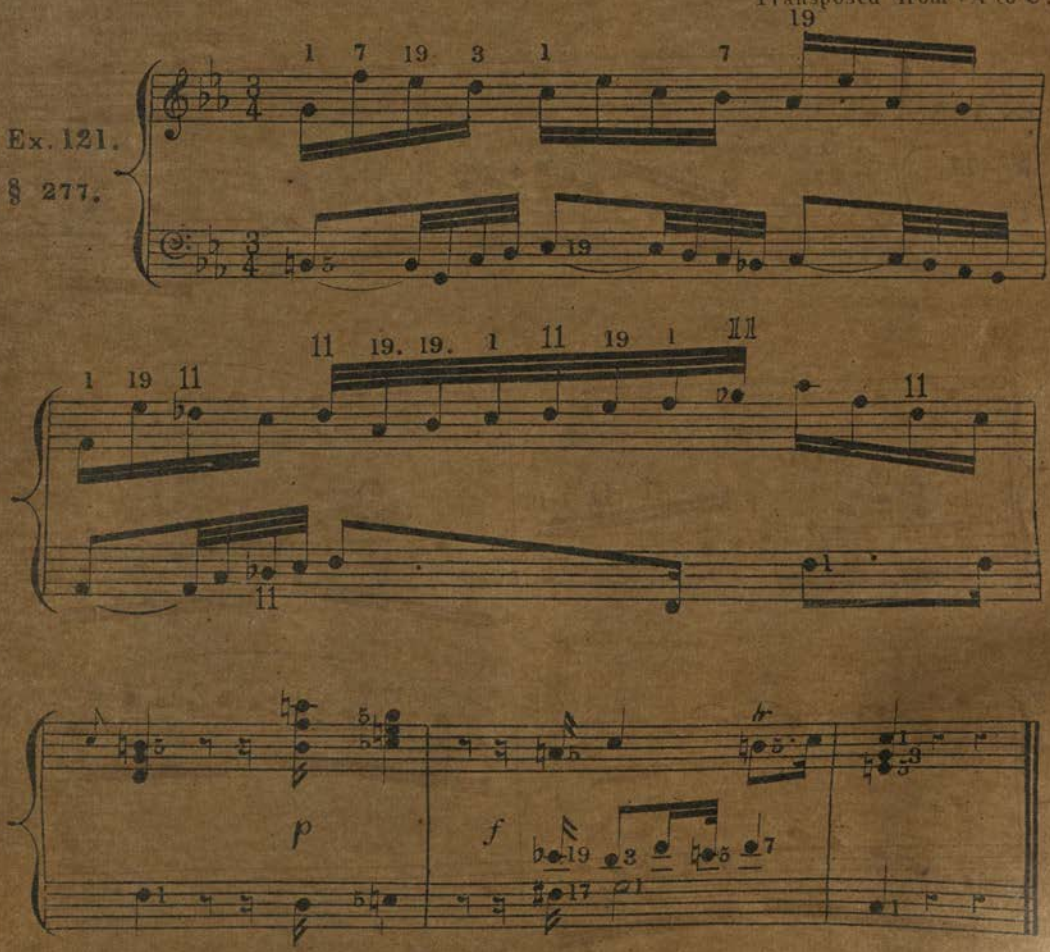
Ex. 120.
§ 277.



S. BACH'S 41st PRELUDE.

Transposed from A to C.

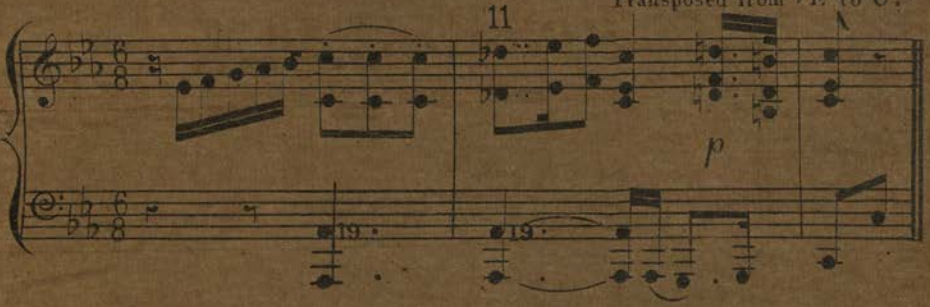
Ex. 121.
§ 277.



BEETHOVEN'S VARIATIONS TO THE MANLY HEART

Transposed from E to C.

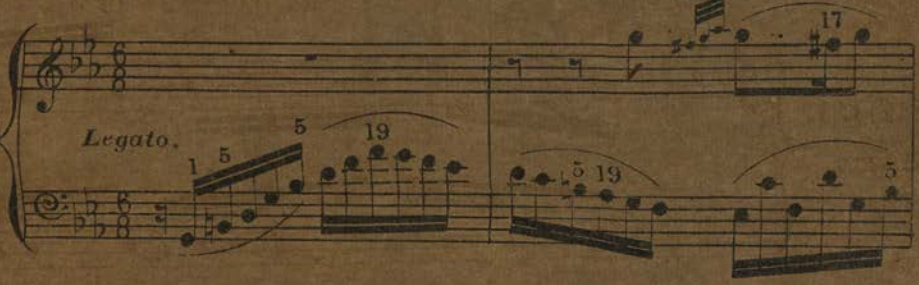
Ex. 122.
§ 277.

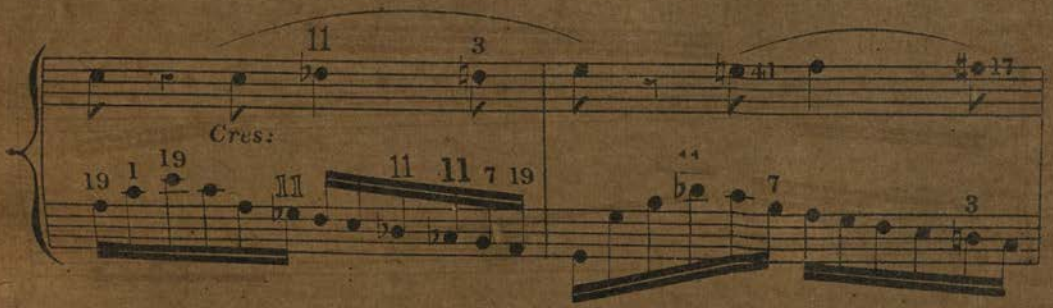


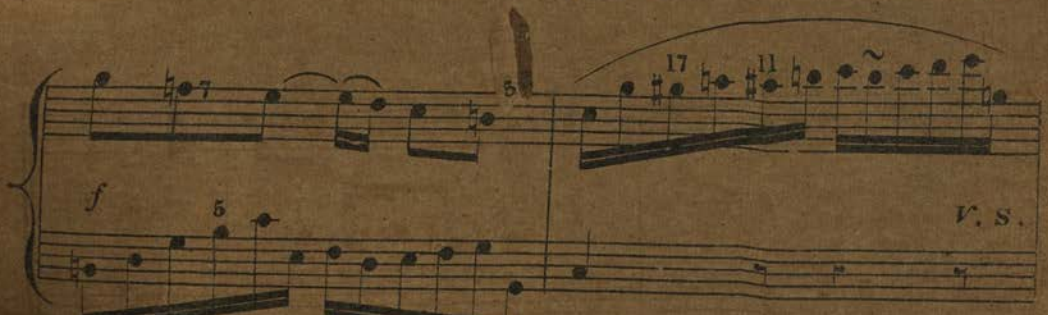
RONDO BY MOZART.

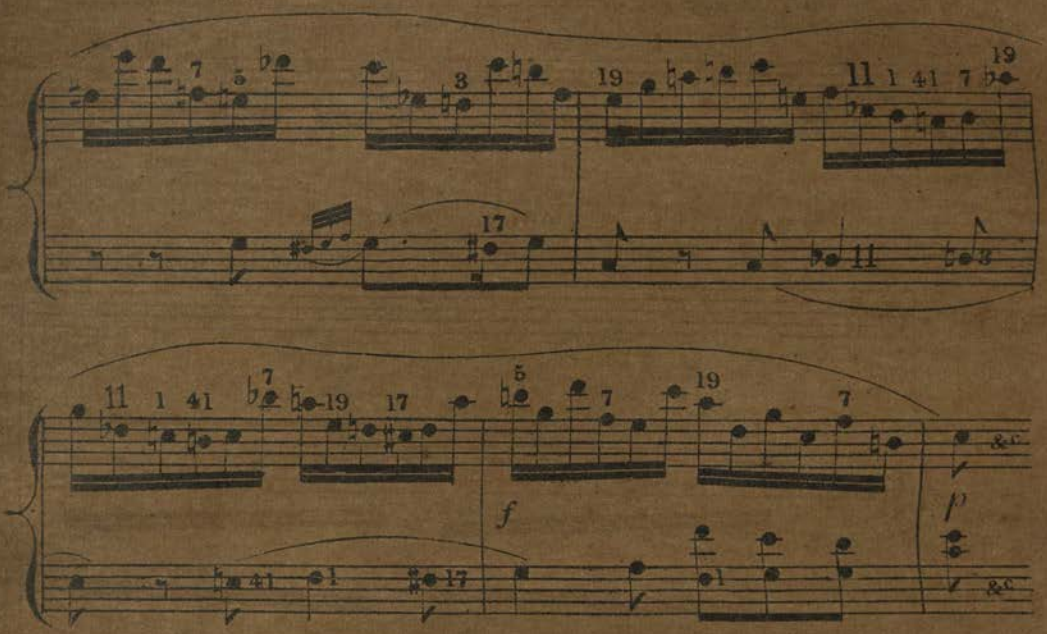
Transposed from A to C.

Ex. 123.
§ 277.





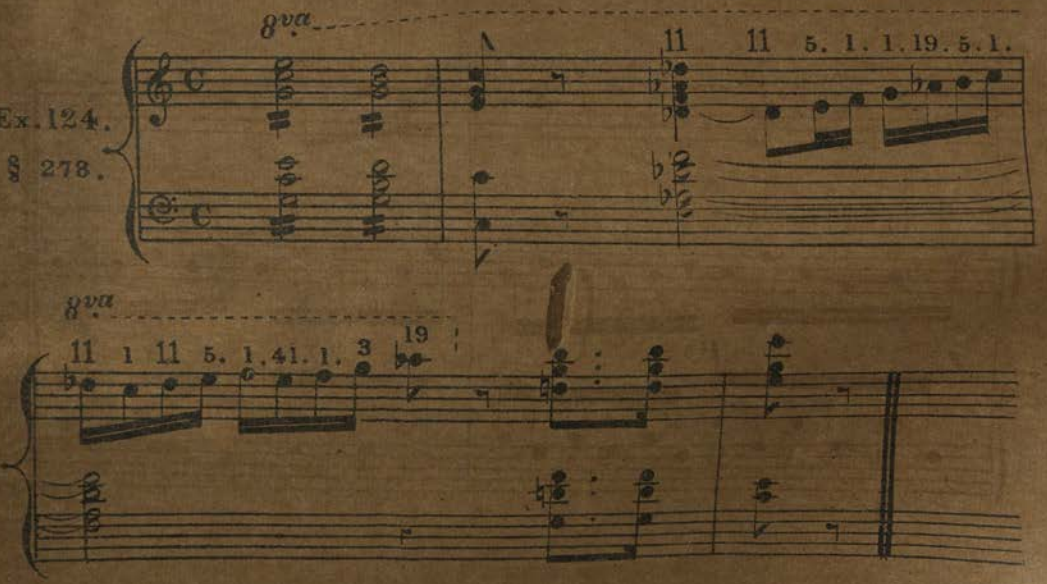




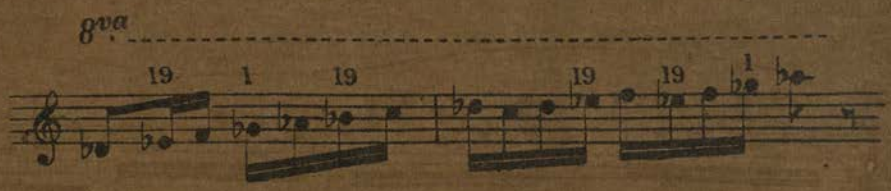
ROSSINI'S OVERTURE TO EDWARD AND CHRISTINA.

Transposed from A to C.

Ex. 124.
§ 278.



Ex. 125.
§ 278.

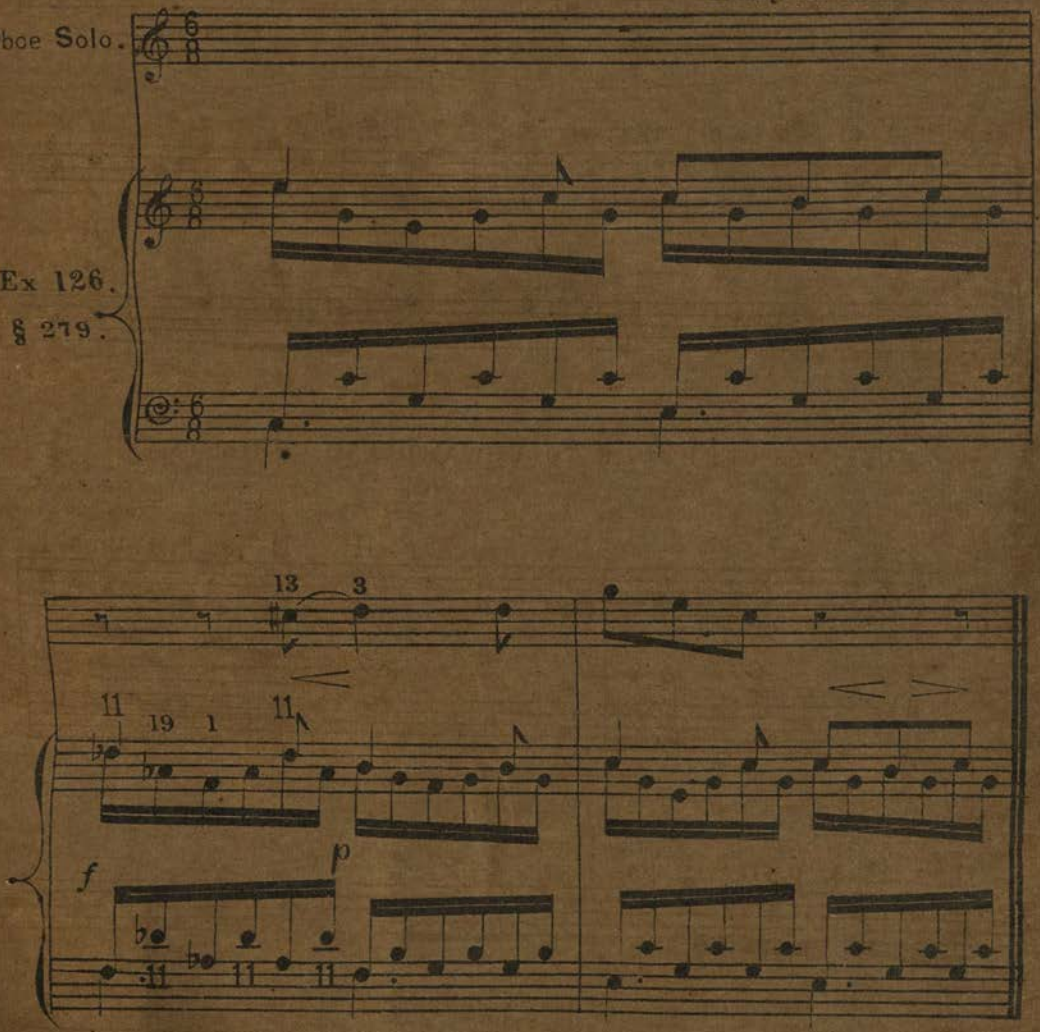


STEIBELT'S STORM CONCERTO.

Transposed from B to C.

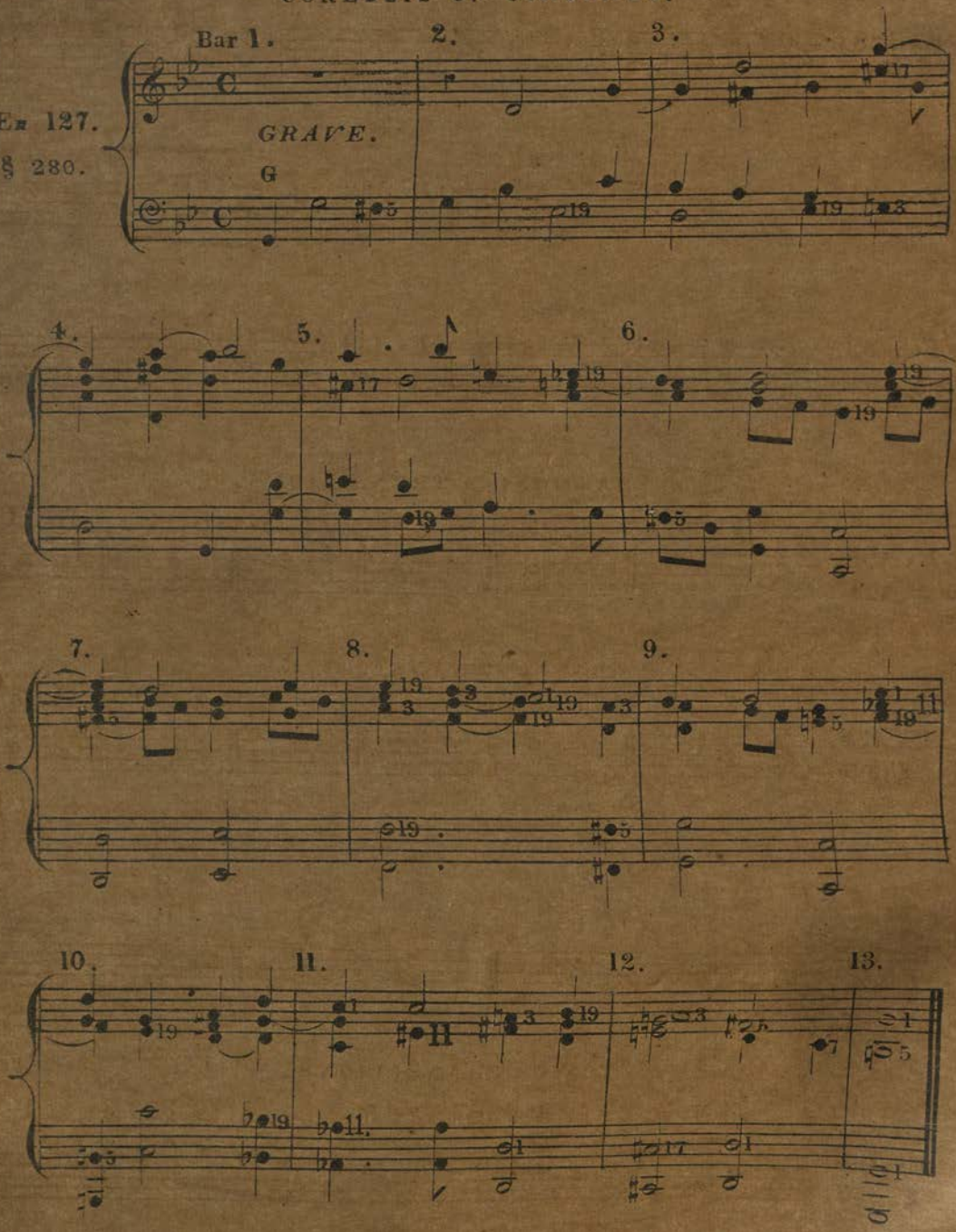
Oboe Solo.

Ex 126.
§ 279.



CORELLI'S 8th CONCERTO.

Ex 127.
§ 280.



Ex.128. § 284. 1 17 1

Ex.129. § 284. 1 7 5

Ex.130. § 289

Ex.131. § 293.

Ex.132. § 297.

Ex.133. § 297.

Ex.134. § 298.

HARMONIC TRANSITIONS.

Ex.135. § 324. 1 5 1 17 3 5 7 13 5 3 5 11

BASAL TRANSITIONS.

Ex.136. § 325. 1 5 17 3 3 5 13 5 5 1 11 5

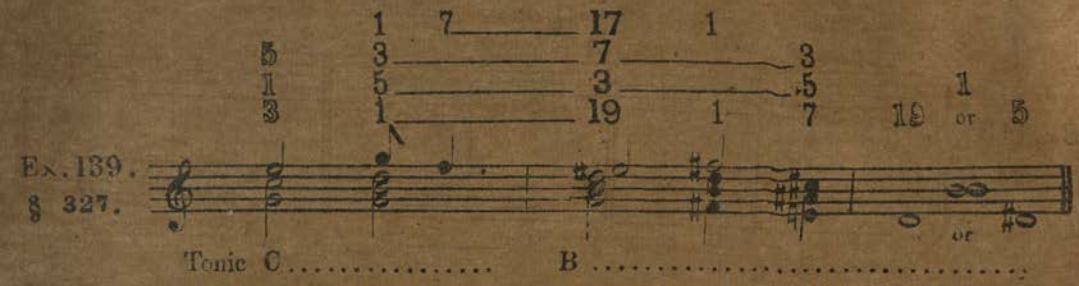
Ex.137. § 325.

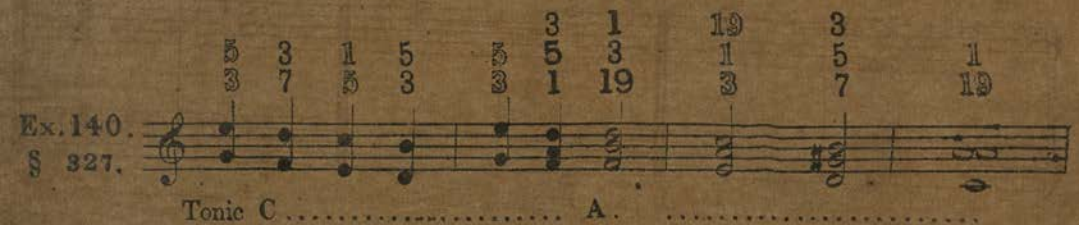
5	3	7		17	3
1	5	3	5	13	5
3	1	5	1	3	1
		1	3	19	3

TONIC TRANSITIONS.

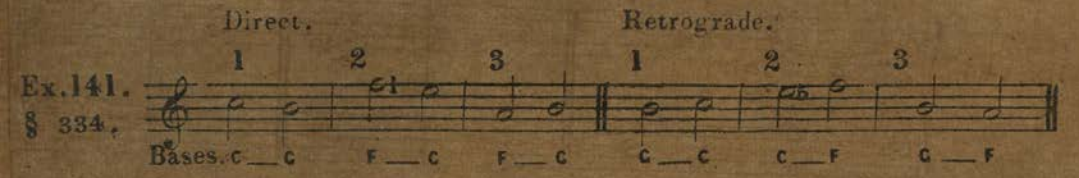
Ex.138. § 326. 1. 3. 5. 3. 1 3 5 5 1. 3. 19. 1. 19 3 1

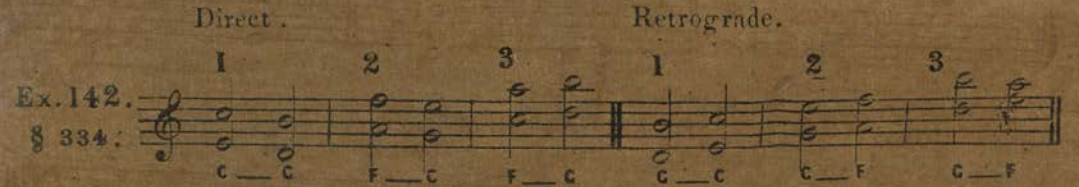
Tonic C..... E.....

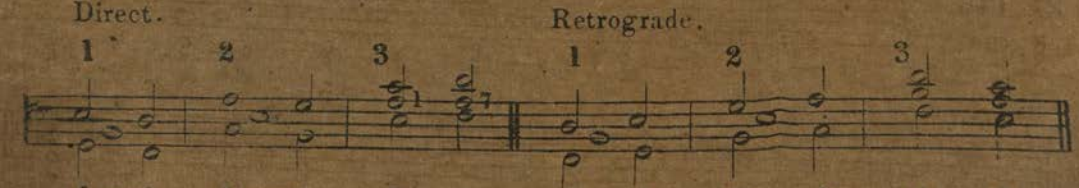




SINGLE BASAL TRANSITIONS.

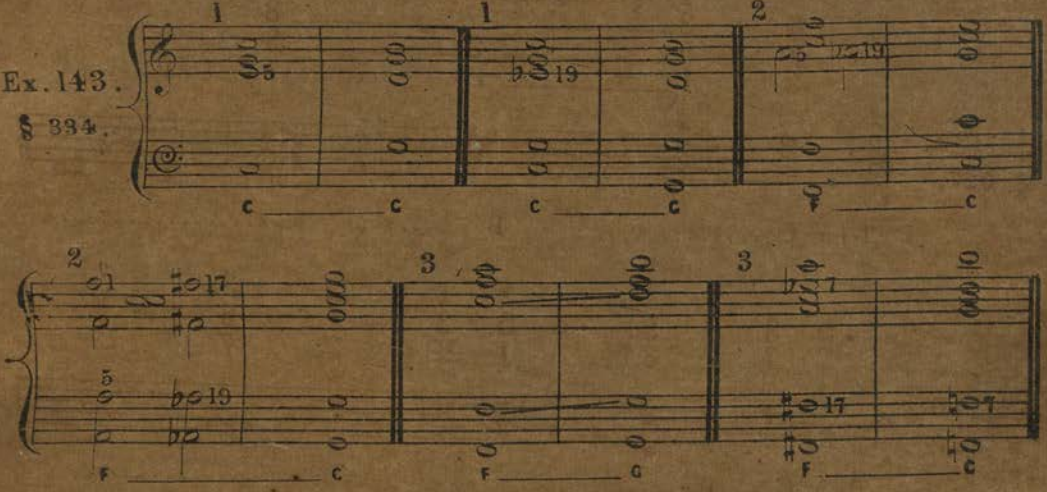






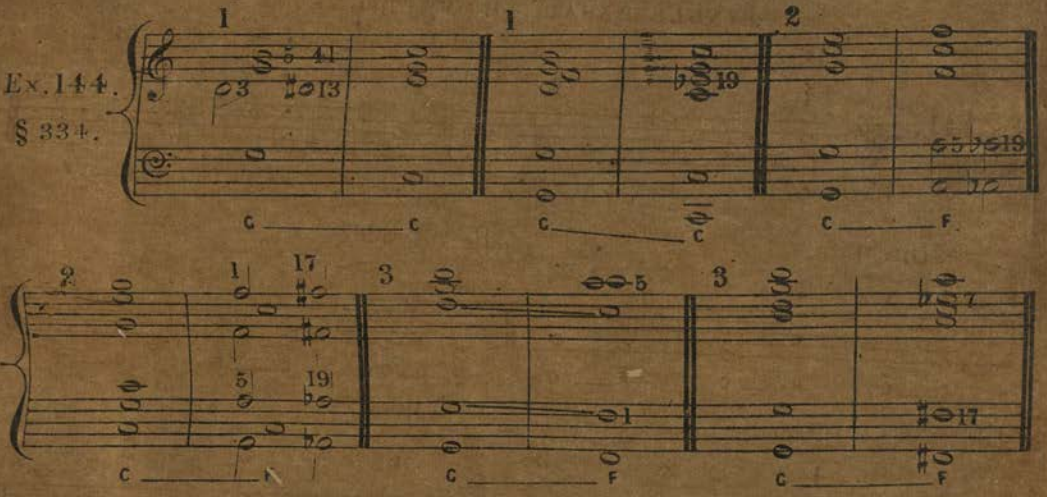
Direct.

Ex. 143.
§ 334.



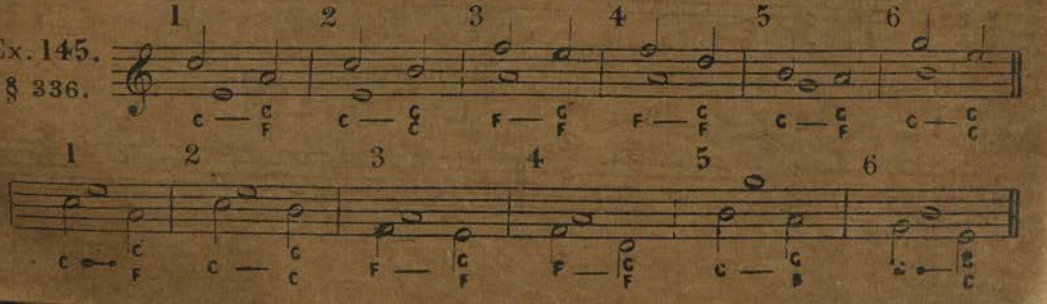
Retrograde.

Ex. 144.
§ 334.

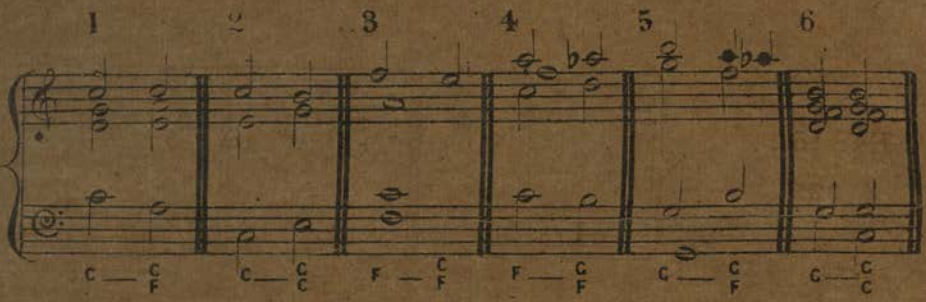


Class 1 Double basal transitions

Ex. 145.
§ 336.

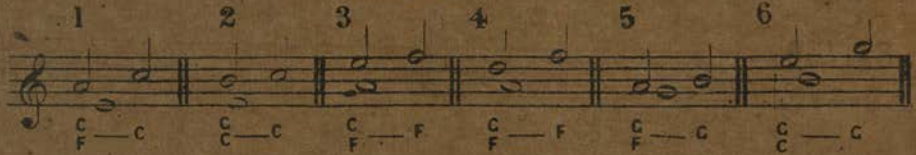


Ex. 146.
§ 336.

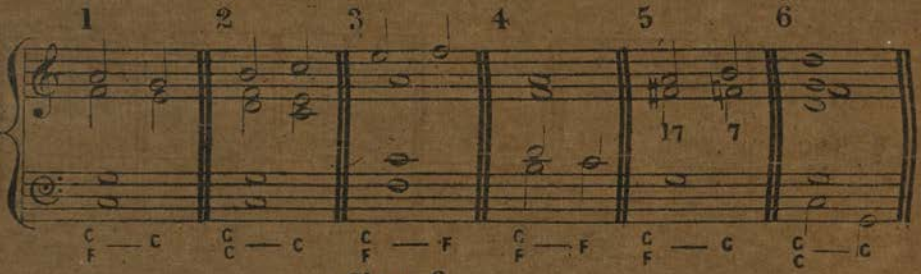


Class 2.

Ex. 147.
§ 337.

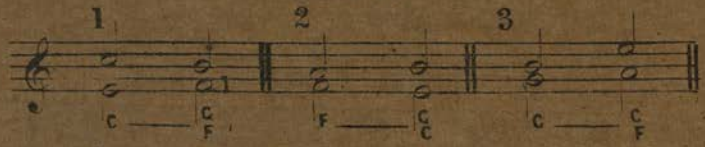


Ex. 148.
§ 337.

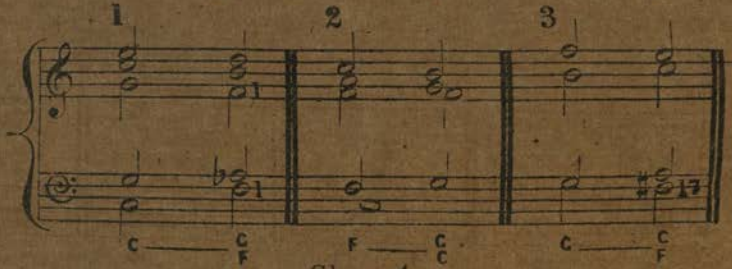


Class 3.

Ex. 149.
§ 338.

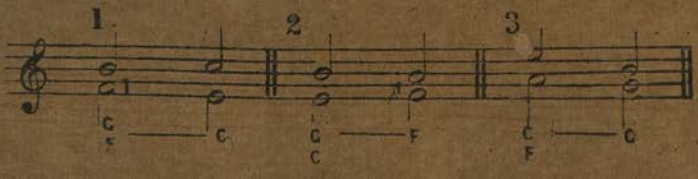


Ex. 150.
§ 338.

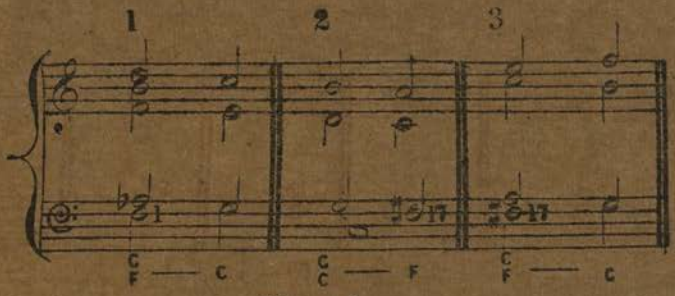


Class 4.

Ex. 151.
§ 339.

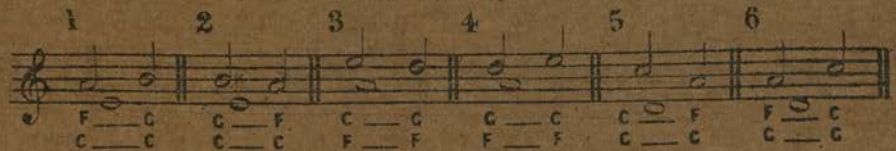


Ex. 152.
§ 839.

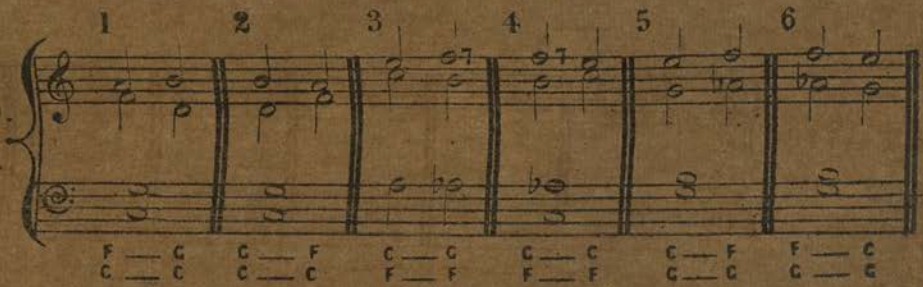


Class 5.

Ex. 153.
§ 340.

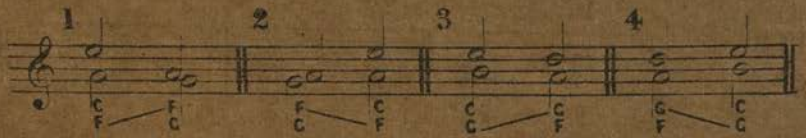


Ex. 154.
§ 340.

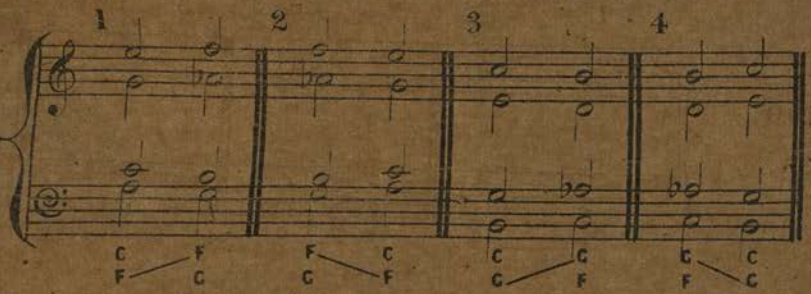


Class 6.

Ex. 155.
§ 341.

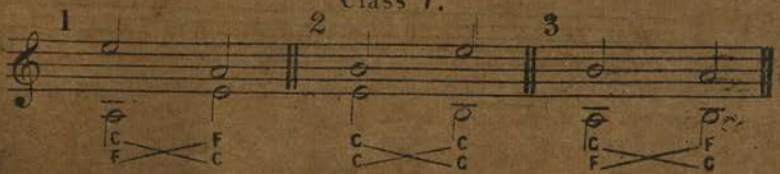


Ex 156.
§ 341.

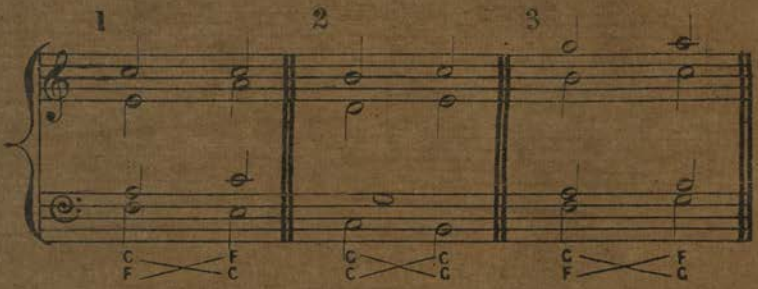


Class 7.

Ex. 57.
§ 342.

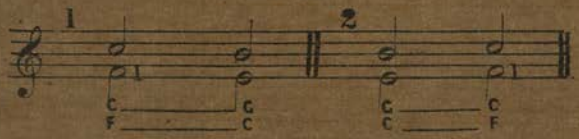


Ex. 158.
§ 342.

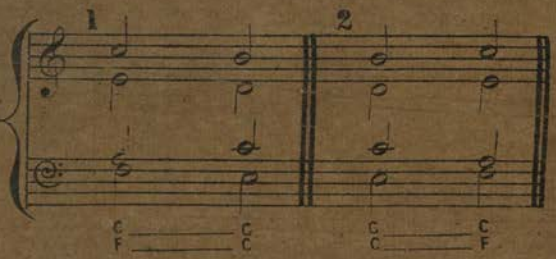


Class 8.

Ex. 159.
§ 343.

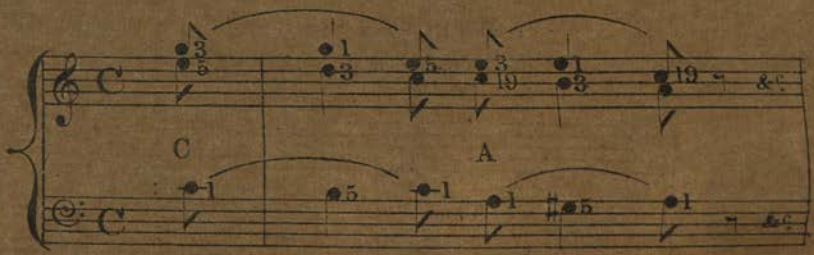


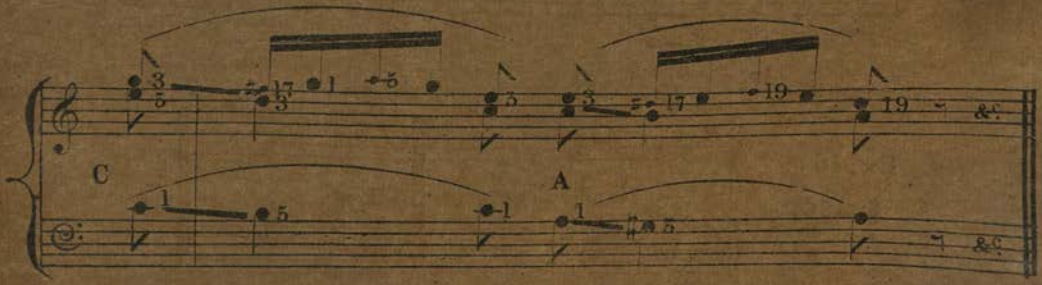
Ex. 160.
§ 343.



Ex. 161.
§ 344.

Ex. 162.
§ 360.

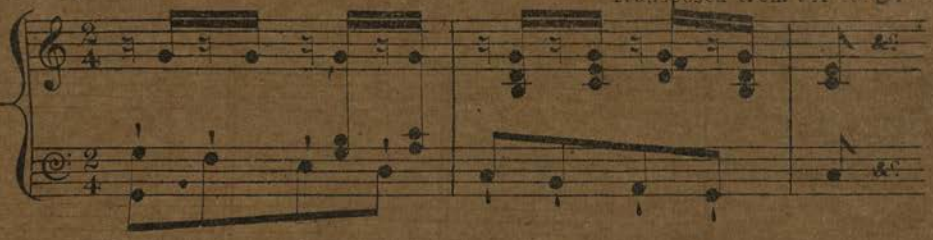




BEETHOVEN Op: 53.

Transposed from ♭A to C.

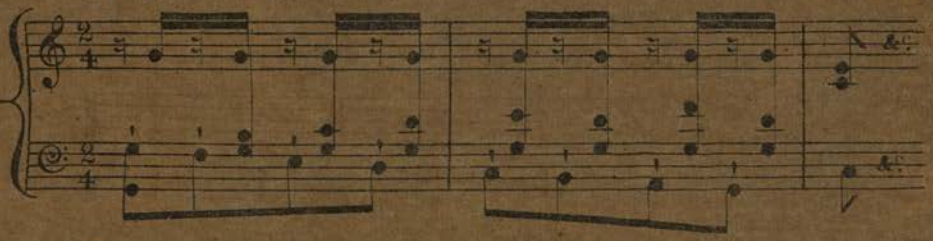
Ex: 163.
§ 354.



The above corrected.

Transposed from ♭A to C.

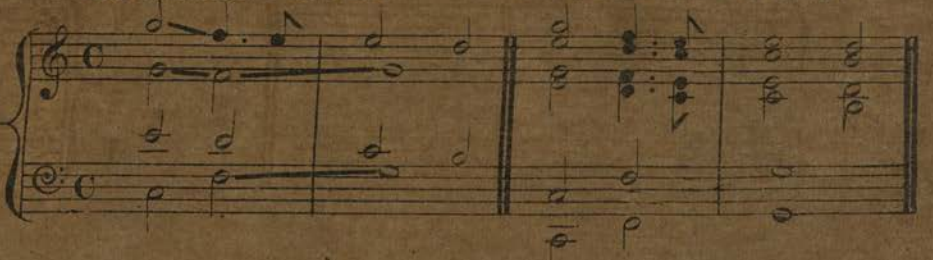
Ex. 164.
§ 354.



Bad consecutive octaves.

Good consecutive octaves.

Ex. 165.
§ 357.

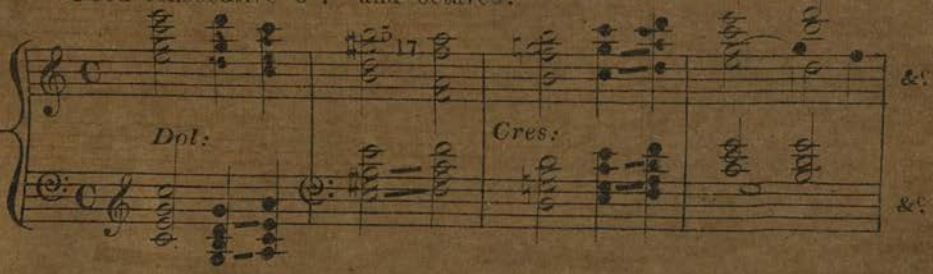


BEETHOVEN'S GRAND SONATA Op: 53.

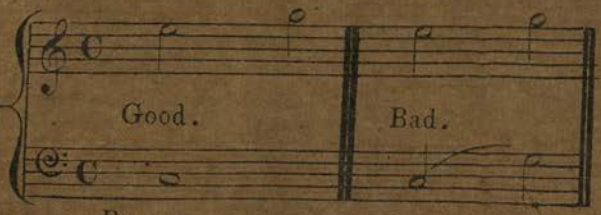
Good consecutive 5ths and octaves.

Transposed from A to C.

Ex. 166
§ 357.

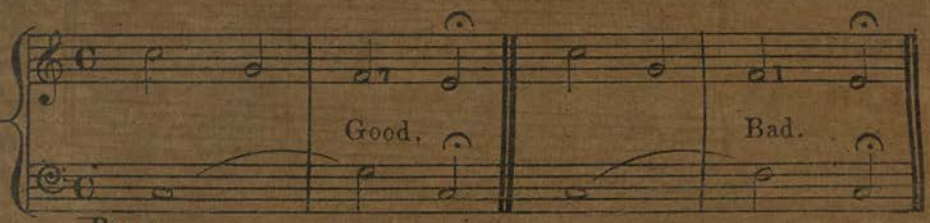


Ex. 167.
§ 366.

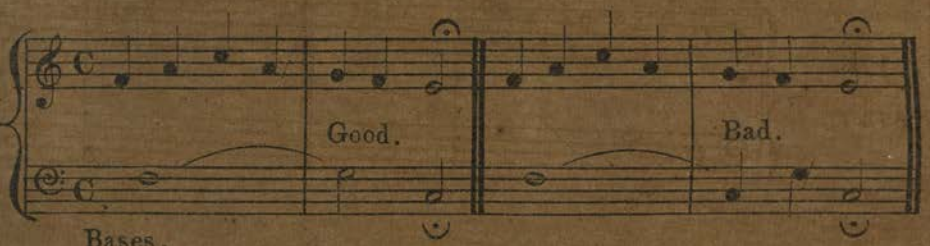


Bases.

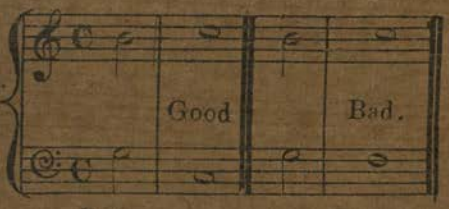
Ex. 168.
§ 367.



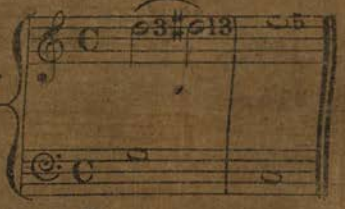
Ex. 169.
§ 368.



Ex. 170.
§ 369

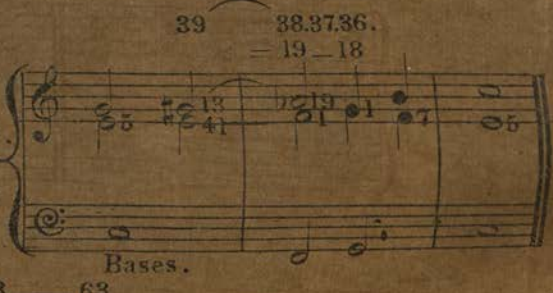


Ex. 171.
§ 370.

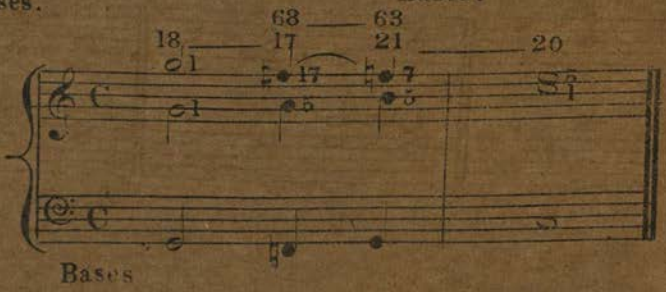


Ex. 172.
§ 371.

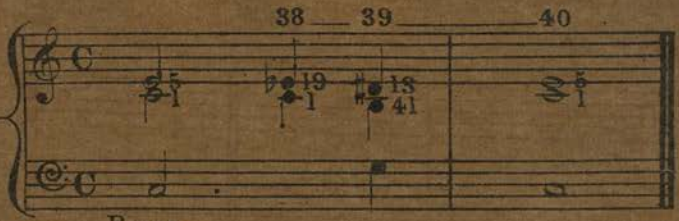
Ex. 173.
§ 372.



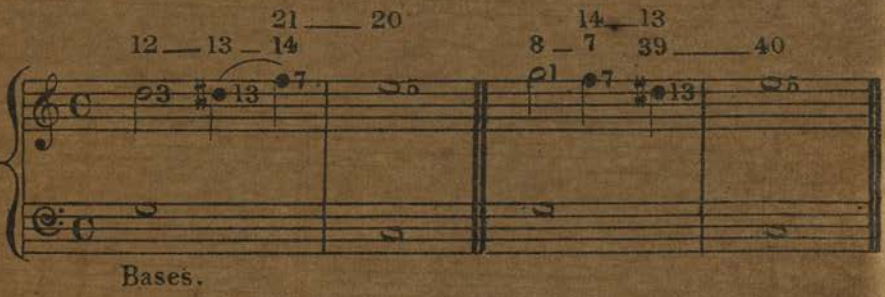
Ex. 174.
§ 372.



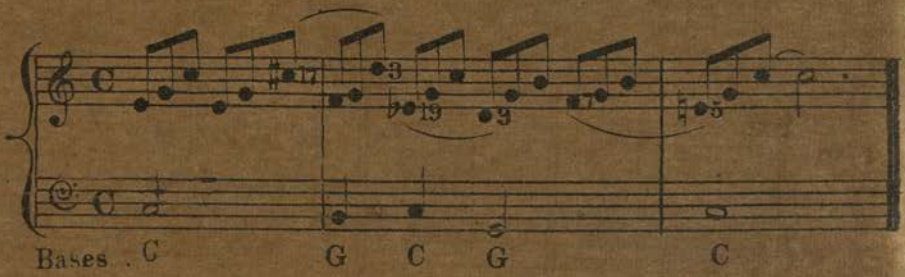
Ex. 175.
§ 373.



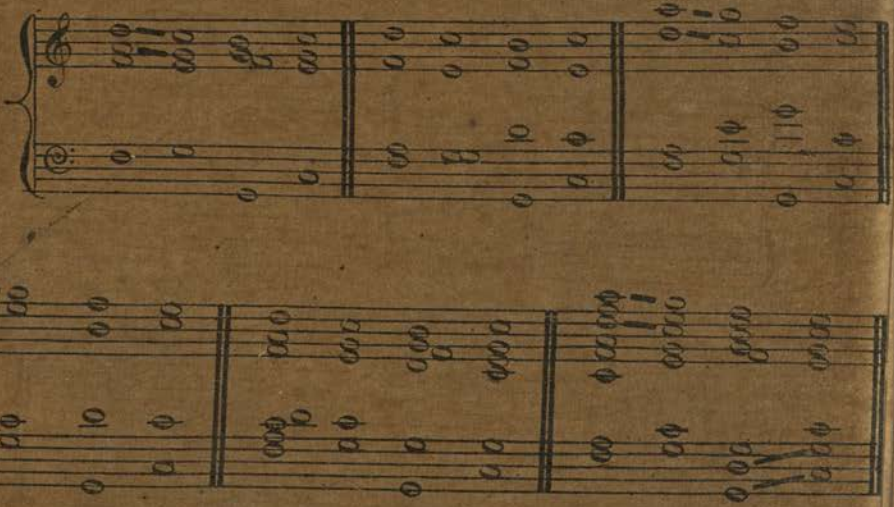
Ex. 176.
§ 374.



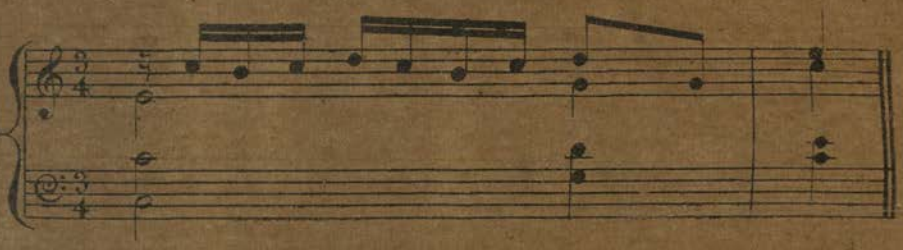
Ex. 177.
§ 375.



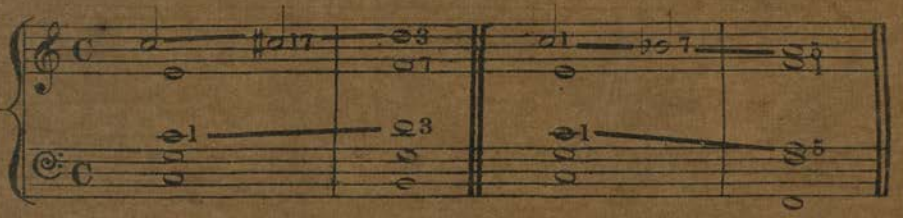
Ex. 178.
§ 386.



Ex. 179.
§ 333.



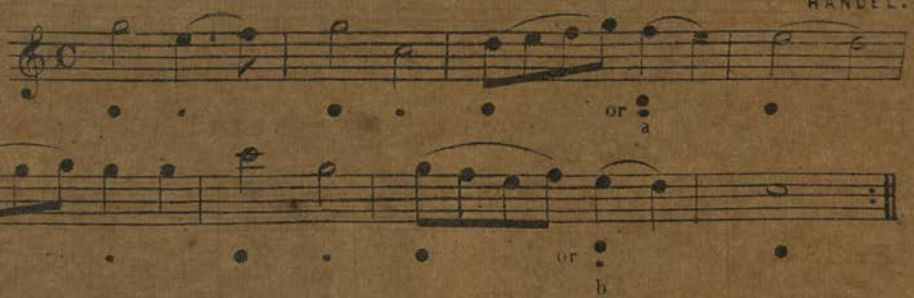
Ex. 180.
§ 333.



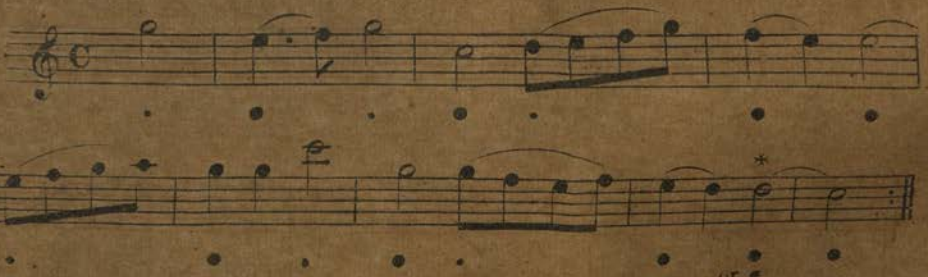
SEE THE CONQUERING HERO COMES.

HANDEL.

Ex. 181.
§ 425.

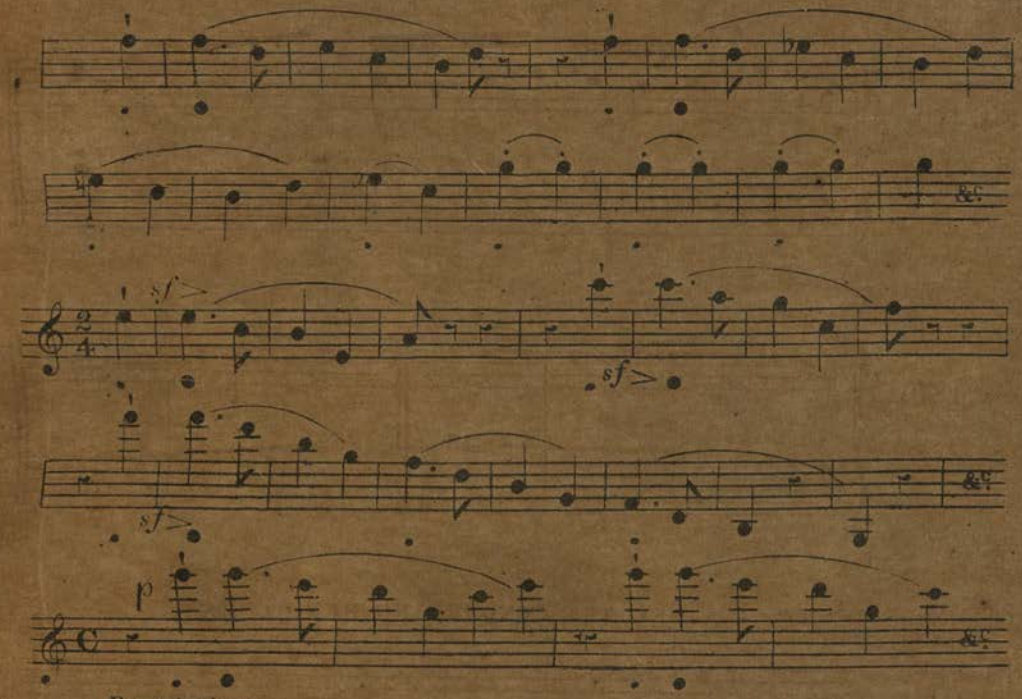


Ex. 182.
§ 426.



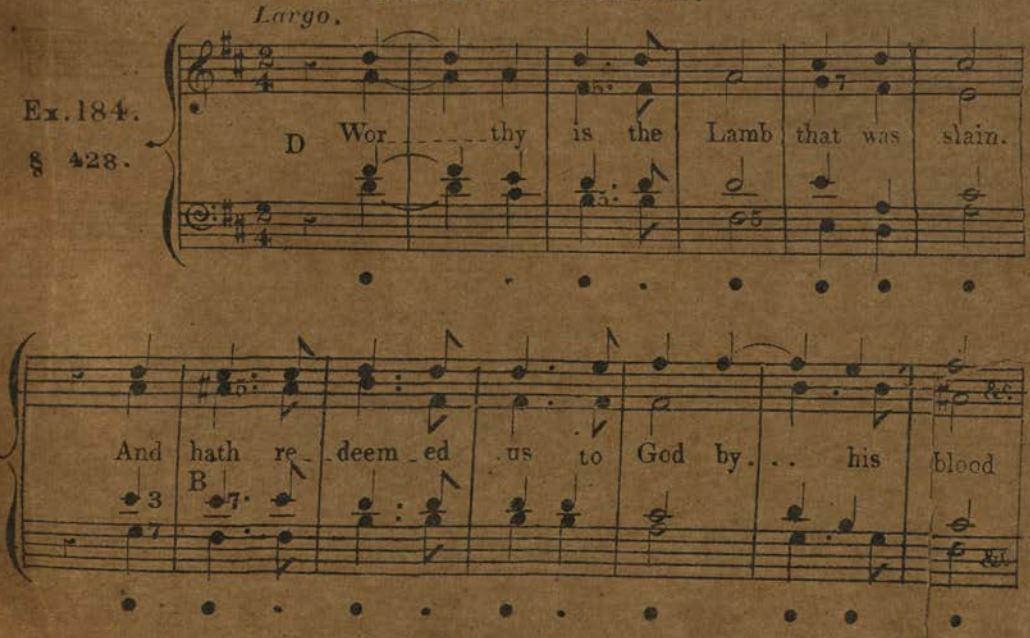
BEETHOVEN'S SONATA Op: 53.

Ex. 183.
§ 427.



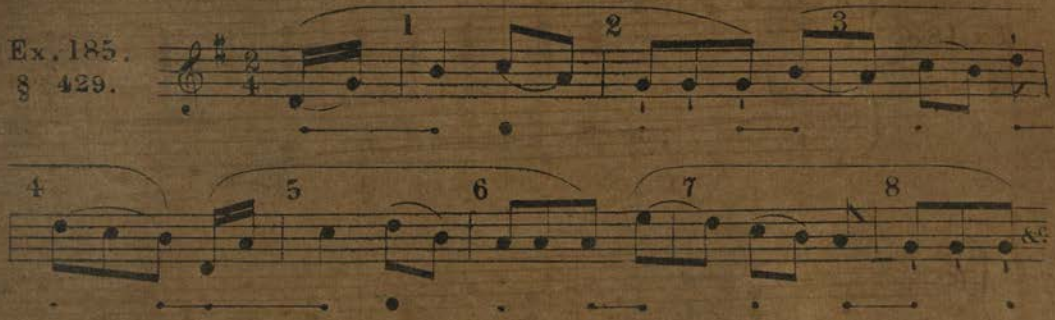
HANDEL'S MESSIAH.

Ex. 184.
§ 428.



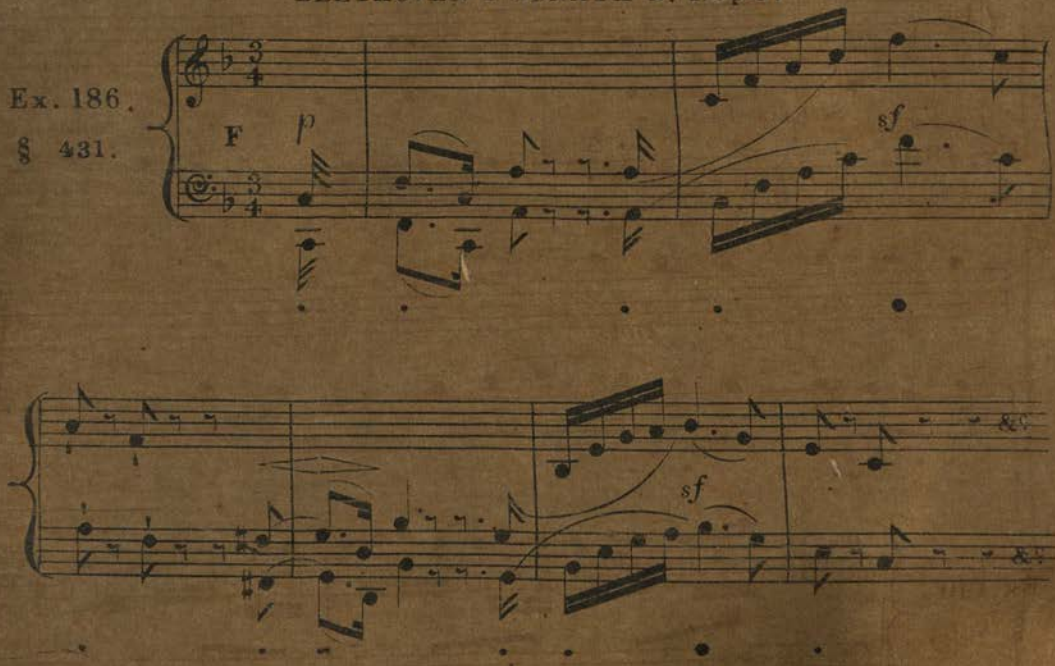
HAYDN'S SYMPHONY Nº 3.

Ex. 185.
§ 429.



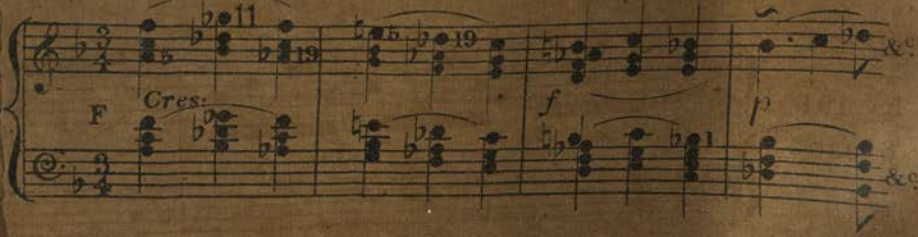
BEETHOVEN'S SONATA Nº 1. Op: 5.

Ex. 186.
§ 431.



From the same.

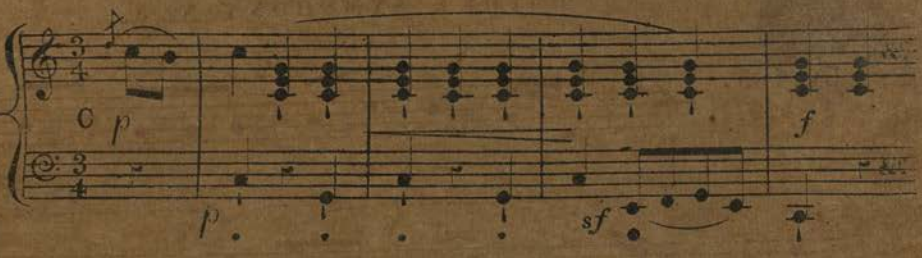
Ex. 187.
§ 431.

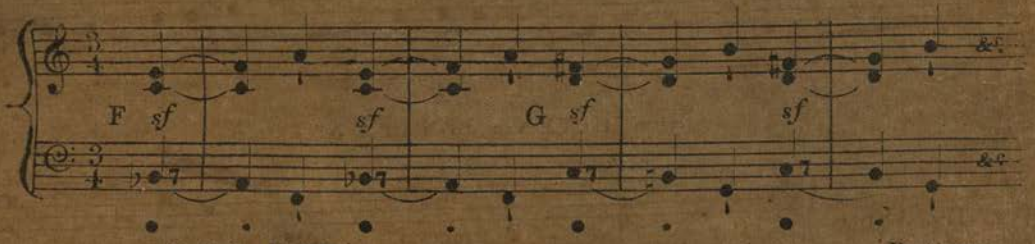


WALTZ.

BEETHOVEN

Ex. 188.
§ 432.

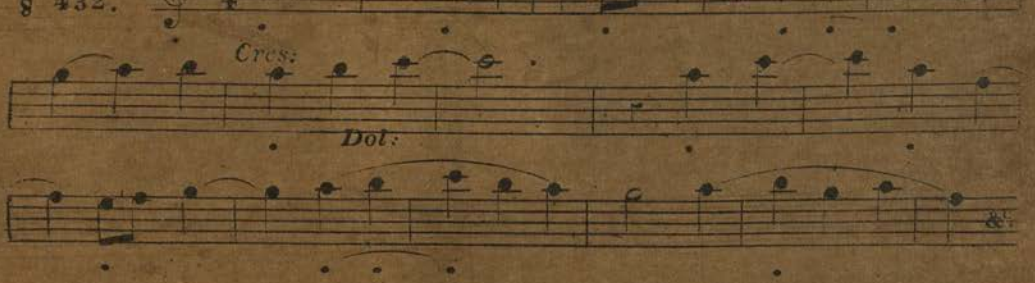




Sinfonia Pastorale

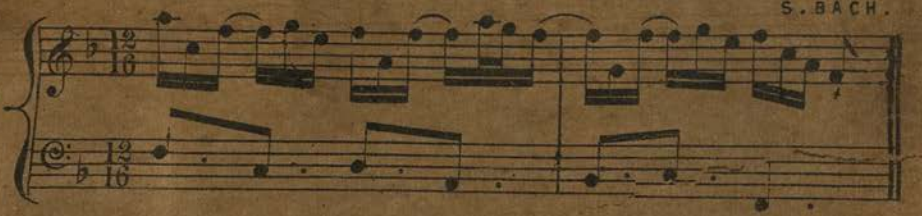
Beethoven.

Ex. 189.
§ 432.

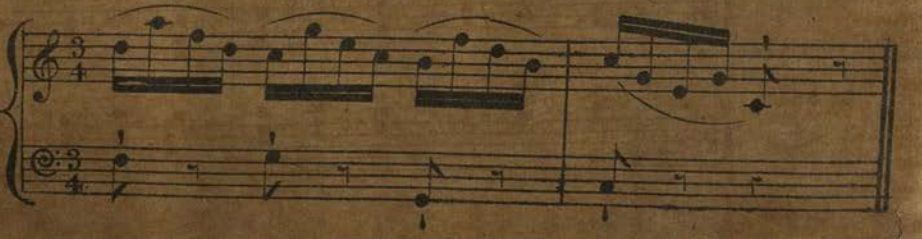


S. BACH.

Ex. 190.
§ 436.

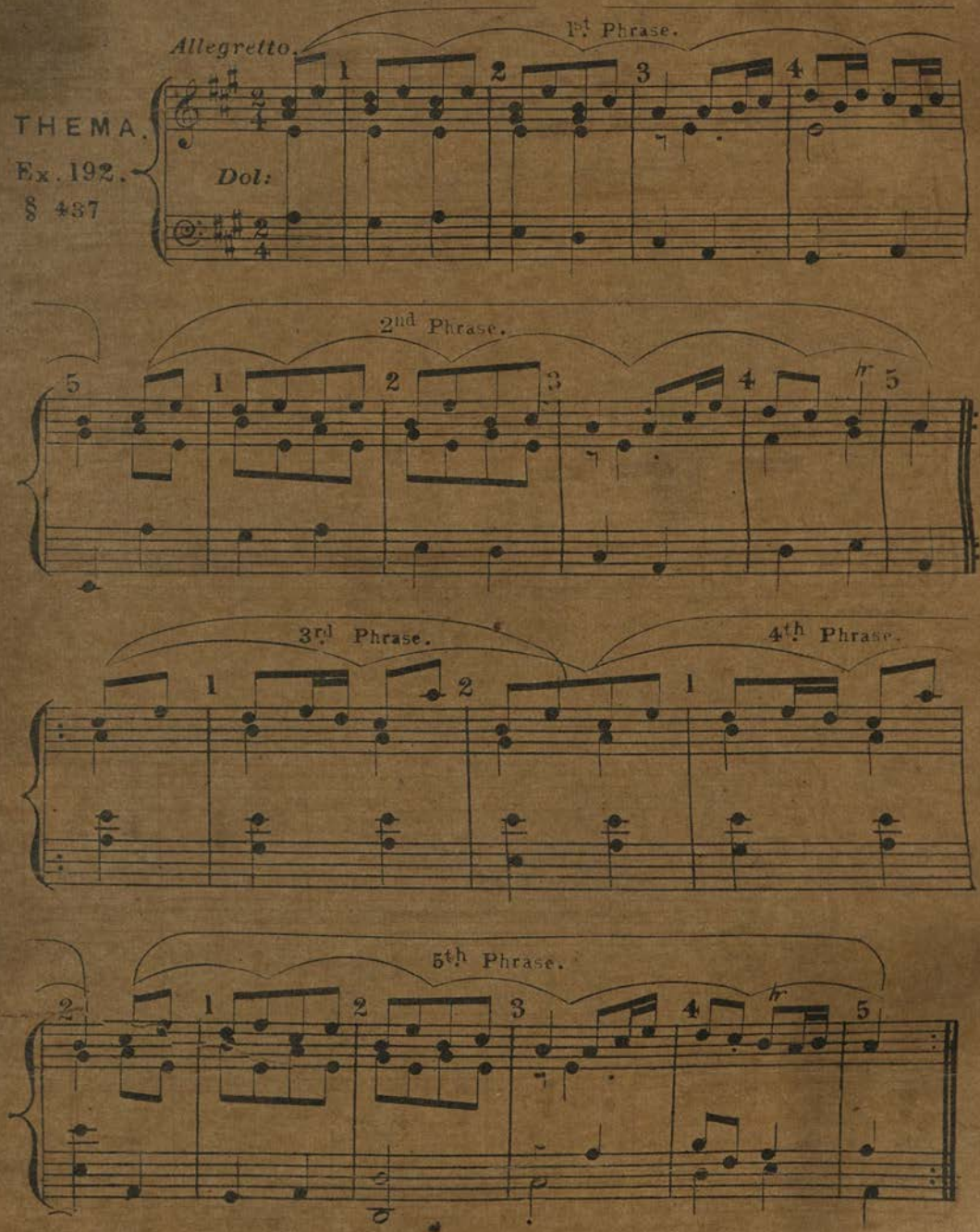


Ex. 191.
§ 336.

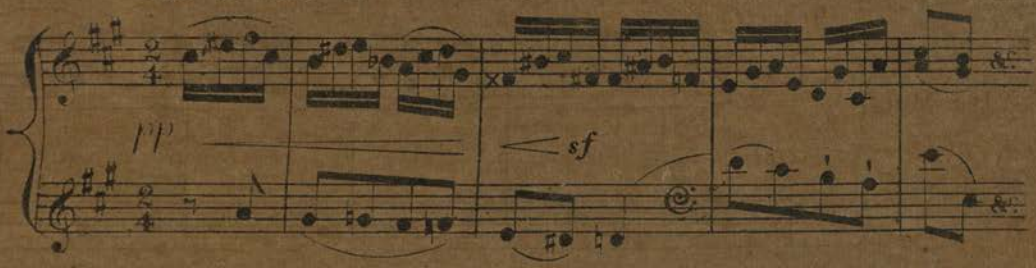


RUSSIAN AIR WITH VARIATIONS.

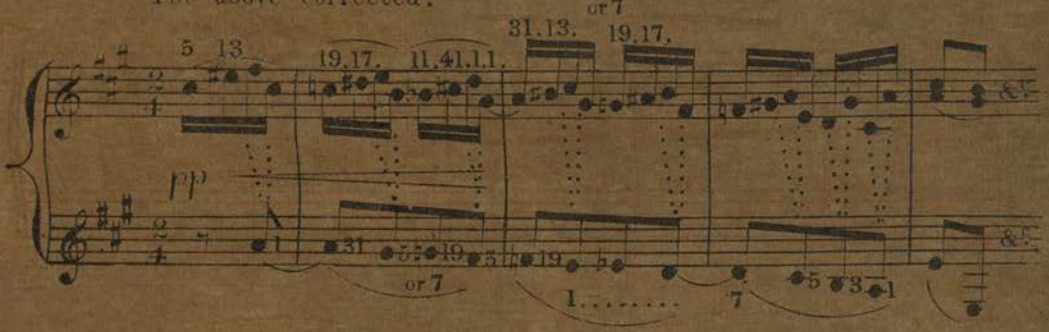
BEETHOVEN.



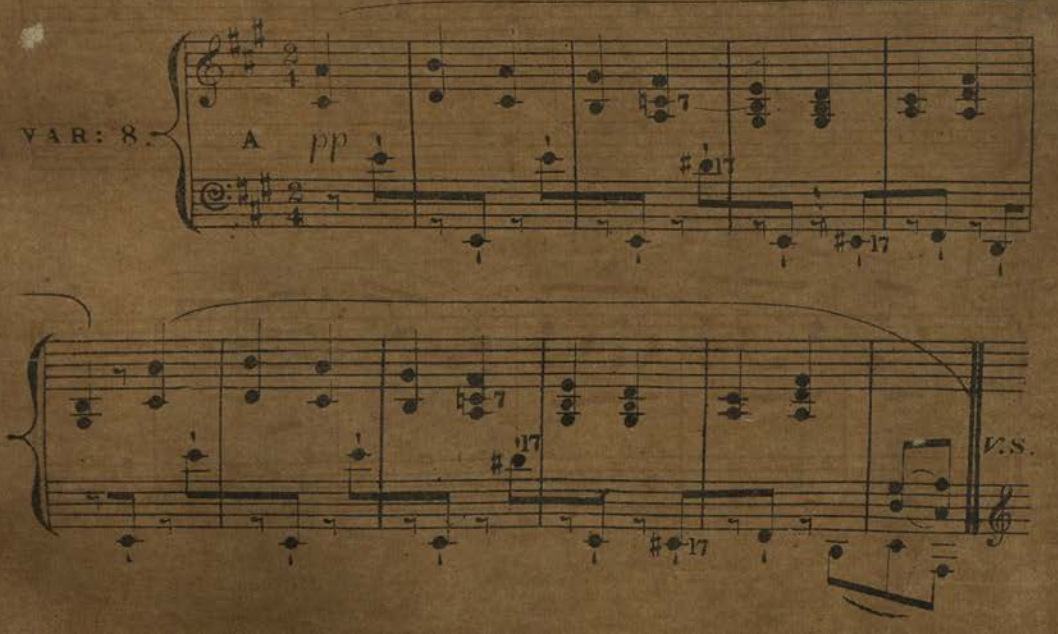
Part of Var: 1.

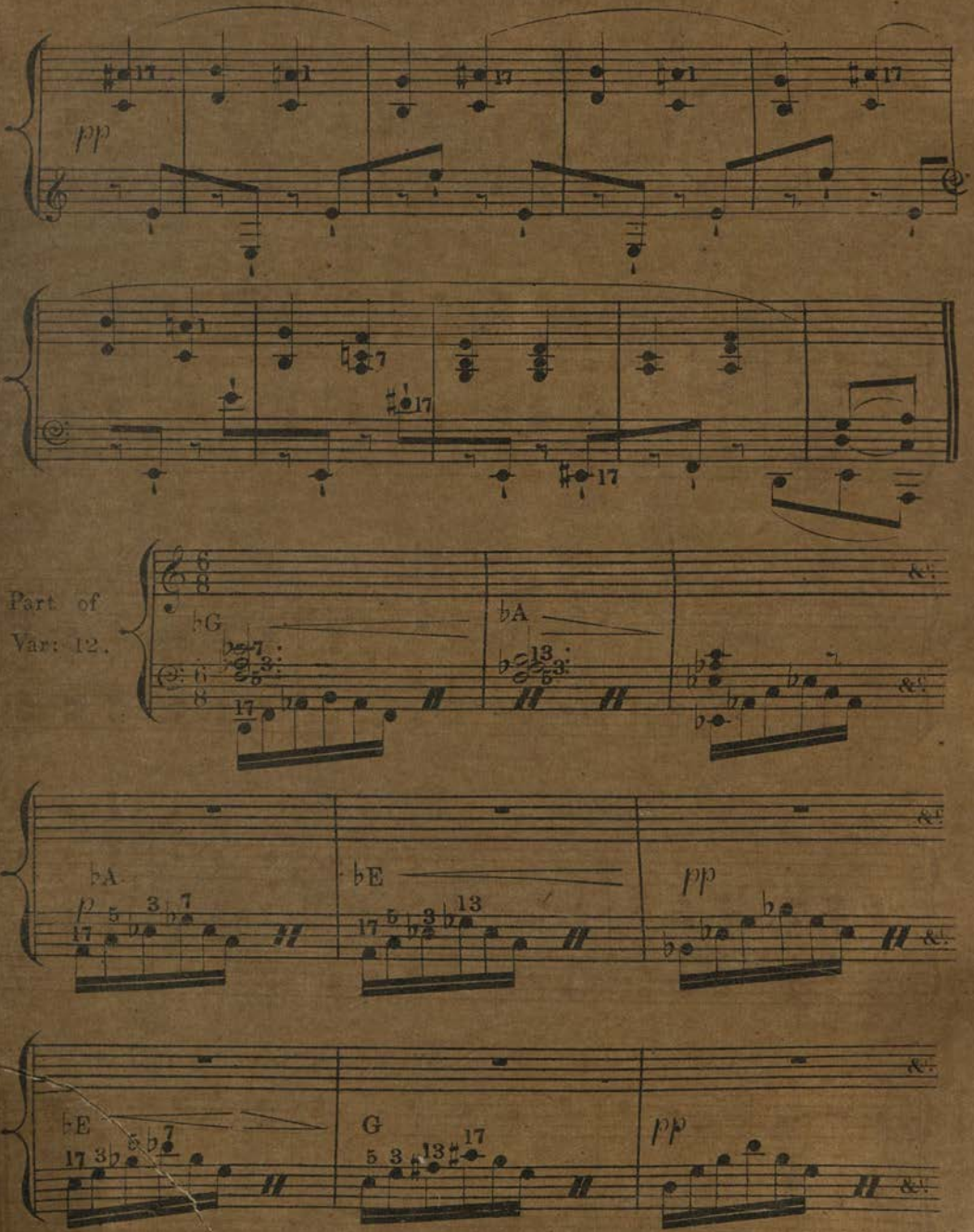


The above corrected.



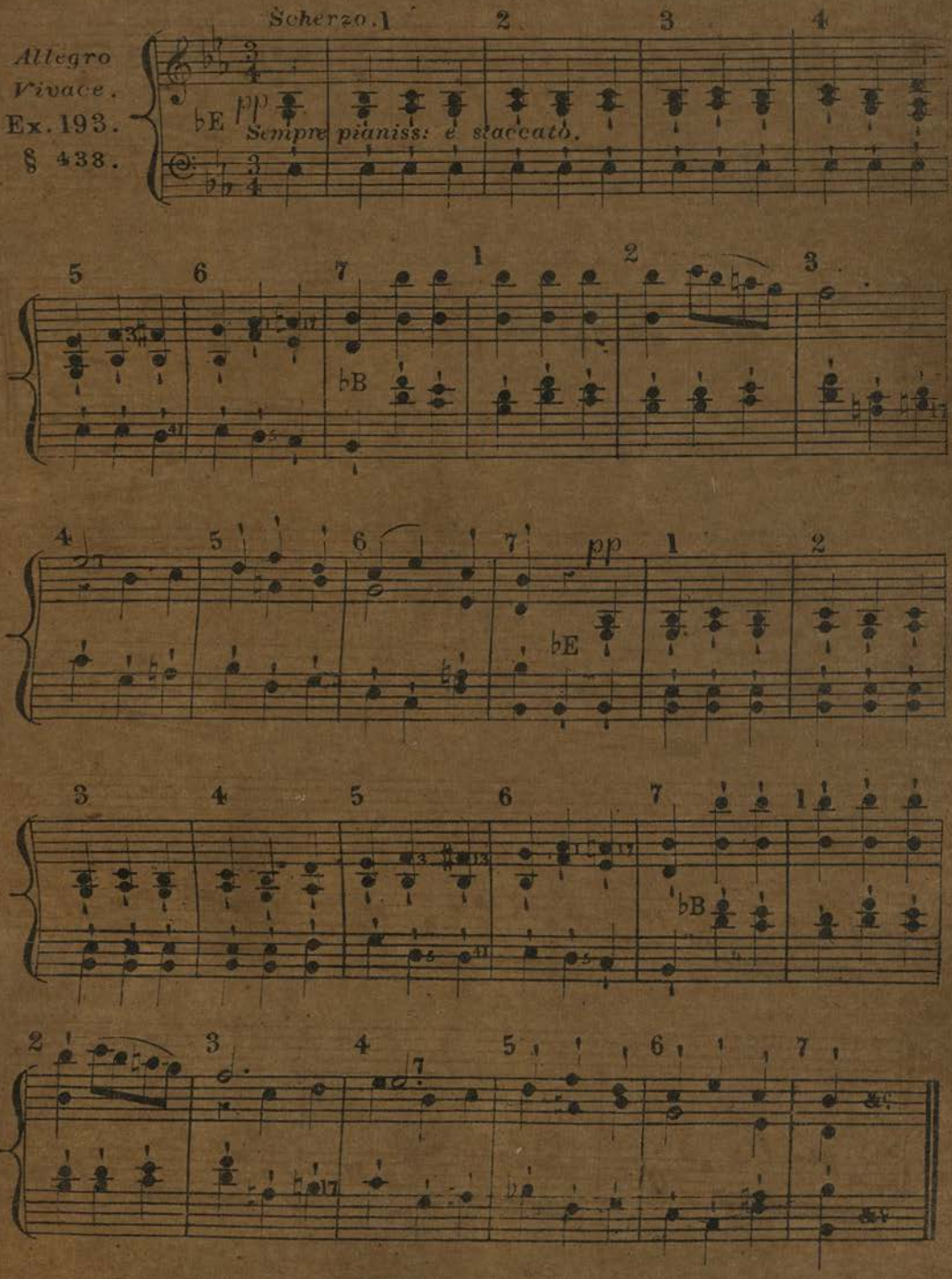
VAR: 8.




pp
Part of Var: 12.
bG
bA
bA
bE
pp
bE
G
pp

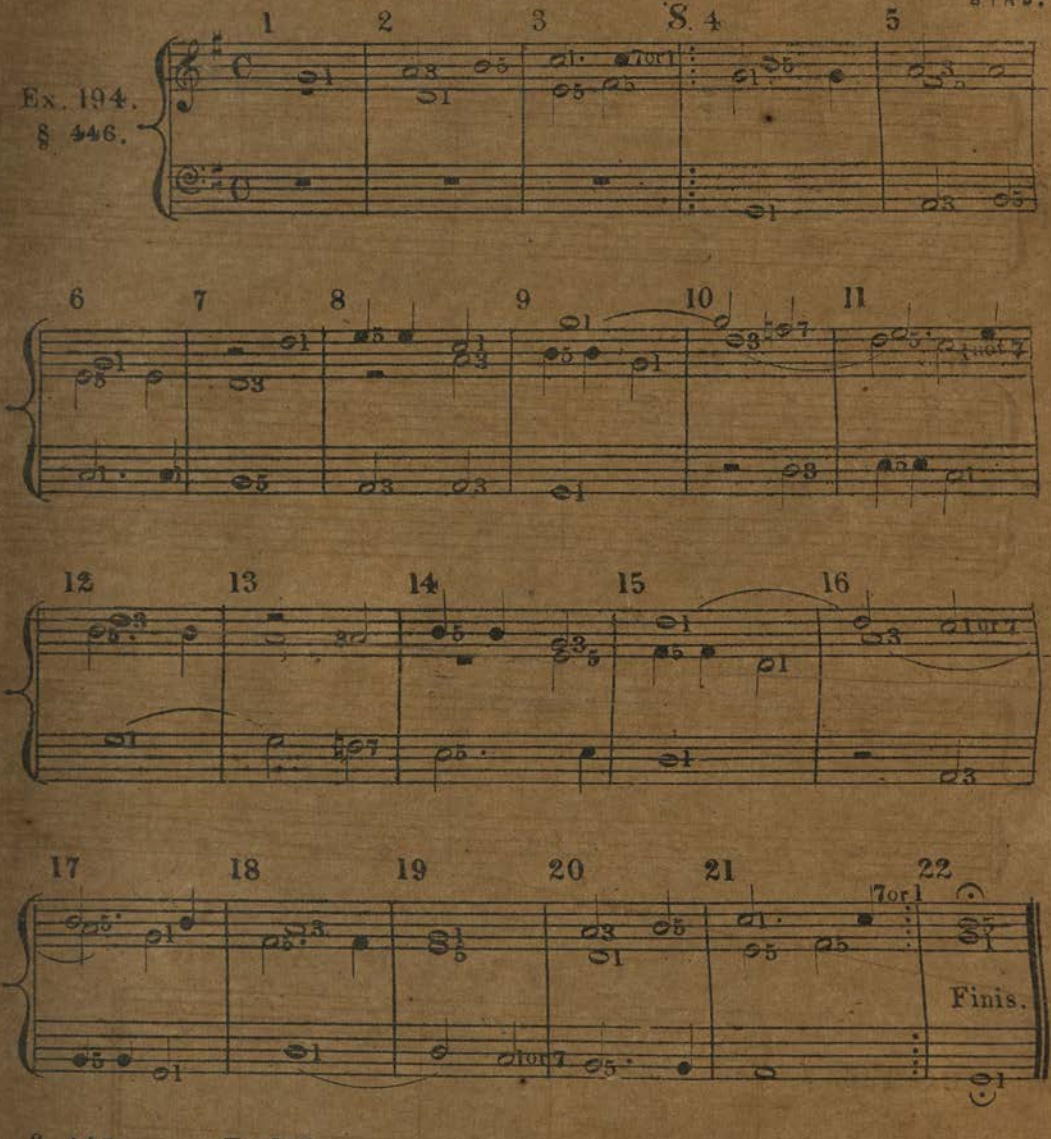
BEETHOVEN'S GRAND SYMPHONY EROICA Nº 6.

Allegro Vivace.
Ex. 193.
§ 438.



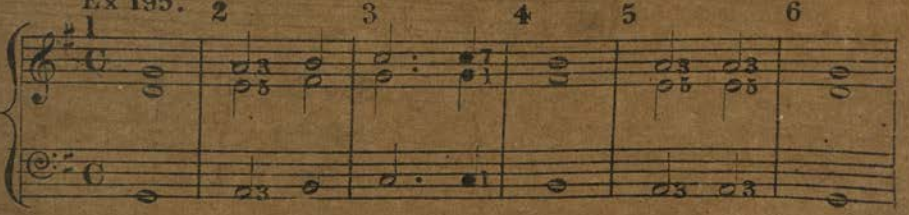
NON NOBIS DOMINE. BIRD.

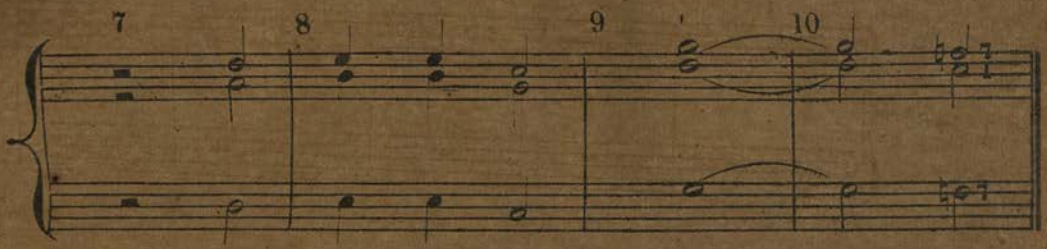
Ex. 194. § 446.

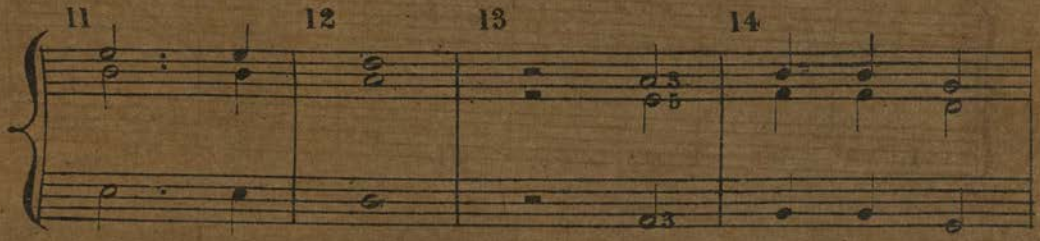


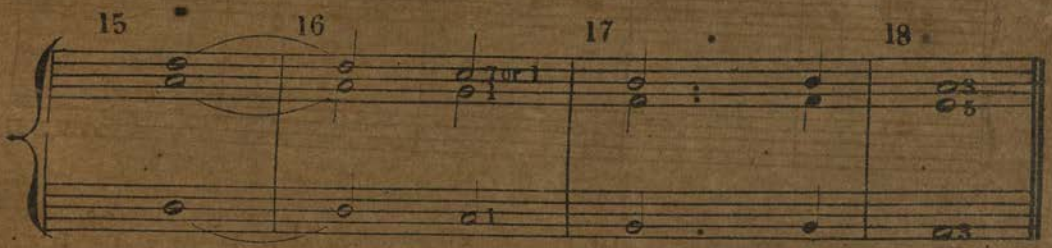
§ 447. Ex 195.

Treble.
Tenor
Base.



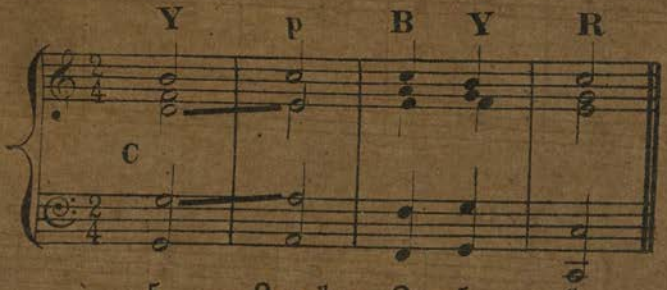




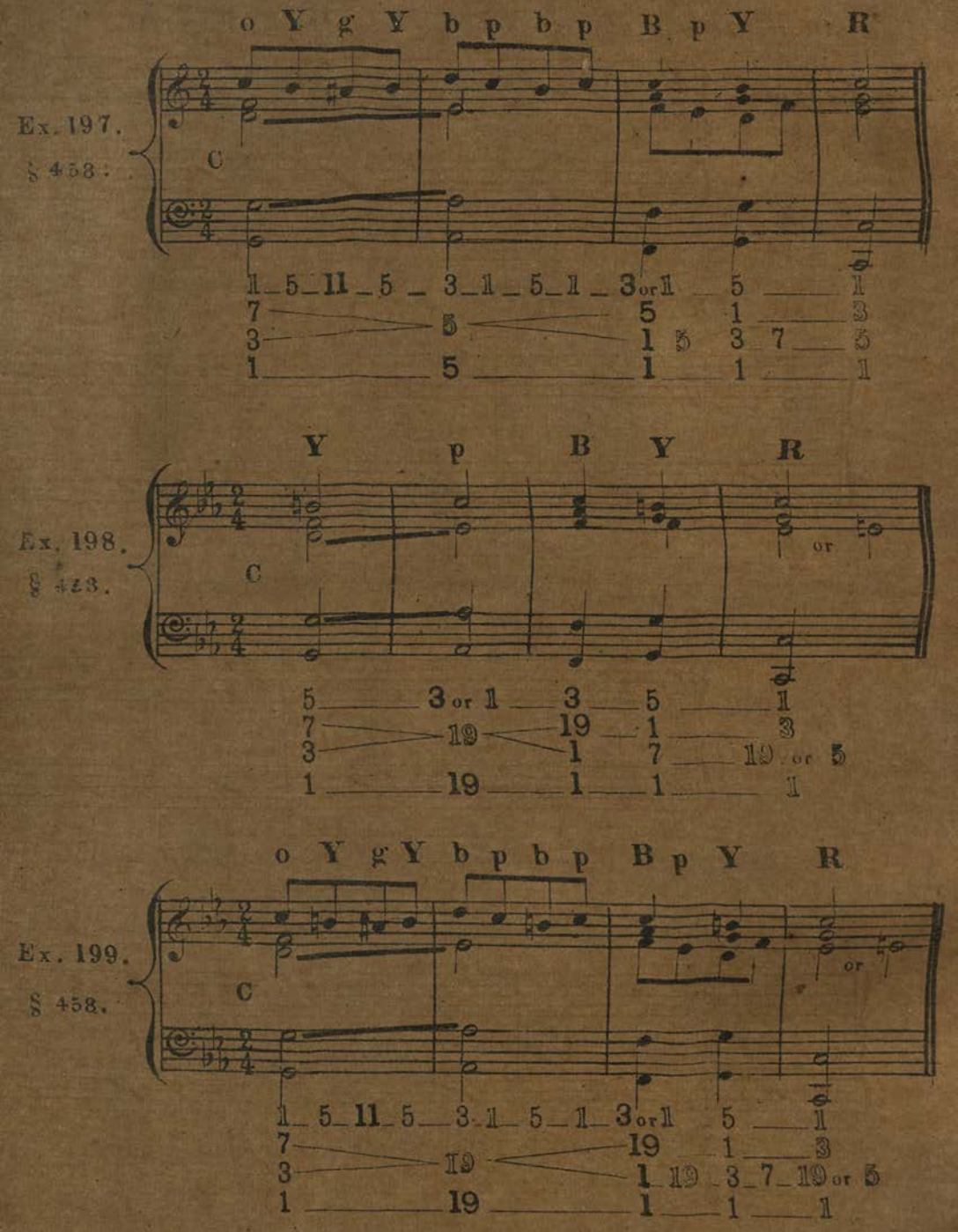


Ex: 196.

§ 458.

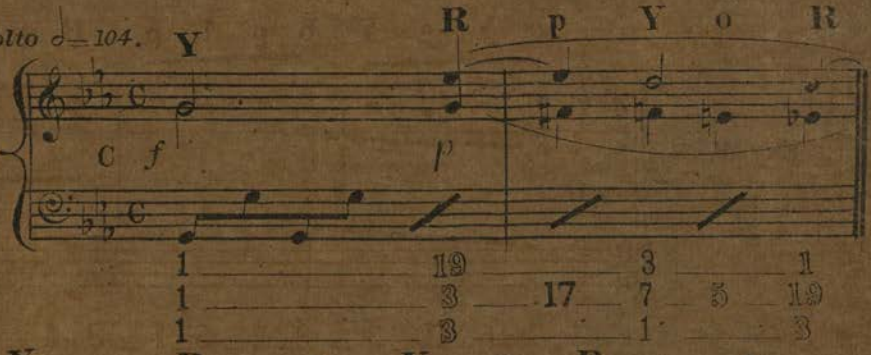


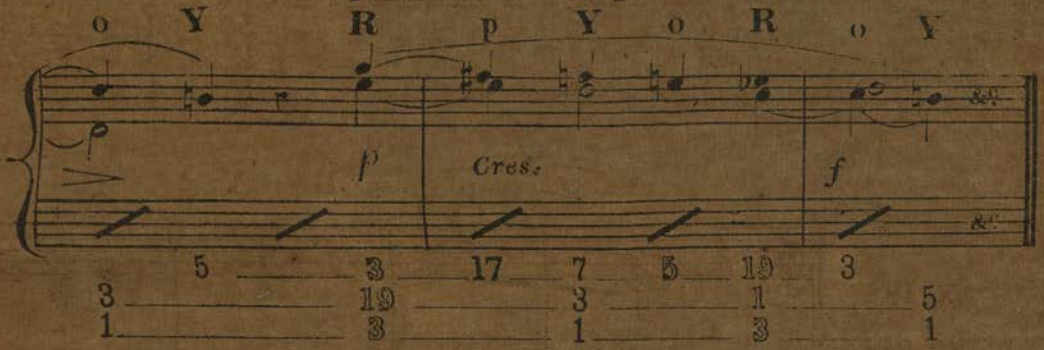
5 3 or 1 3 5 1
7 5 5 1 3
3 1 7 5
1 5 1 1 1


o Y g Y b p b p B p Y R
Ex. 197.
§ 458.
C
1 5 11 5 3 1 5 1 3 or 1 5 1
7 5 5 1 3
3 1 5 3 7 5
1 5 1 1 1
Y p B Y R
Ex. 198.
§ 423.
C
or
5 3 or 1 3 5 1
7 19 19 1 3
3 1 7 19 or 5
1 19 1 1 1
o Y g Y b p b p B p Y R
Ex. 199.
§ 458.
C
or
1 5 11 5 3 1 5 1 3 or 1 5 1
7 19 19 1 3
3 19 1 19 3 7 19 or 5
1 19 1 1 1

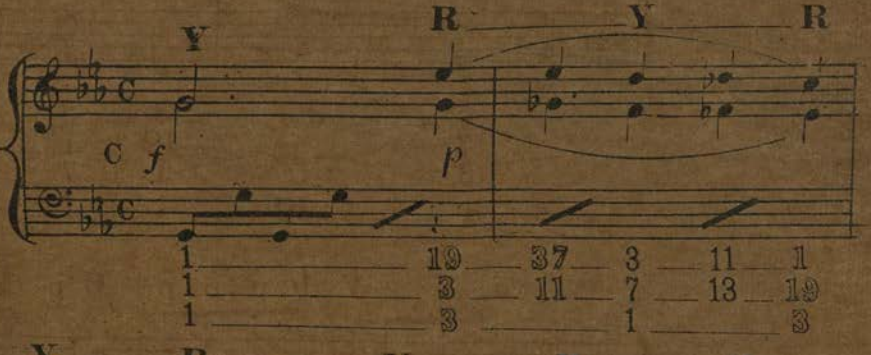
Allegro molto ♩ = 104.

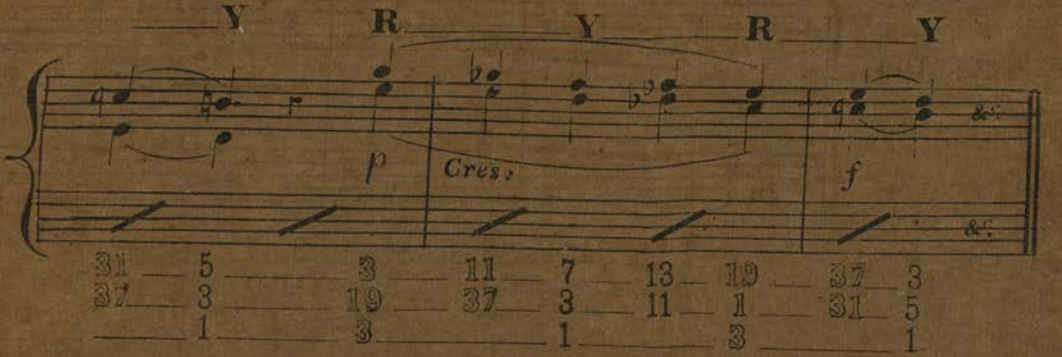
Ex. 200.
§ 360



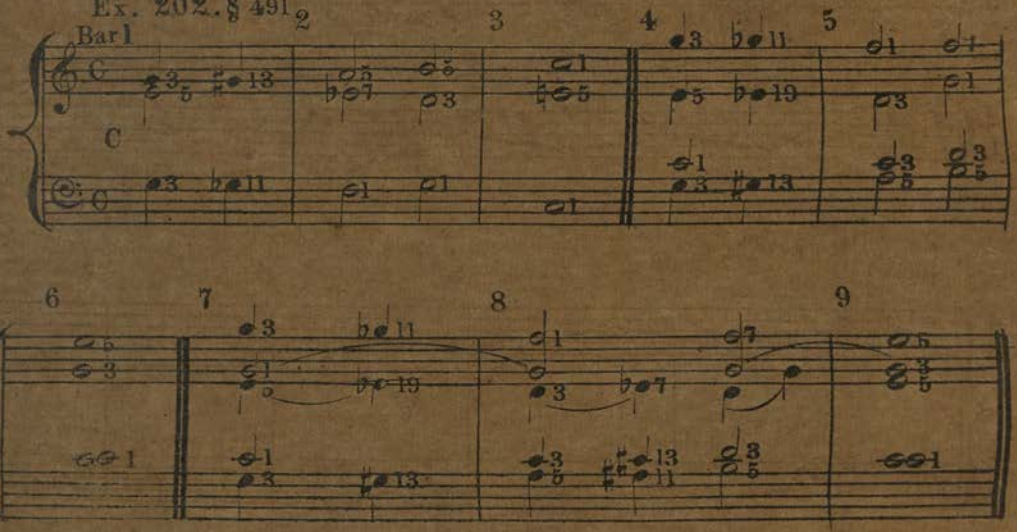


Ex. 201.
§ 360.

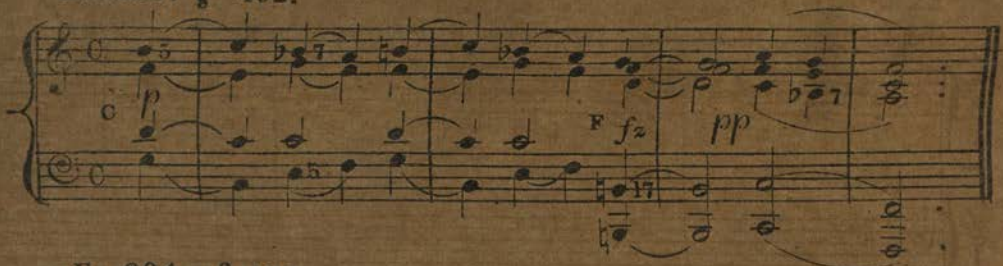




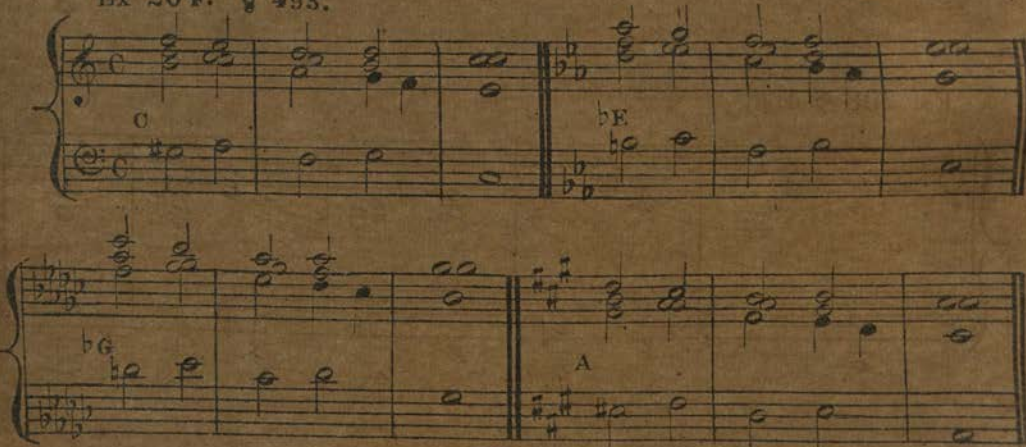
Ex. 202. § 491.



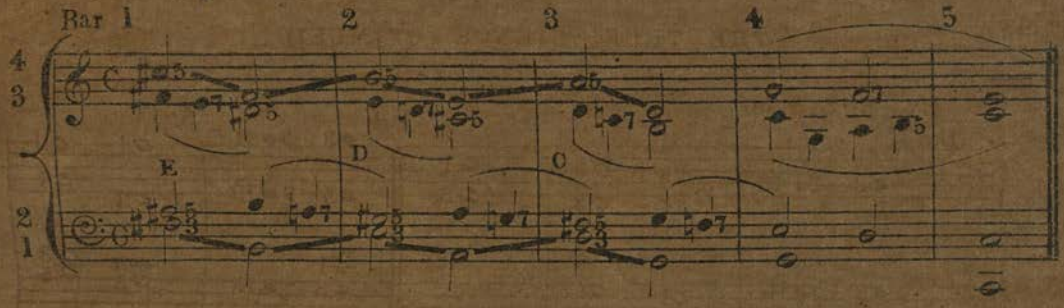
Ex. 203. § 492.



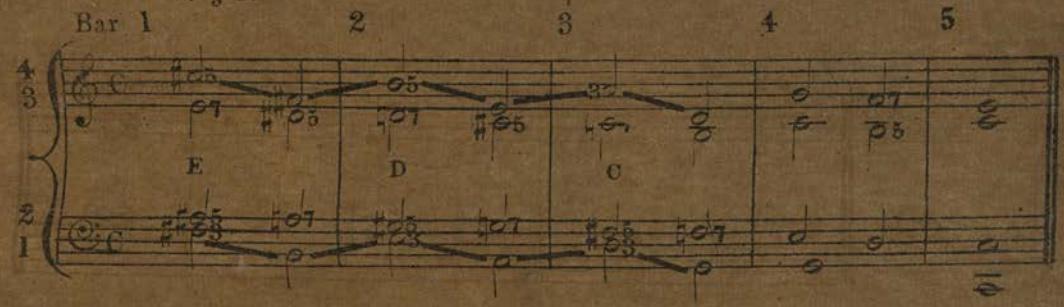
Ex 204. § 493.



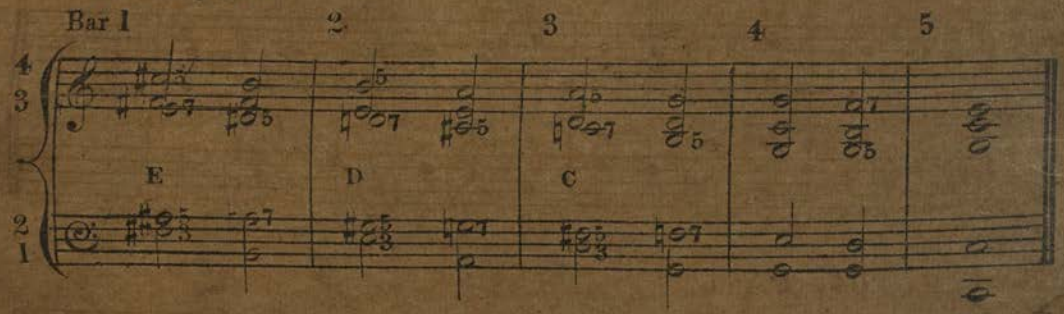
Ex. 205. § 494.



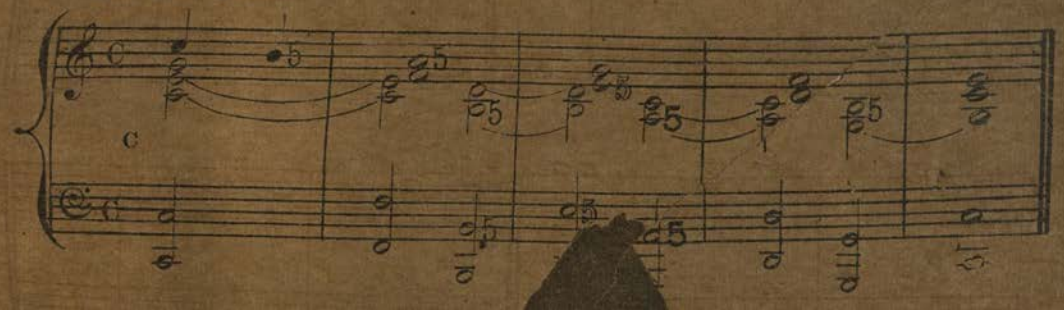
Ex. 206. § 494.



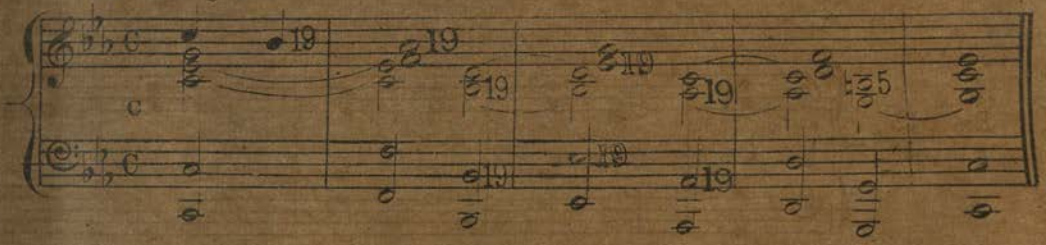
Ex 207. § 494.



Ex 208 § 495.

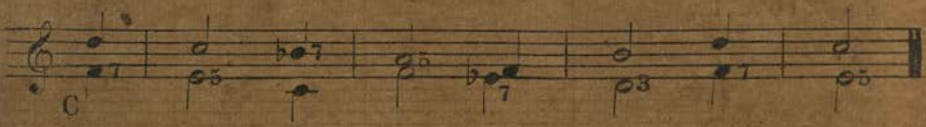


Ex 209 § 495

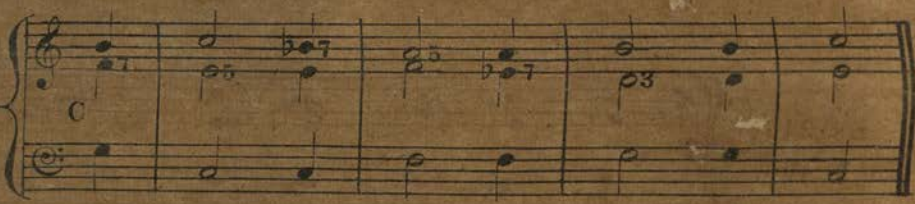


Ex 210. § 496.

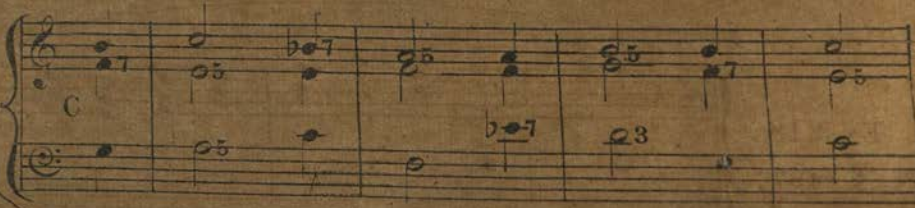
Ex 211 § 496.



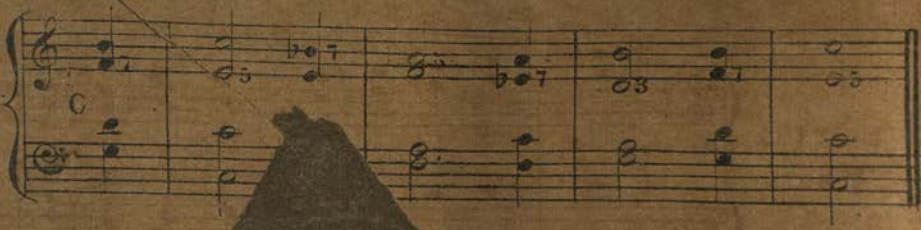
Ex 212. § 496.



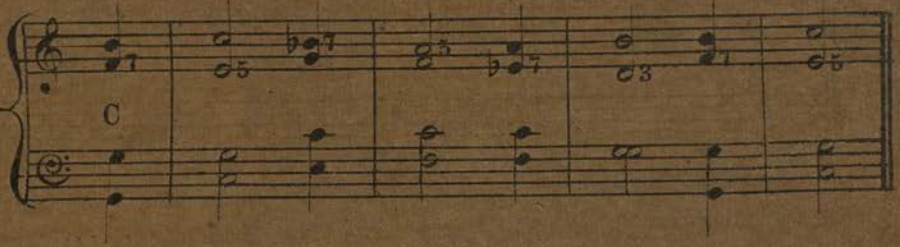
Ex 213. § 496.



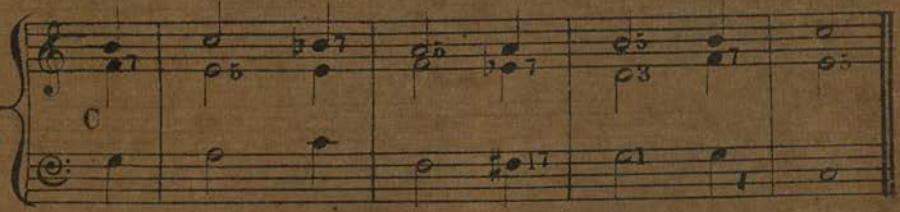
Ex 214. § 496.



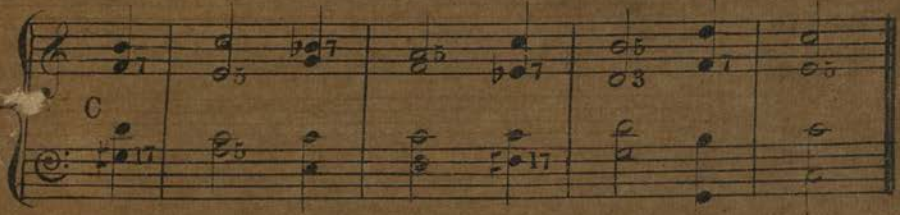
Ex 215.
§ 496.



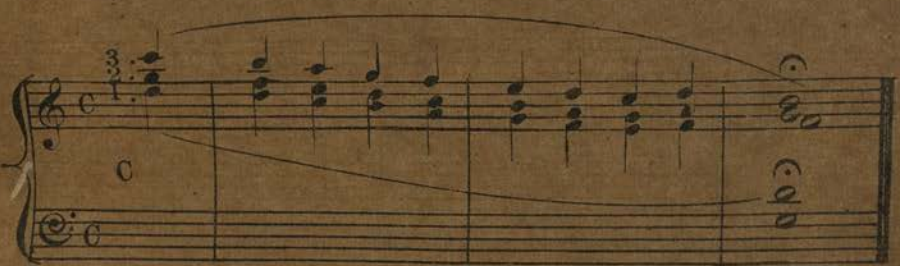
Ex 216.
§ 496.



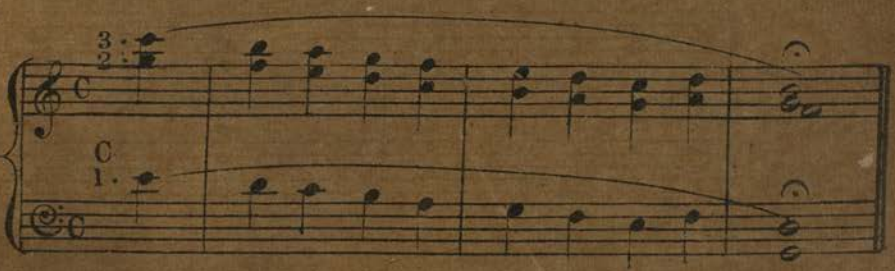
Ex 217.
§ 496.



Ex 218.
§ 497.

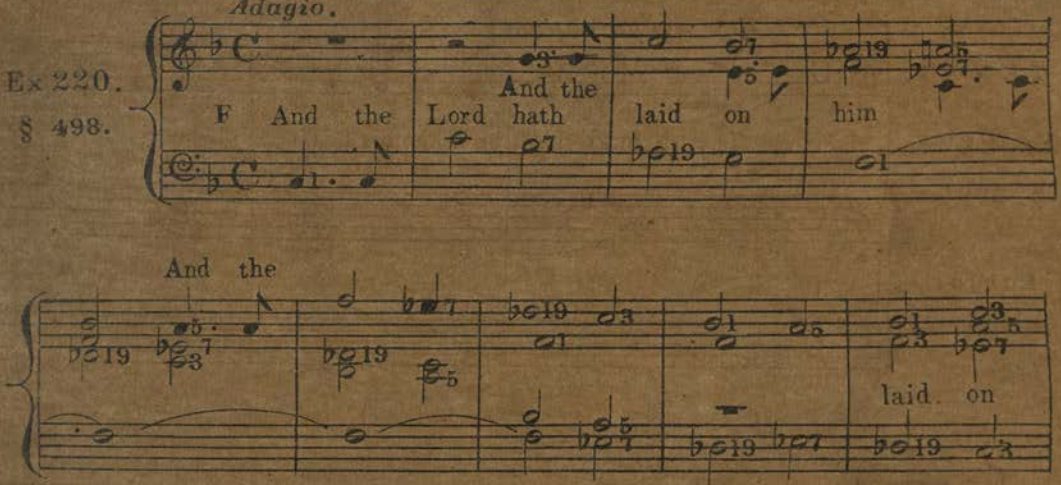


Ex 219.
§ 497.



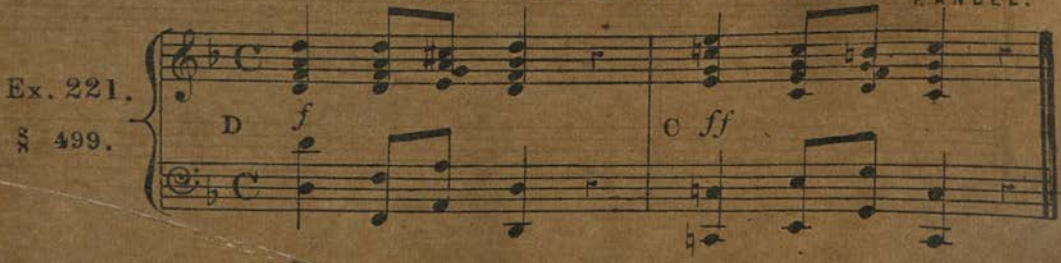
HANDEL'S MESSIAH.

Ex. 220. § 498.

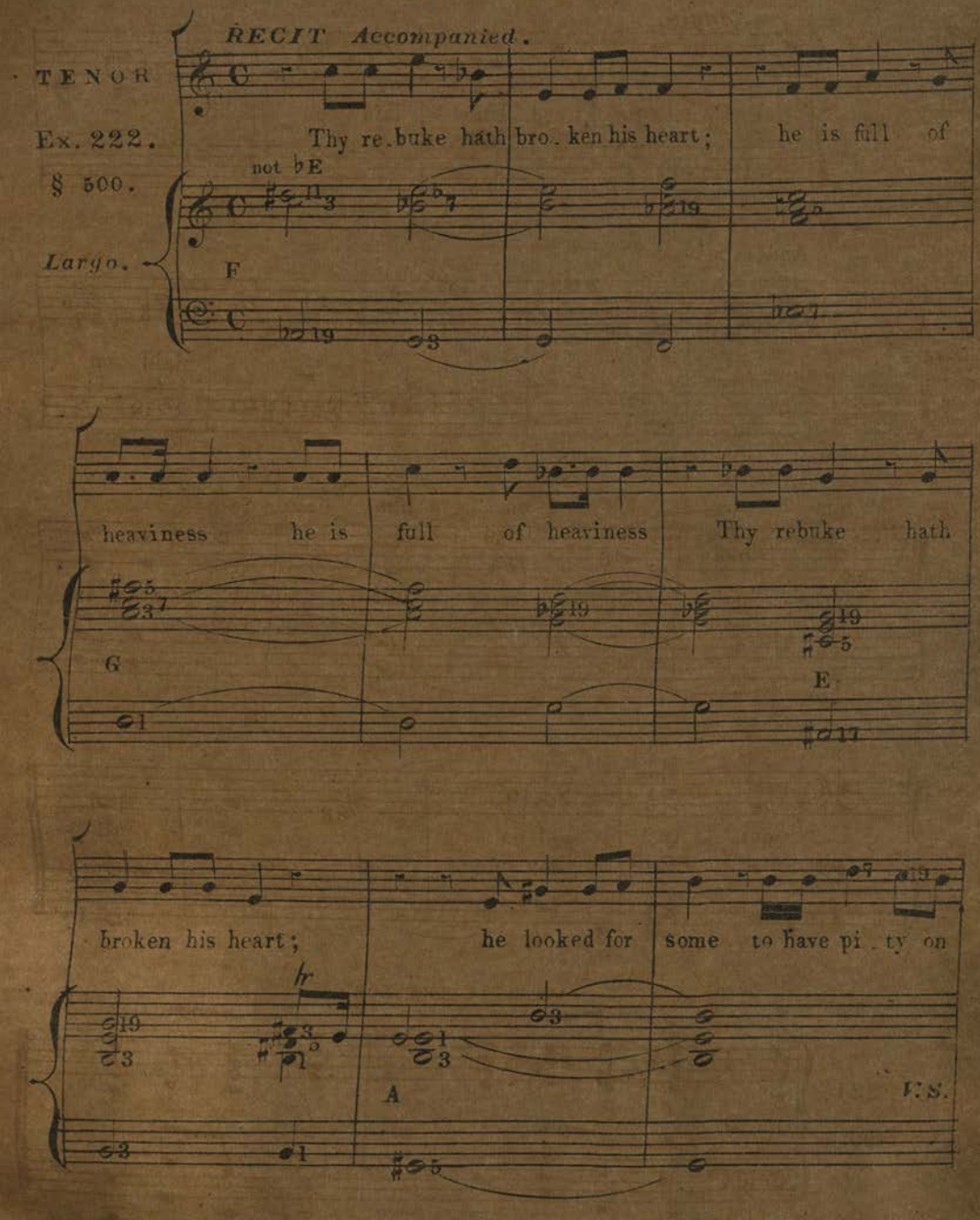


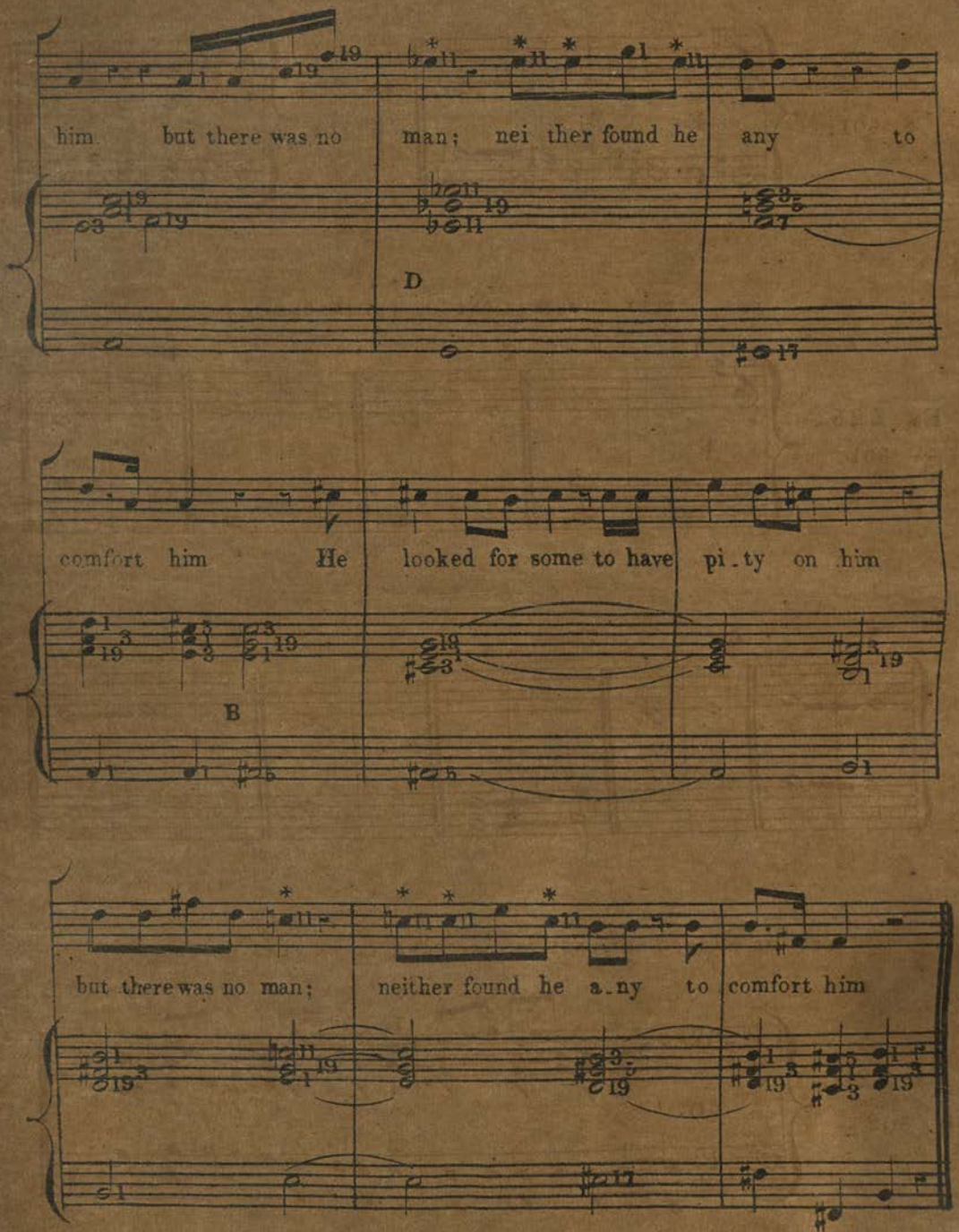
FALL'N IS THE FOE. HANDEL.

Ex. 221. § 499.

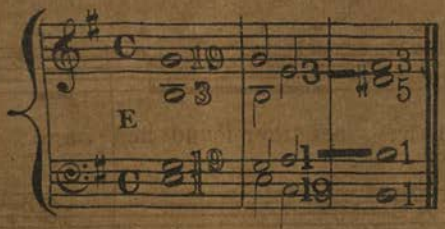


HANDEL'S MESSIAH

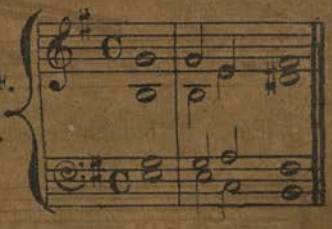



him but there was no man; nei ther found he any to
D
comfort him He looked for some to have pi_ty on him
B
but there was no man; neither found he a_ny to comfort him

Ex: 223. § 501.

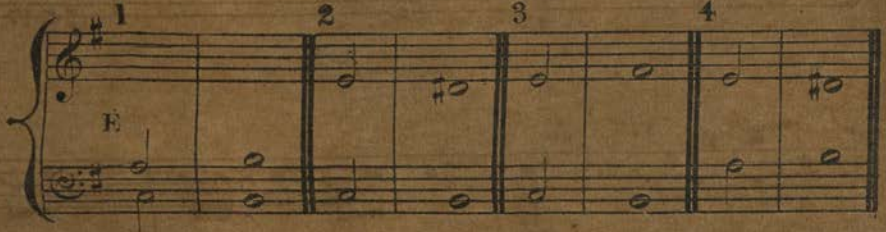


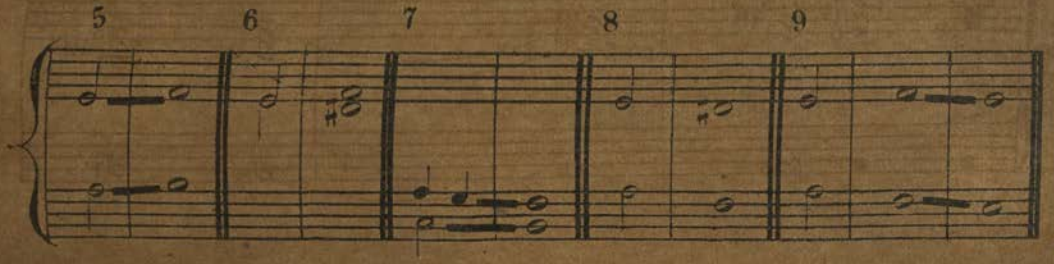
Ex. 224. § 501.



DUET COMBINATIONS IN THE ABOVE.

Ex. 225. § 501.

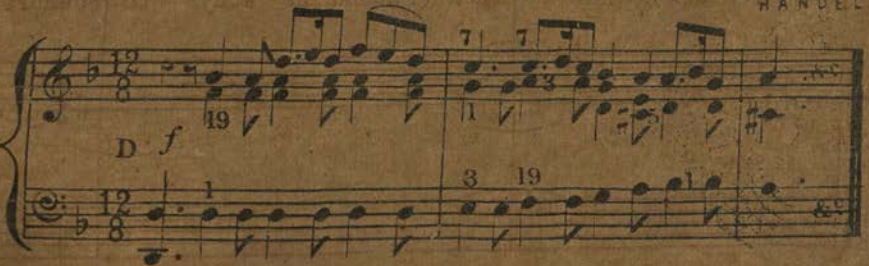




LET ME WANDER NOT UNSEEN. L'ALLEGRO ED IL PENSIEROSO.

HANDEL.

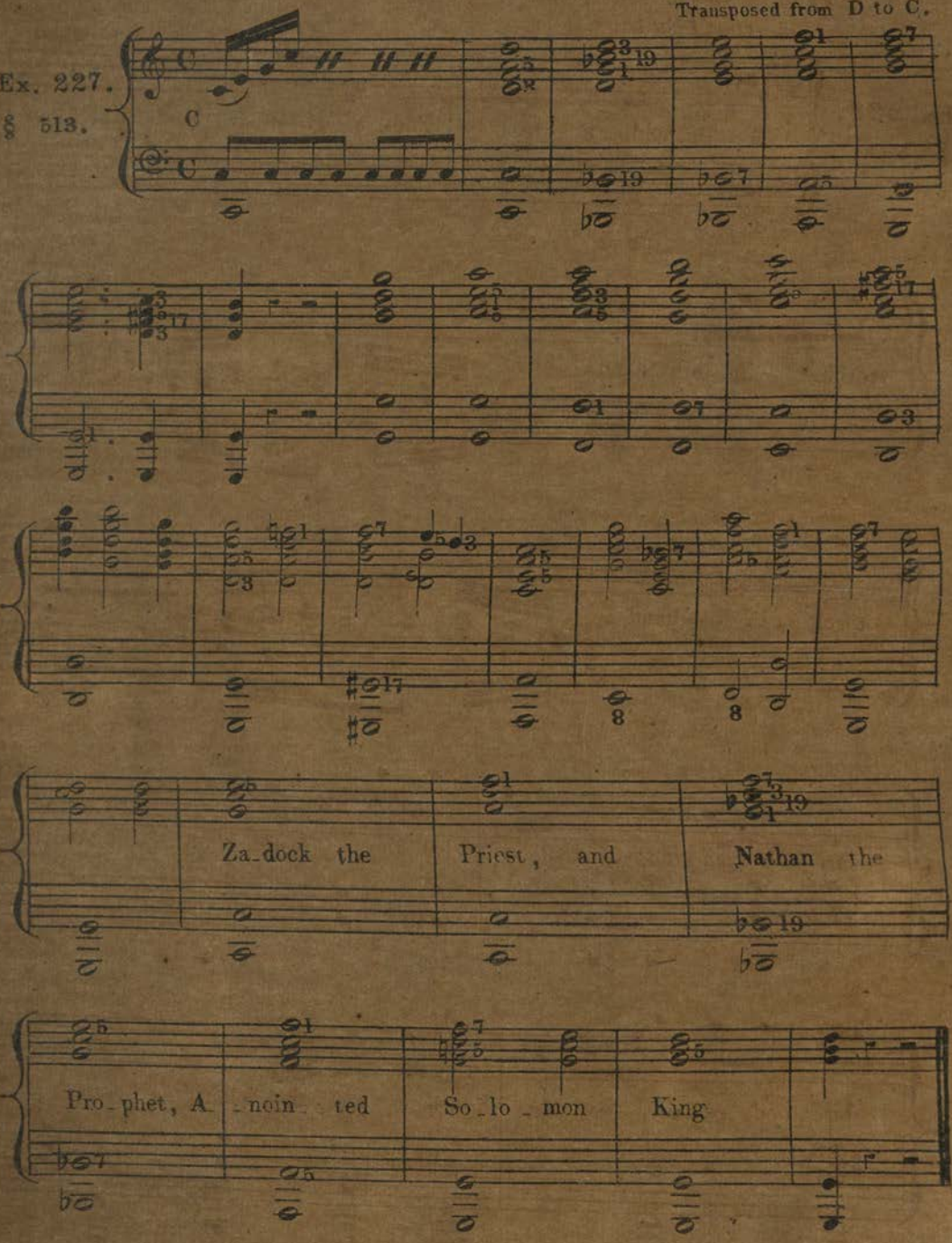
Ex. 226. § 502.



HARMONY OF THE FIRST MOVEMENT OF HANDEL'S CORONATION ANTHEM.

Transposed from D to C.

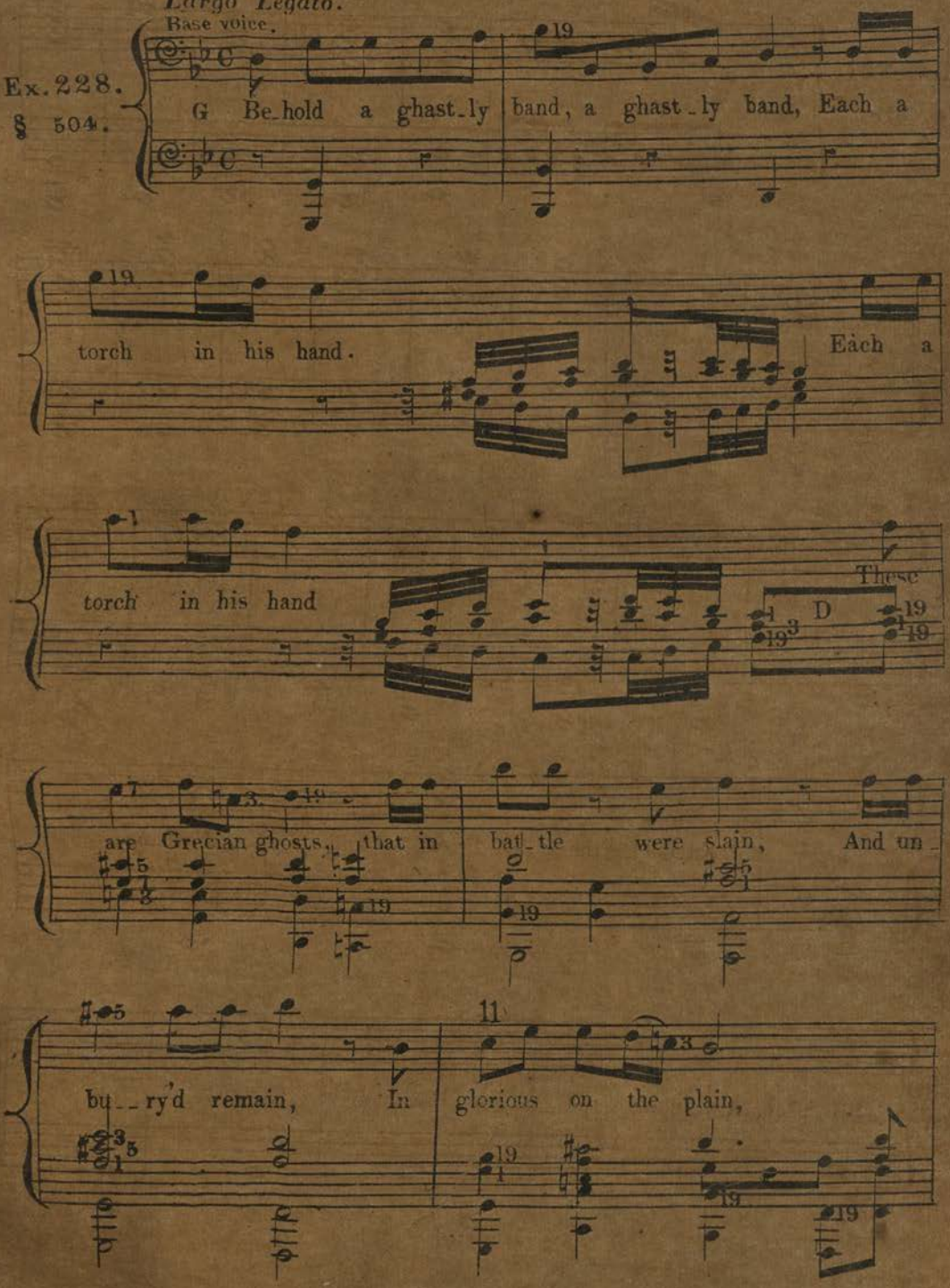
Ex. 227.
§ 513.

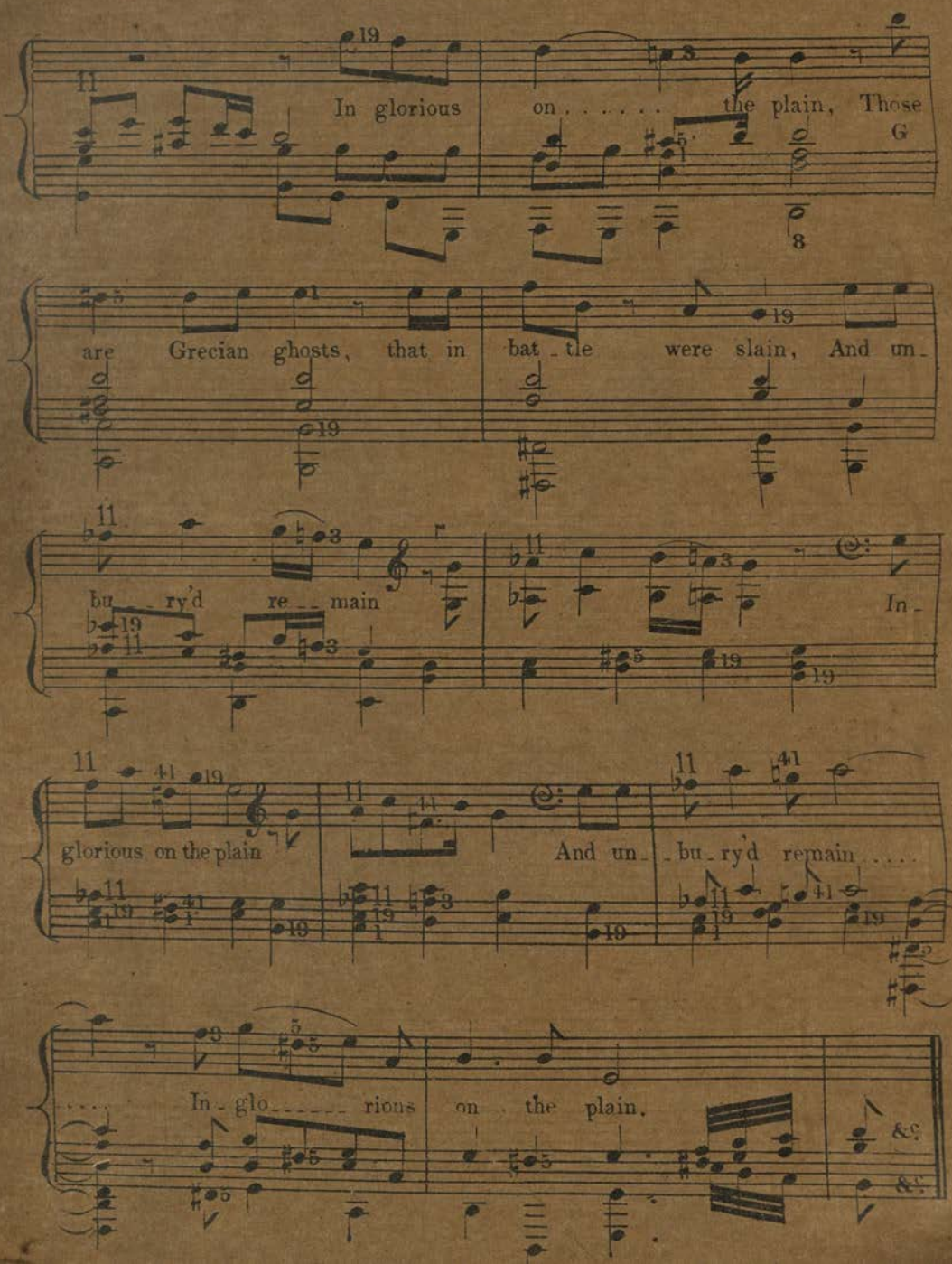


REVENGE, TIMOTHEUS CRIES — *"Alexander's Feast."* HANDEL.

Largo Legato.

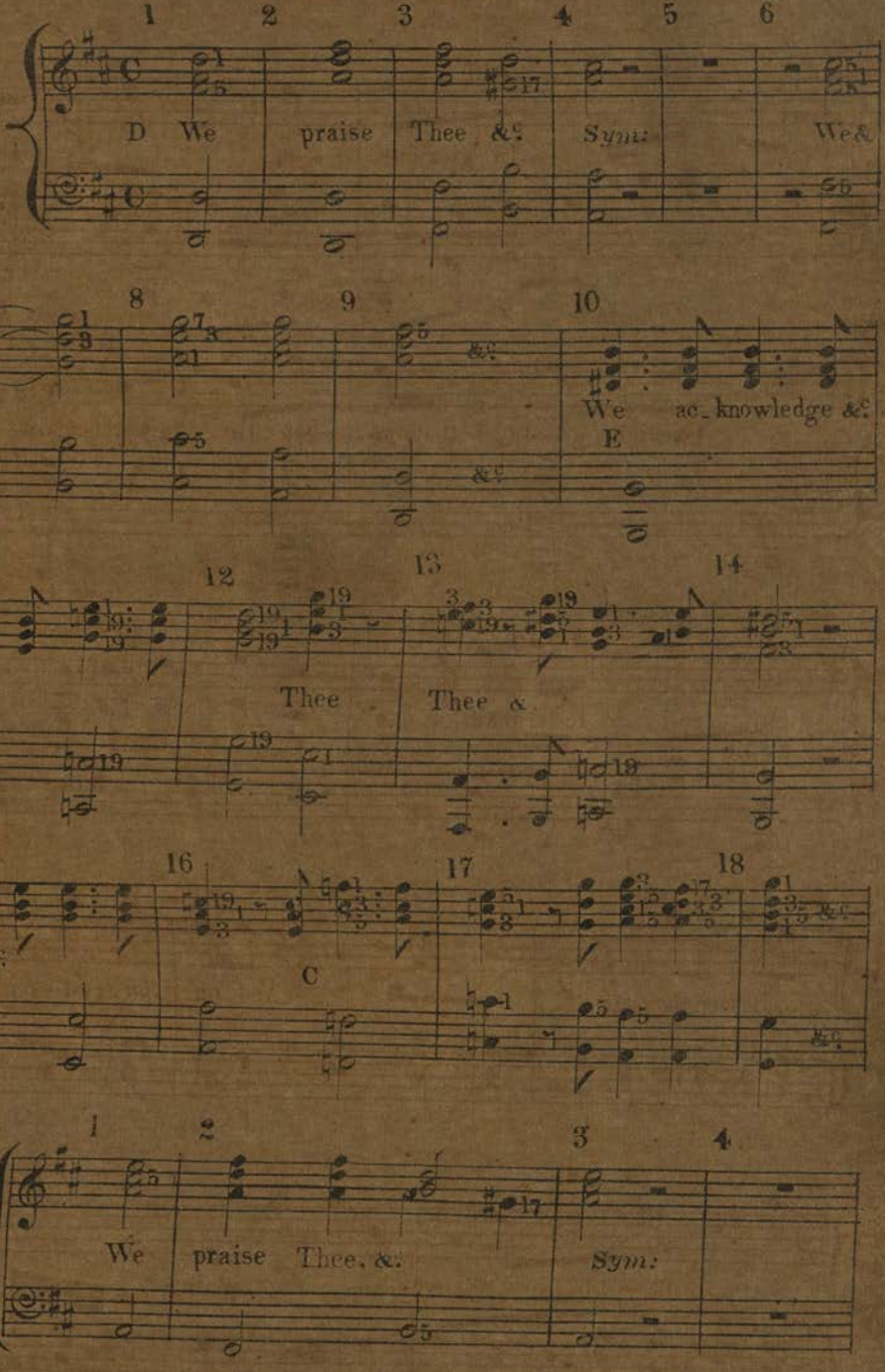
Ex. 228. § 504.


Base voice.
G Be_hold a ghast_ly band, a ghast_ly band, Each a
torch in his hand. Each a
torch in his hand
These
D
are Grecian ghosts, that in bat_tle were slain, And un_
bu_ry'd remain, In
11
glorious on the plain,

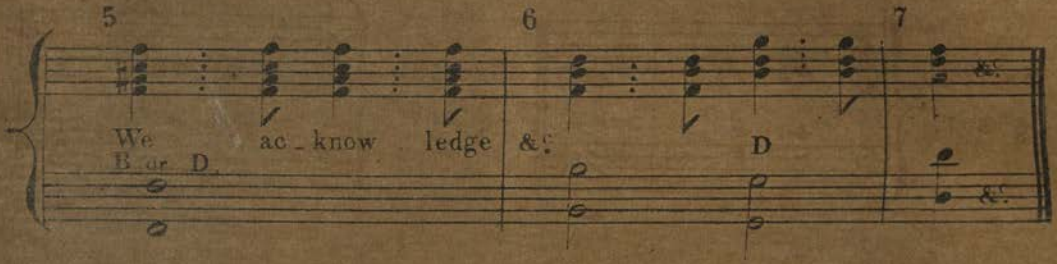

In glorious on . . . . . the plain, Those
are Grecian ghosts, that in bat - tle were slain, And un -
bu - ry'd re - - main In -
glorious on the plain And un - - bu - ry'd remain . . . .
. . . . In - glo - - - - - rious on the plain.

HANDEL'S DETTENGEN TE DEUM.

Ex. 229.
§ 505.

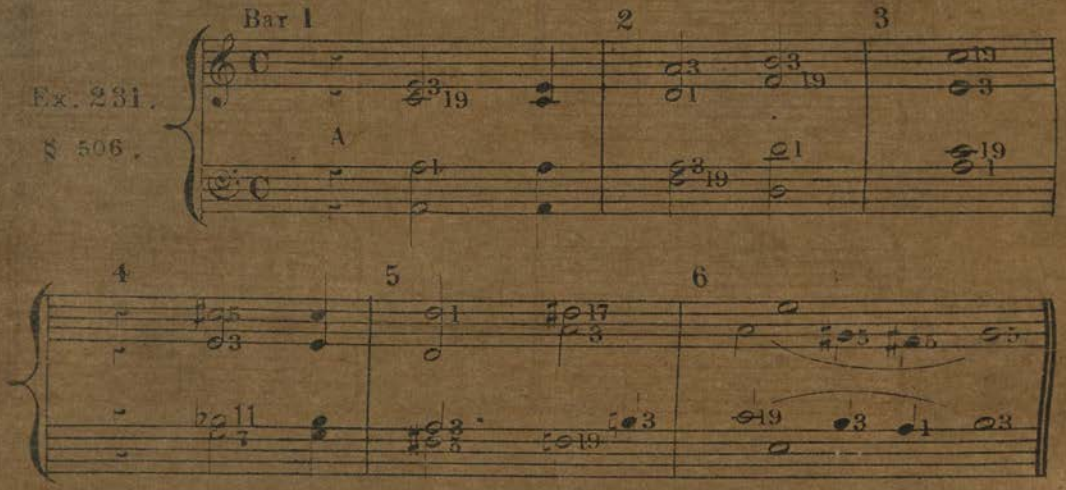


Ex. 230.
§ 505.



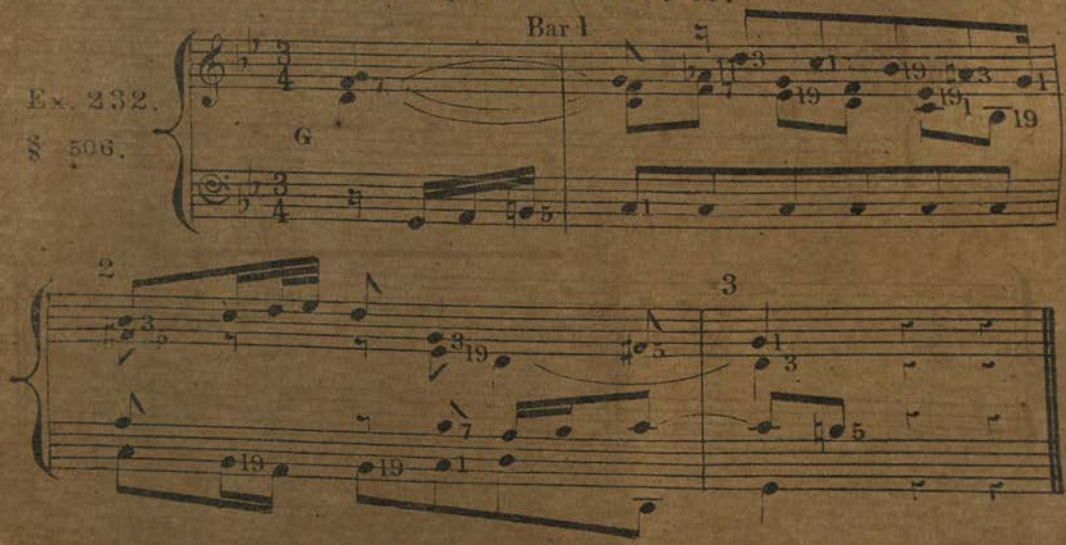
SINCE BY MAN_ MESSIAH. HANDEL.

Ex. 231.
§ 506.

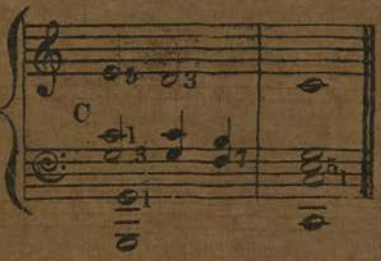


SEB: BACH'S FUGUE No 40.

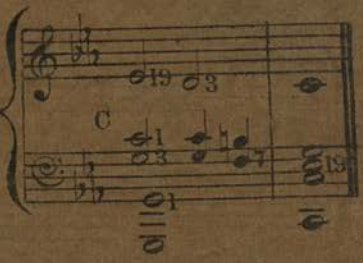
Ex. 232.
§ 506.



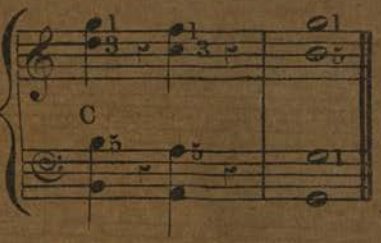
Ex. 233. § 507.



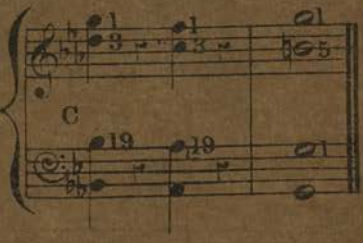
Ex. 236. § 507.



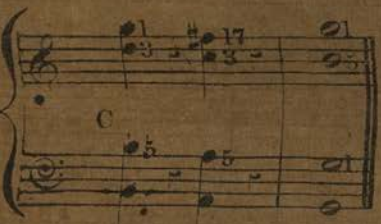
Ex. 234. § 507.



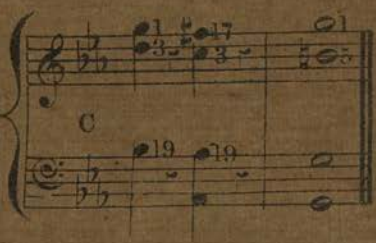
Ex. 237. § 507.



Ex. 235. § 507.

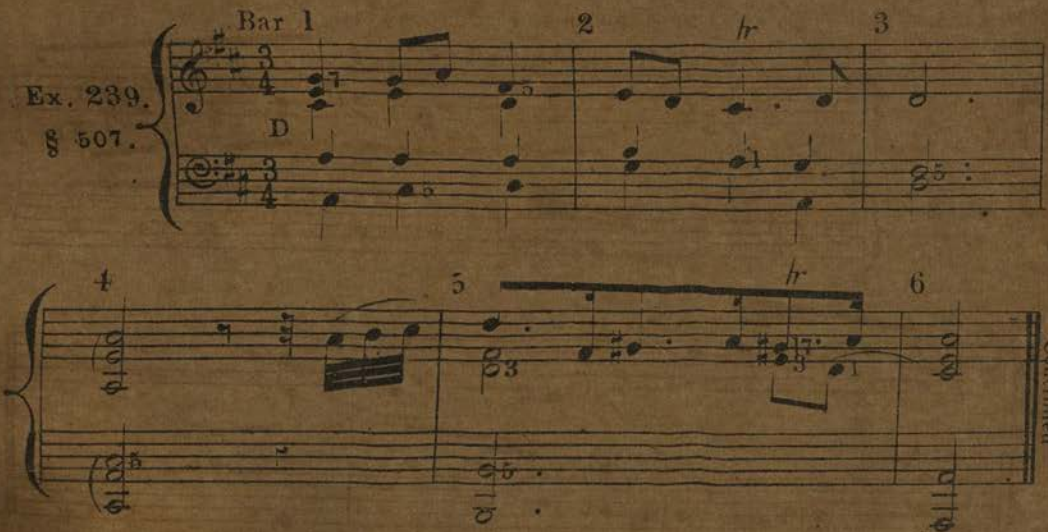


Ex. 238. § 507.

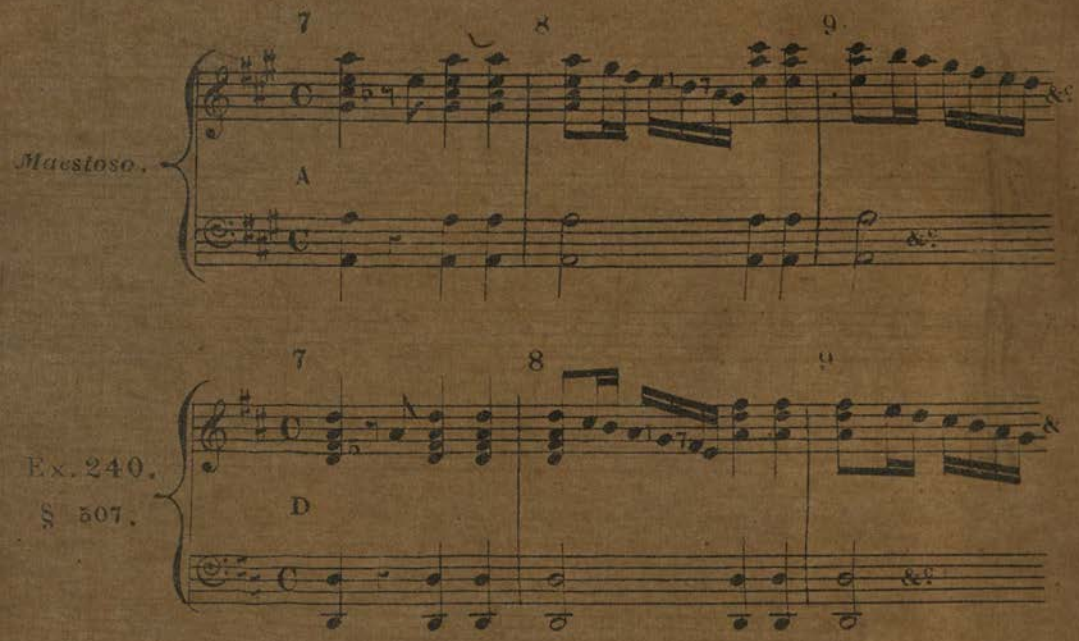


HANDEL'S WATER MUSIC.

Ex. 239. § 507.



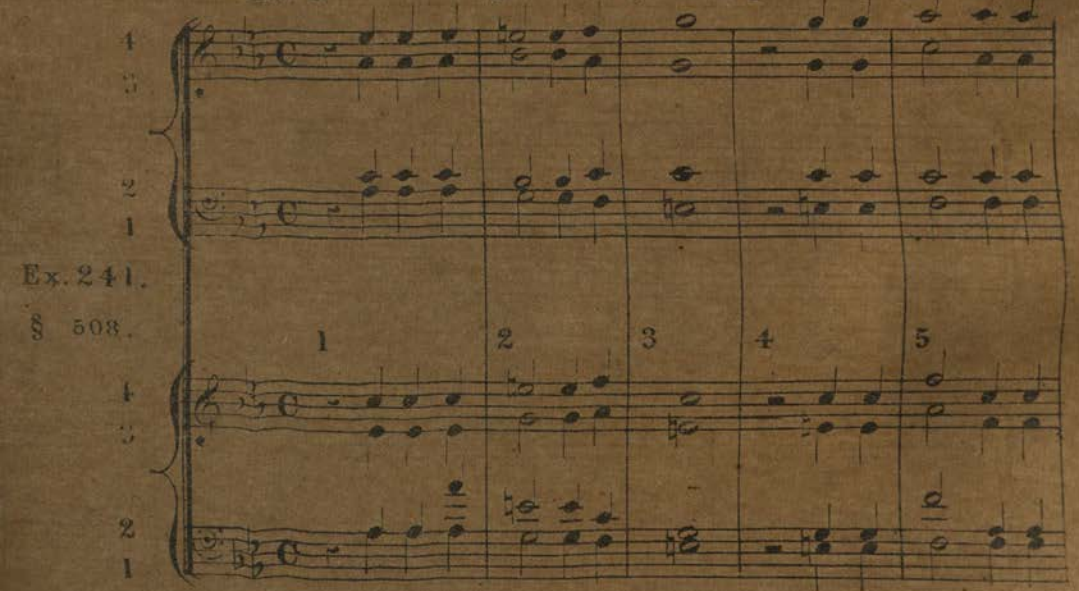
Continued

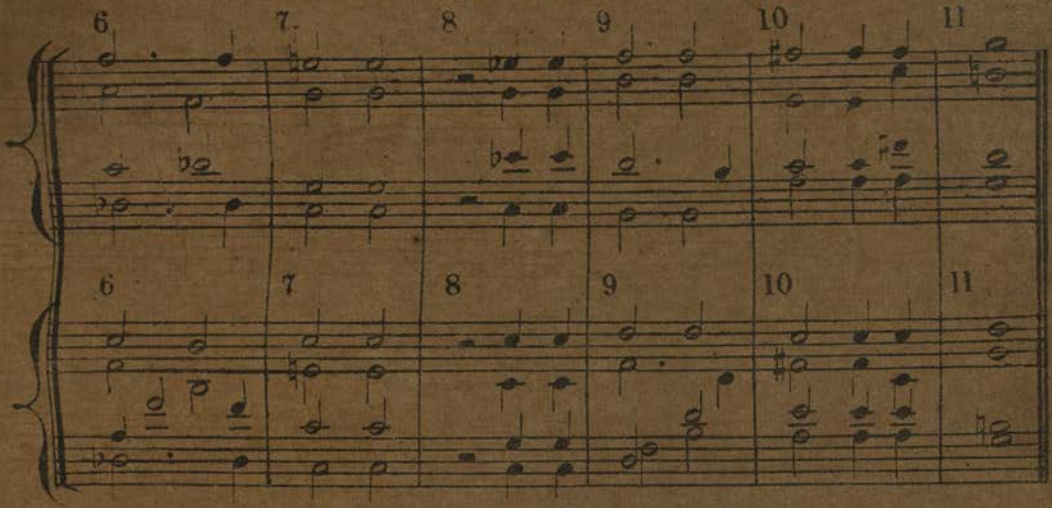


ISRAEL IN ECYPT.

And Israel saw that great work.

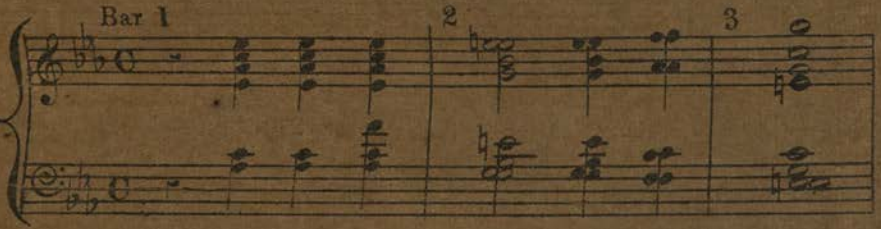
HANDEL.

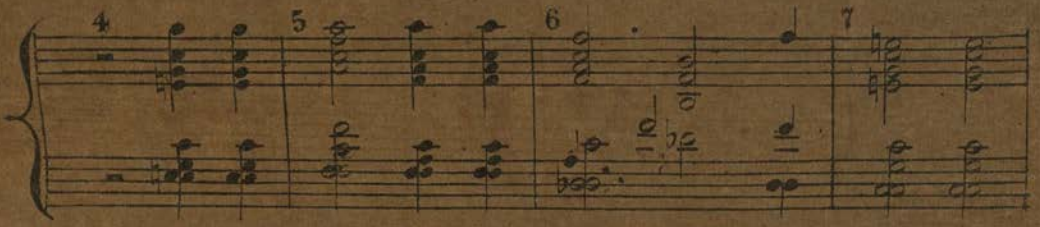


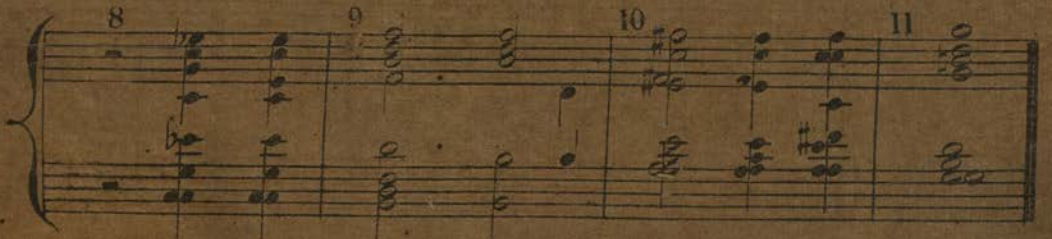


The same.

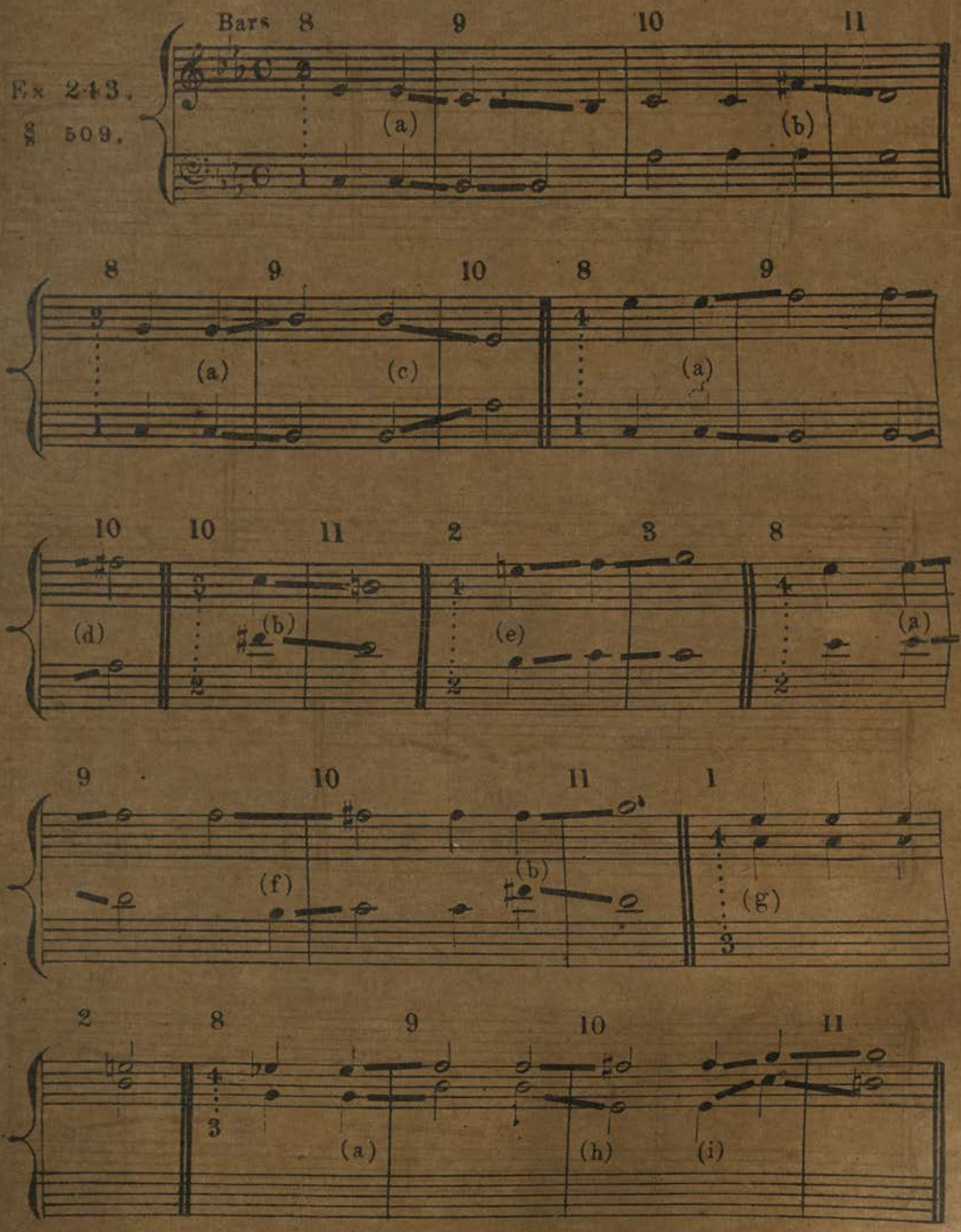
Ex. 242
§ 508





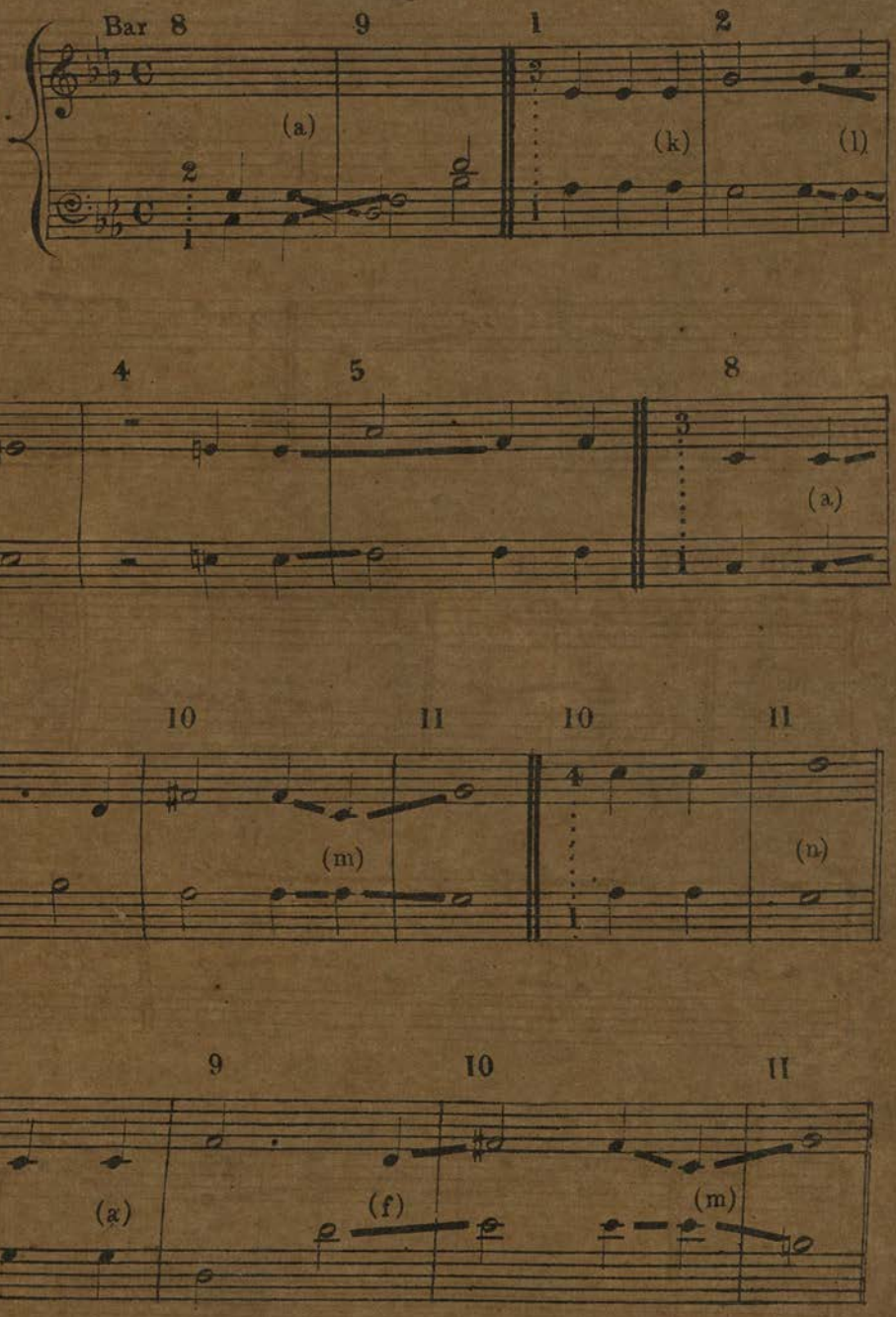


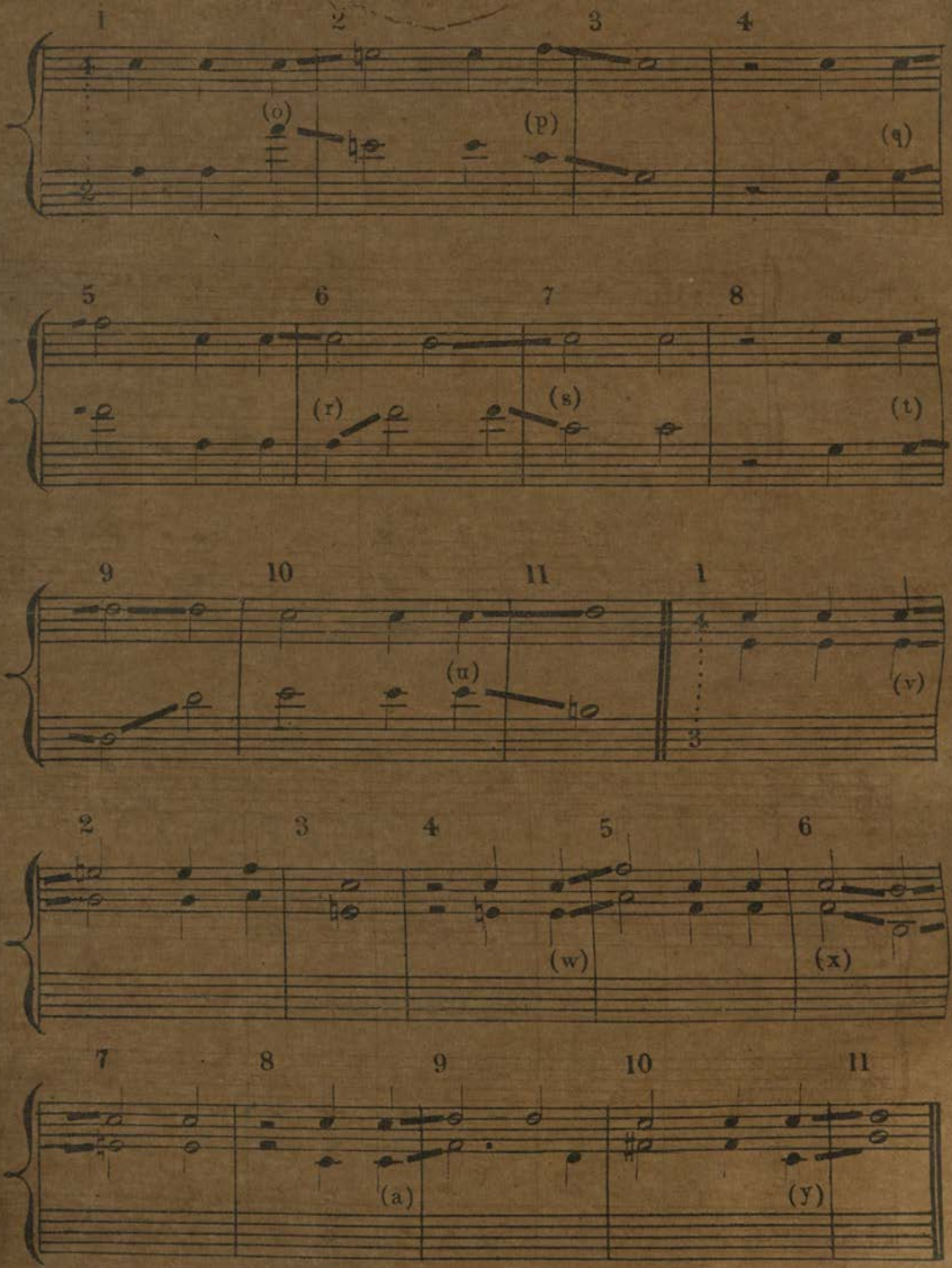
Ex 243.
§ 509.


Bars 8 9 10 11
(a)
(b)
8 9 10 8 9
(a)
(c)
(a)
10 10 11 2 3 8
(d)
(b)
(e)
(a)
9 10 11 1
(f)
(b)
(g)
2 8 9 10 11
(a)
(h)
(i)

Ex: 244.

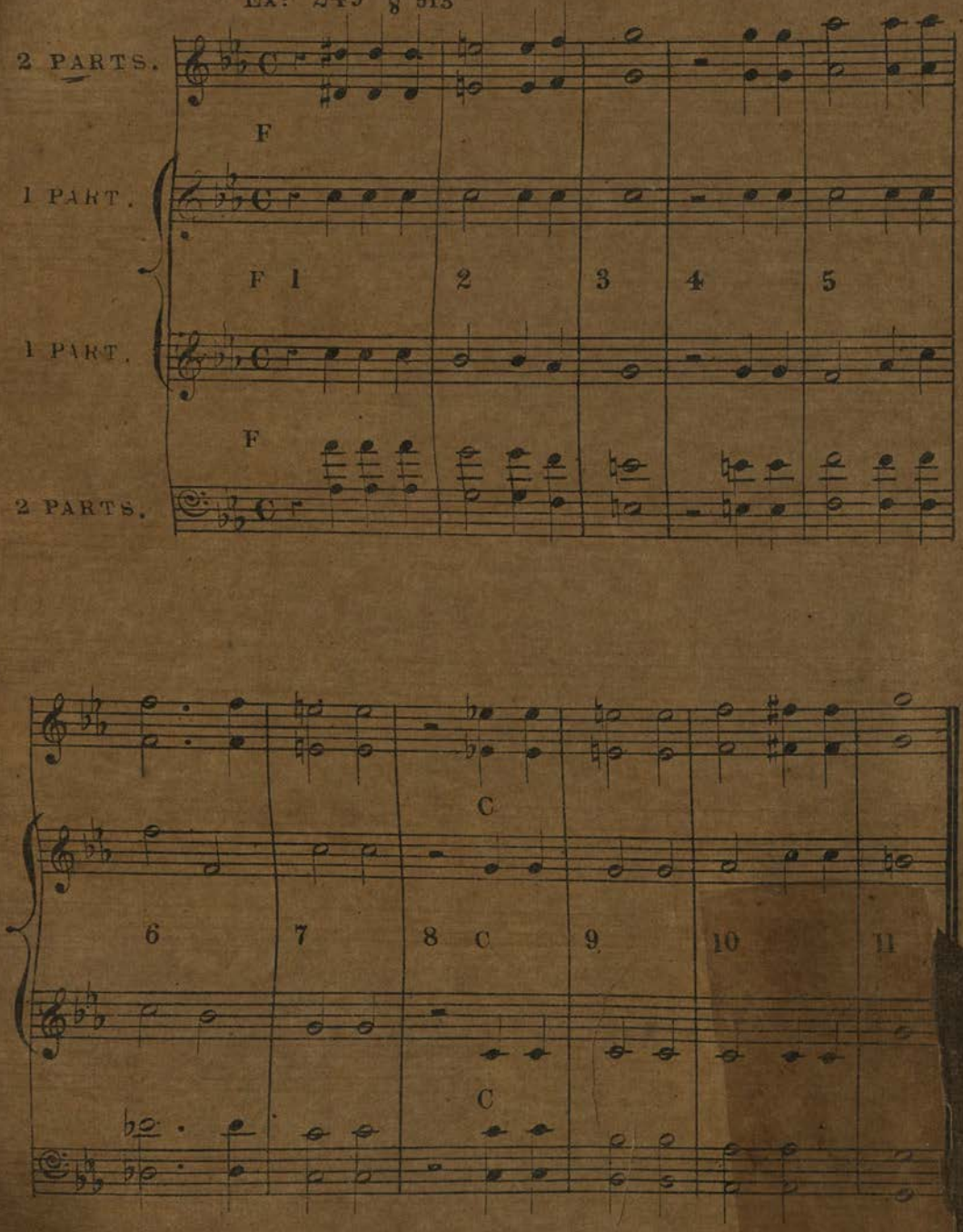
§ 509.




1
2
3
4
(o)
(p)
(q)
5
6
7
8
(r)
(s)
(t)
9
10
11
1
(u)
(v)
2
3
4
5
6
(w)
(x)
7
8
9
10
11
(a)
(y)

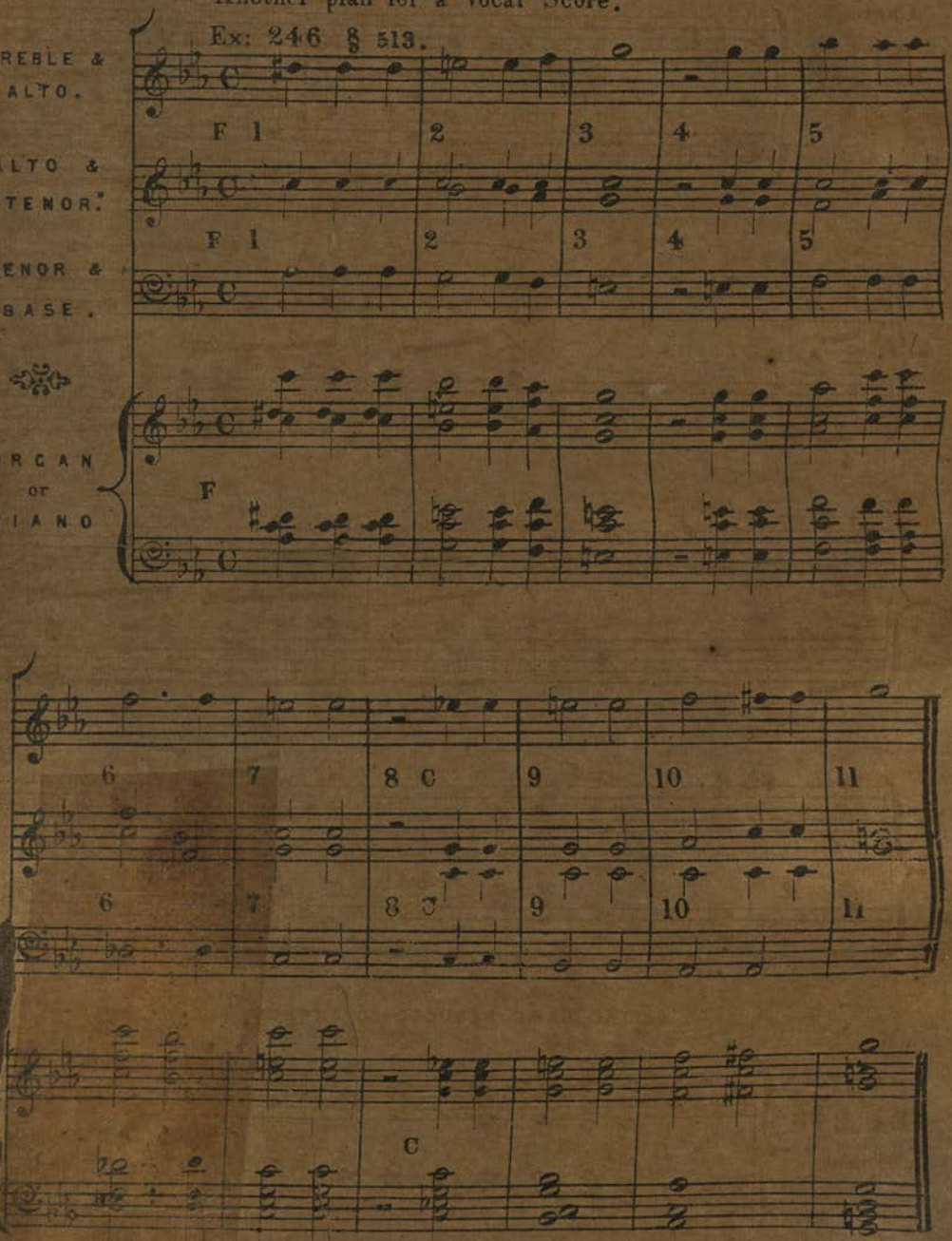
Plan for a Vocal Score.

Ex: 245 § 513



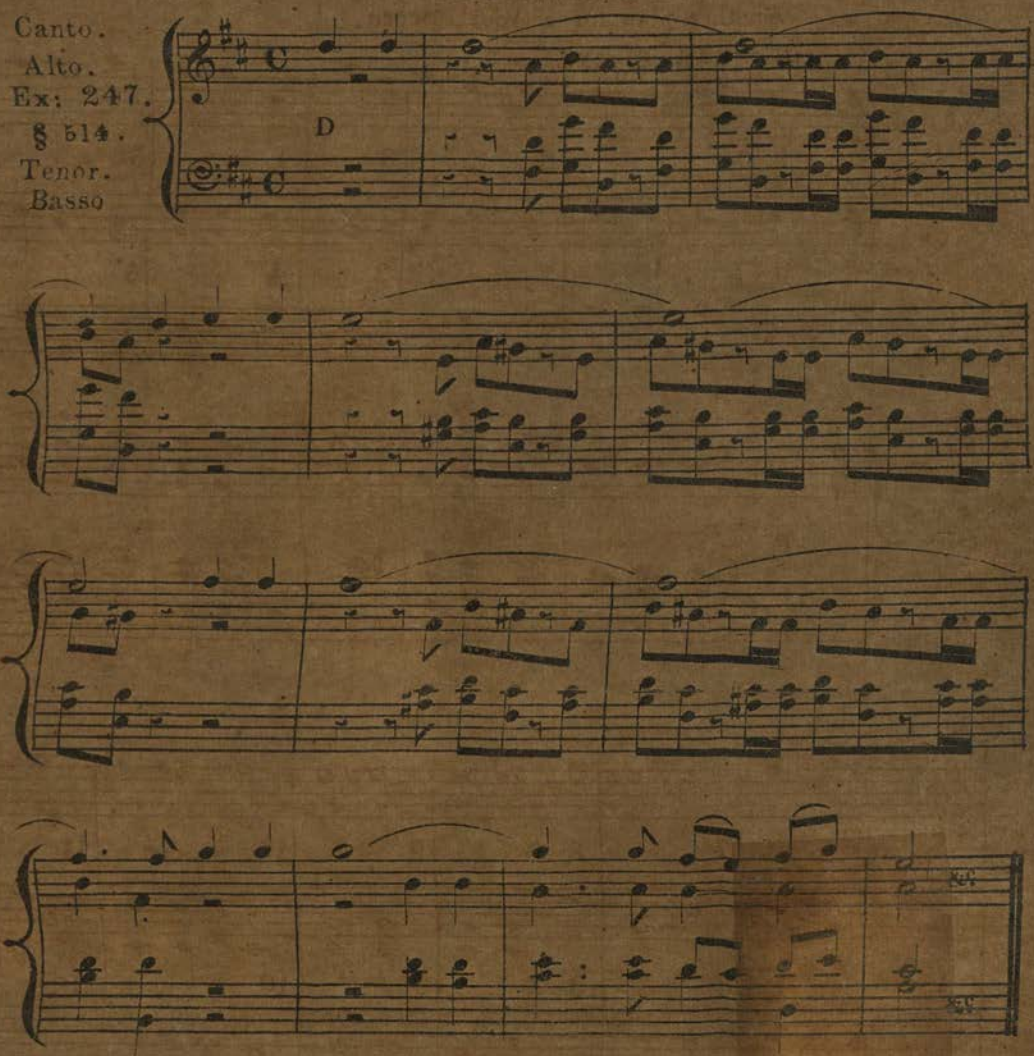
Another plan for a Vocal Score.

Ex: 246 § 513.



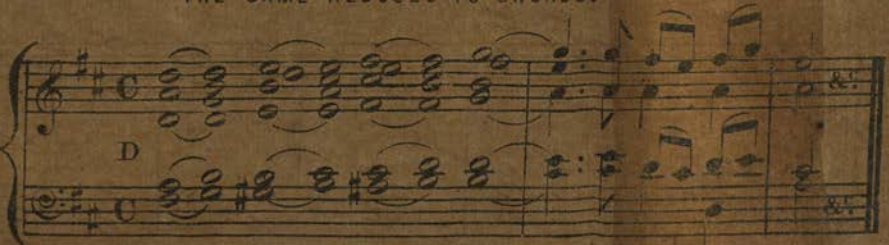
HALLELUJAH CHORUS.

Canto.
Alto.
Ex: 247.
§ 514.
Tenor.
Basso



THE SAME REDUCED TO CHORDS.

Ex. 248.
§ 514.

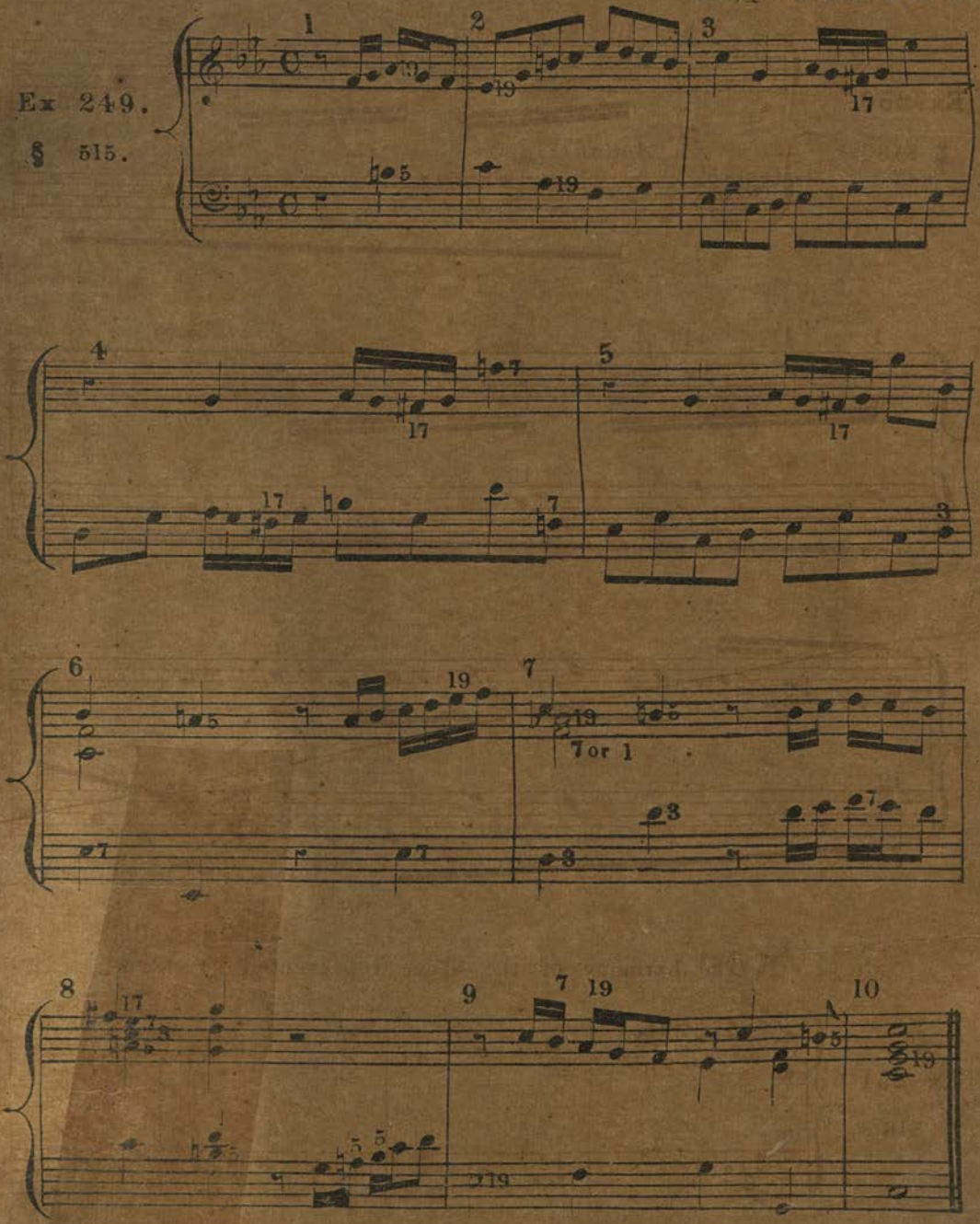


SEB: BACH'S PRELUDES & FUGUES № 48.

Transposed from B to C.

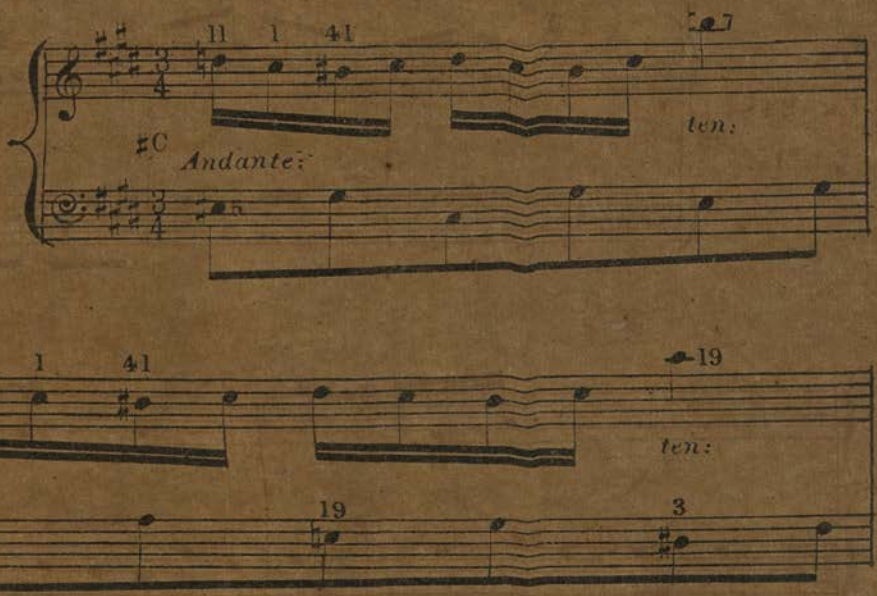
Ex 249.

§ 515.



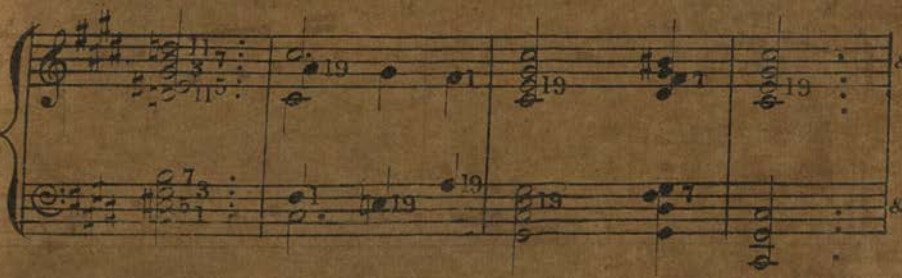
POLONOISE by S. BACH.

Ex: 250.
§ 516.



The harmony of the above represented by chords.

Ex 251
§ 516

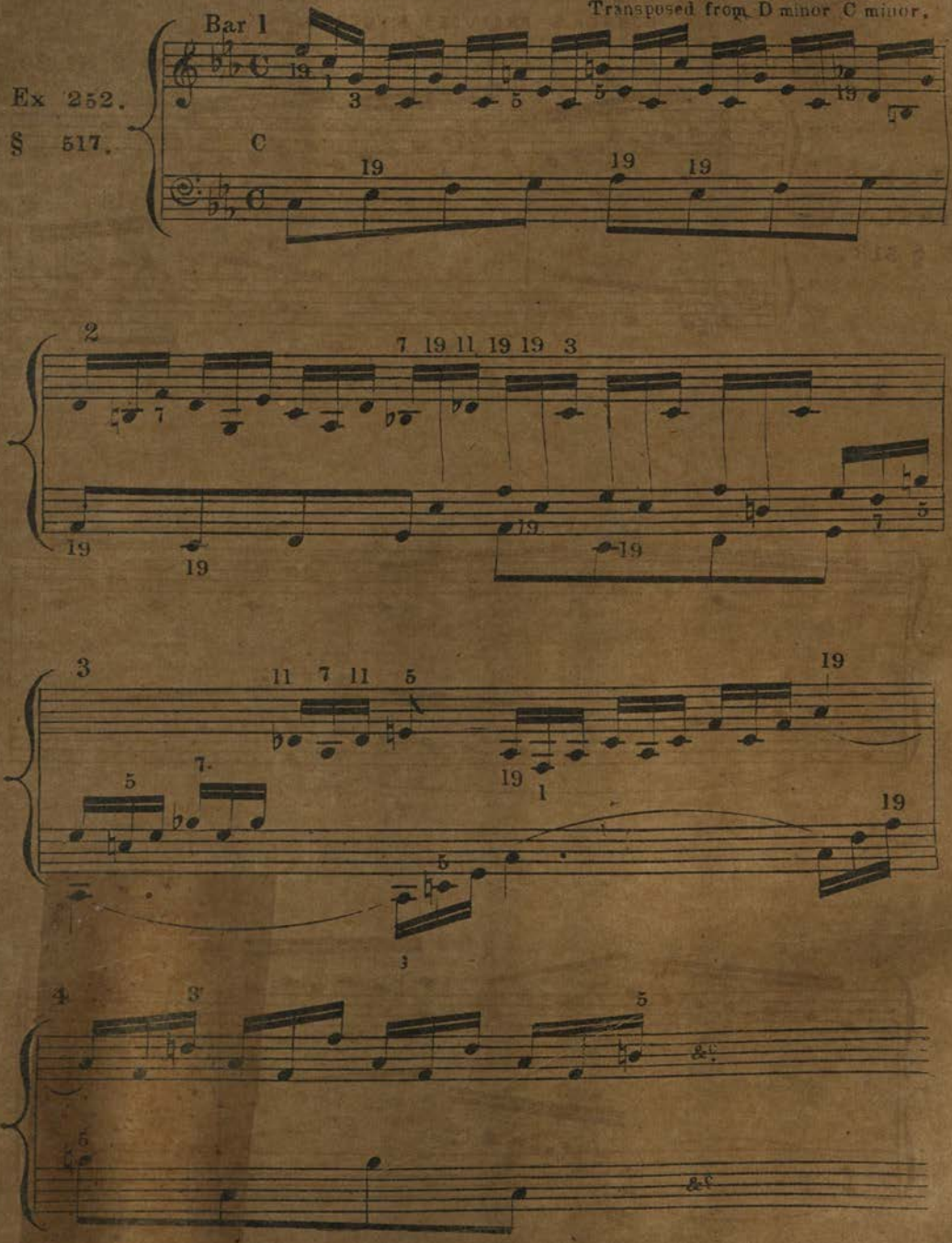


SEB: BACH'S PRELUDES & FUGUES No 6

Transposed from D minor C minor.

Ex 252.

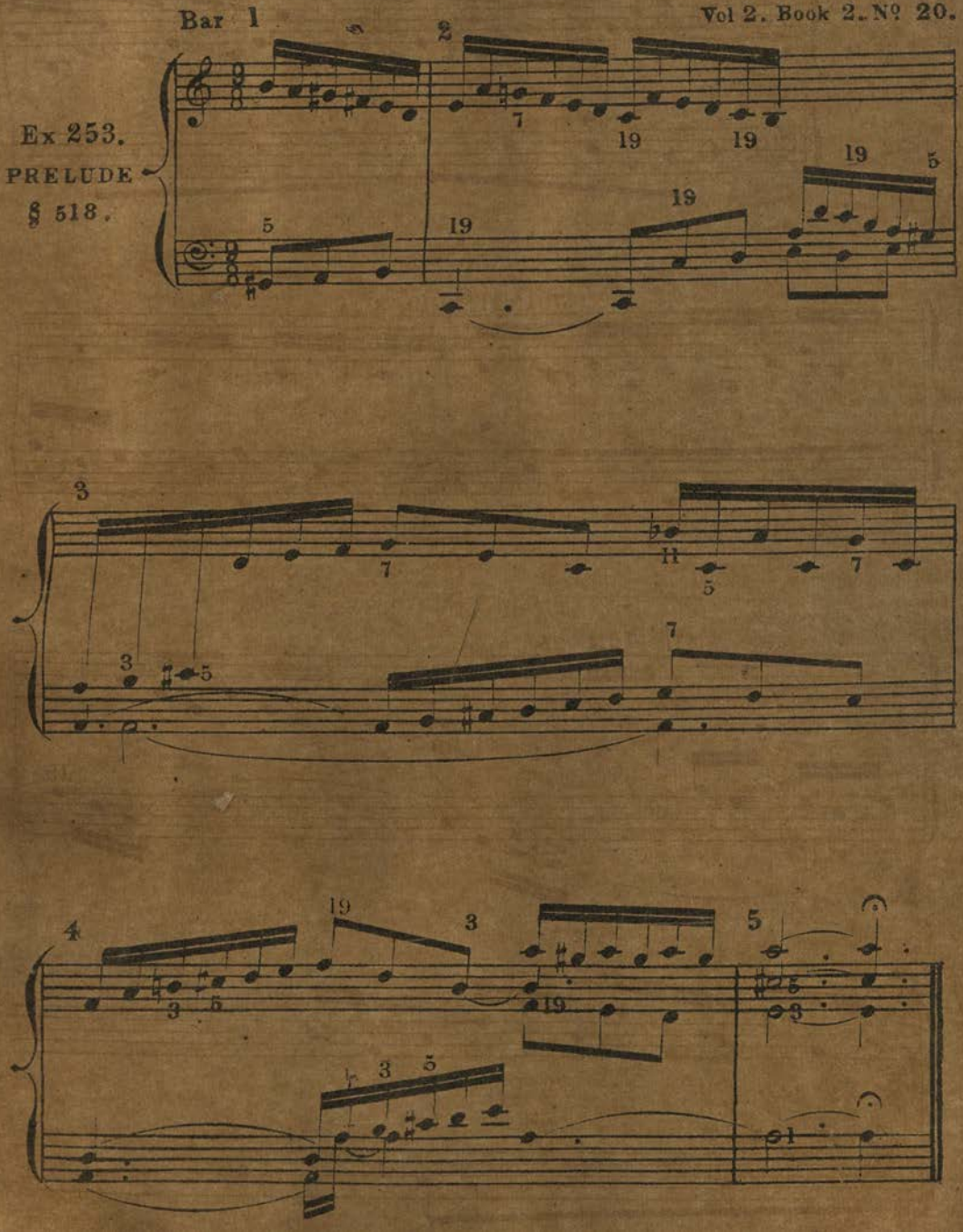
§ 517.



S. BACH'S PRELUDES & FUGUES.

Vol 2. Book 2. Nº 20.

Ex 253.
PRELUDE
§ 518.



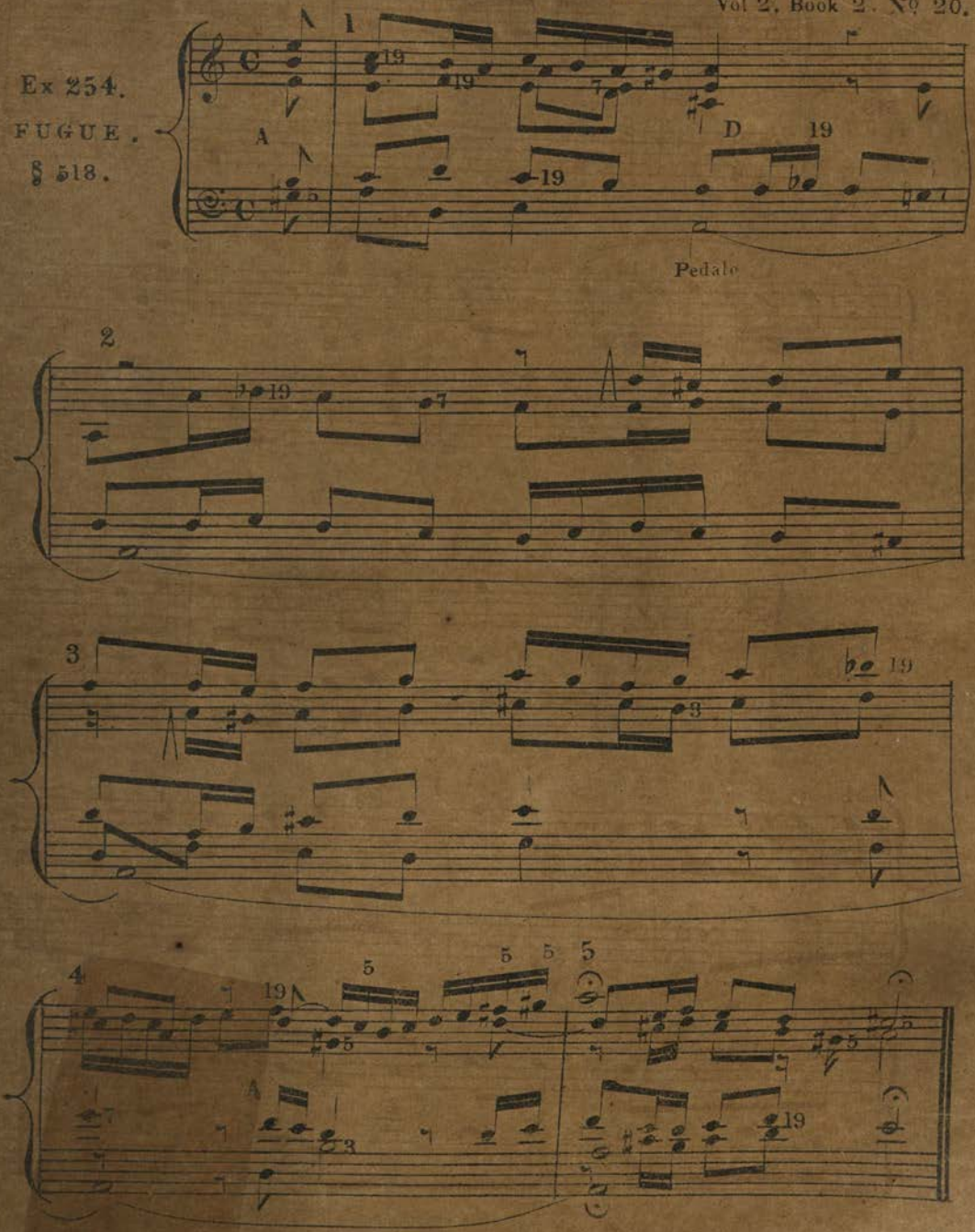
S. BACH'S. PRELUDES & FUGUES.

Vol 2. Book 2. Nº 20.

Ex 254.

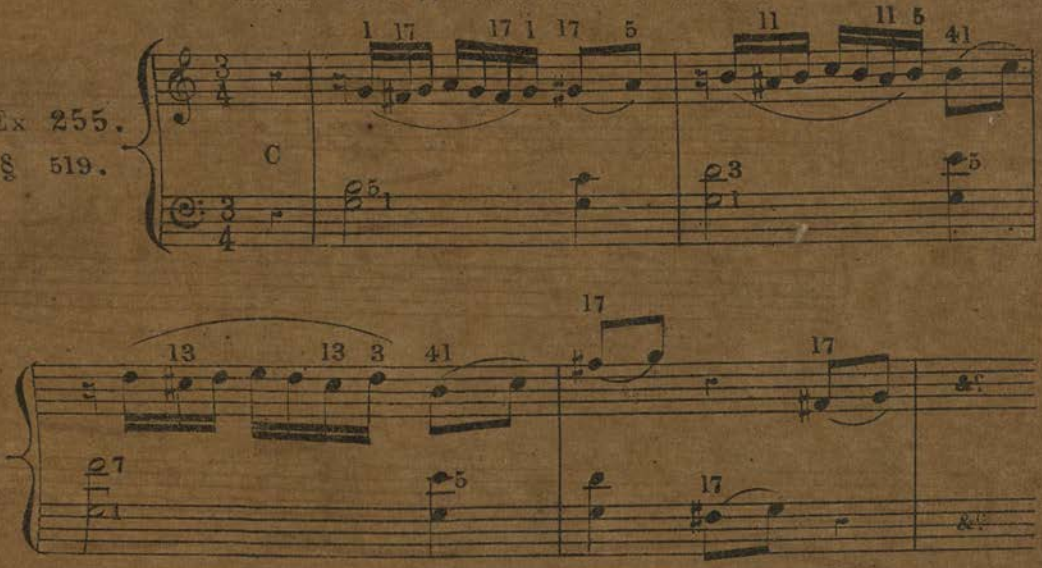
FUGUE.

§ 518.



SONATA WITH 12 VARIATIONS BY HAYDN.

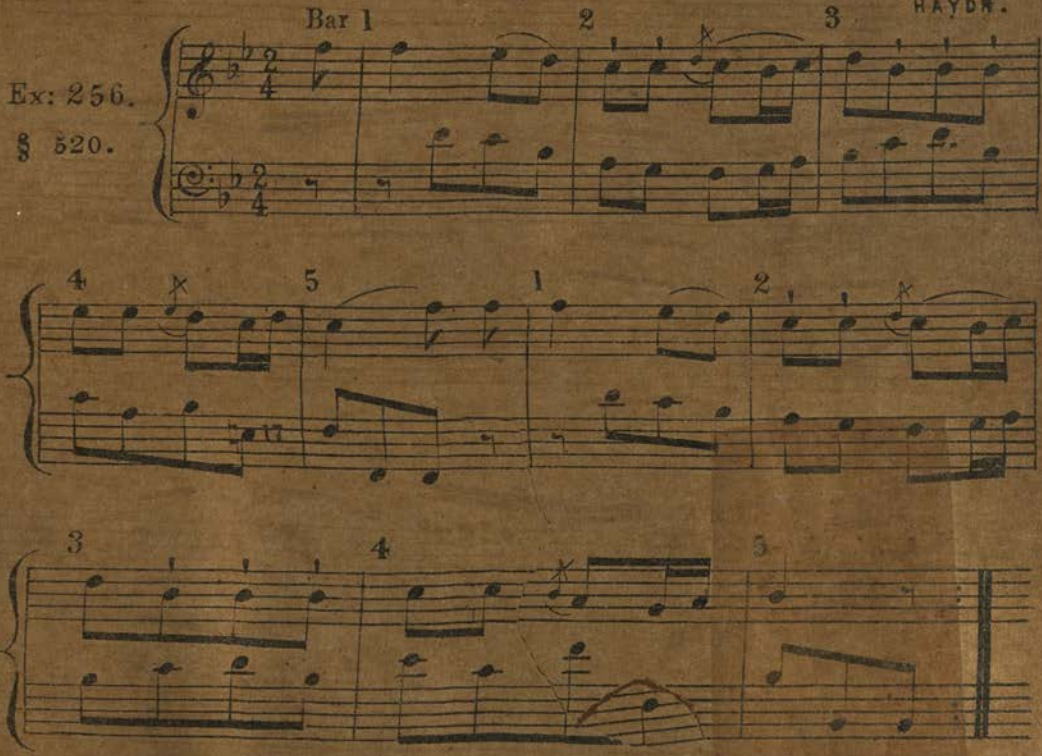
Ex 255.
§ 519.



Scherzando Presto.

HAYDN.

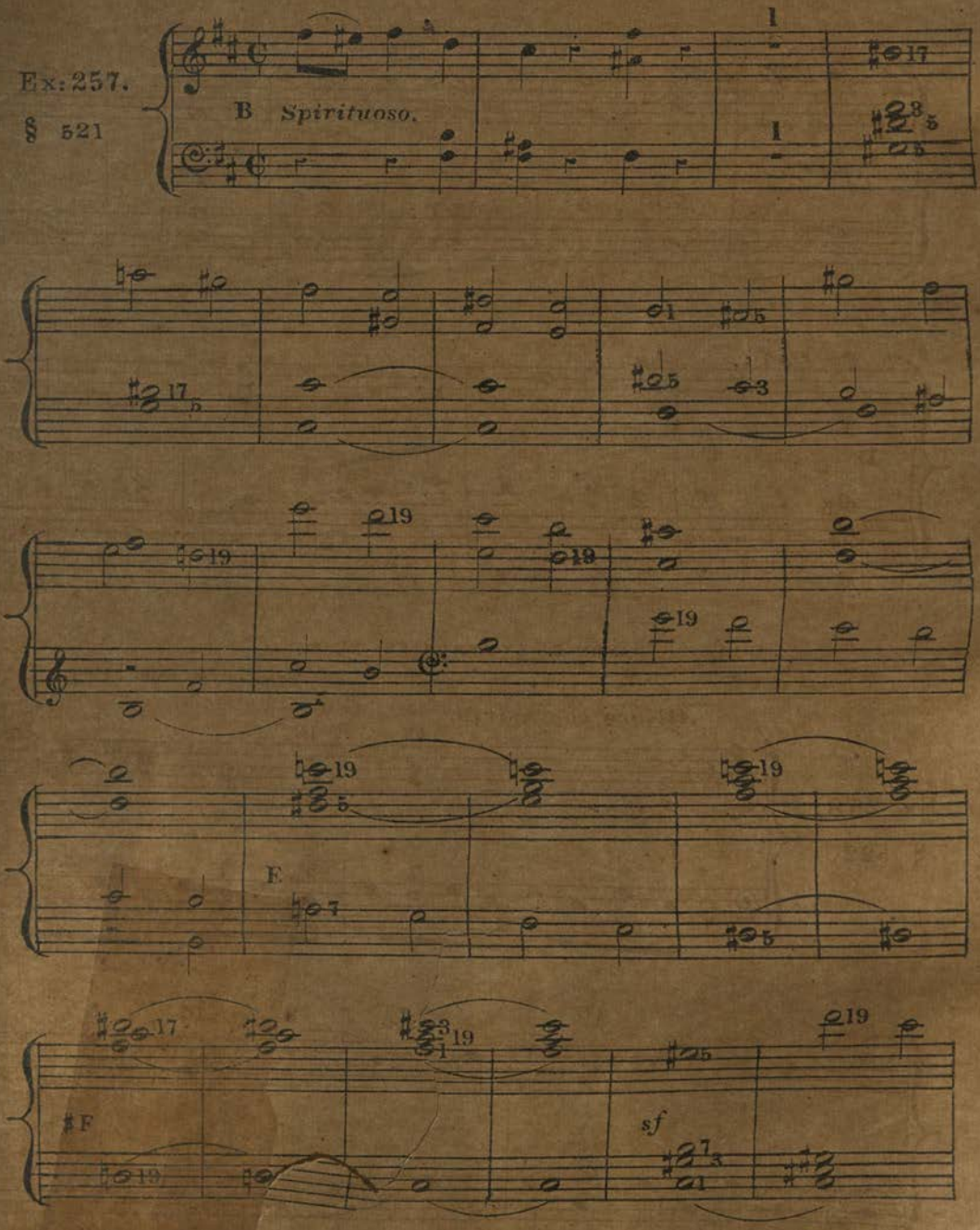
Ex: 256.
§ 520.

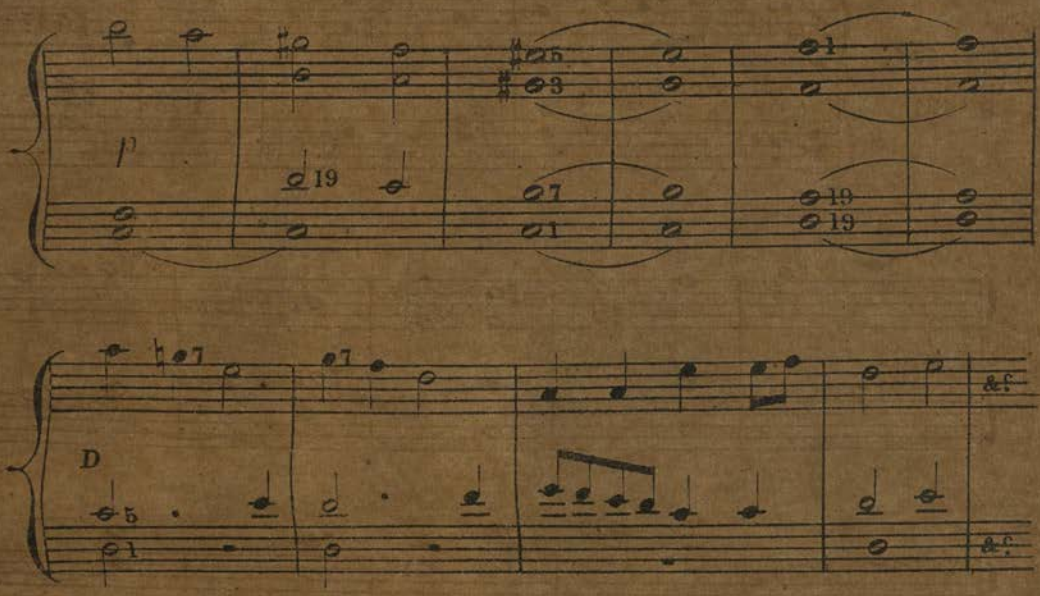


HAYDN'S SYMPHONIES No. 1.

Ex: 257.

§ 521



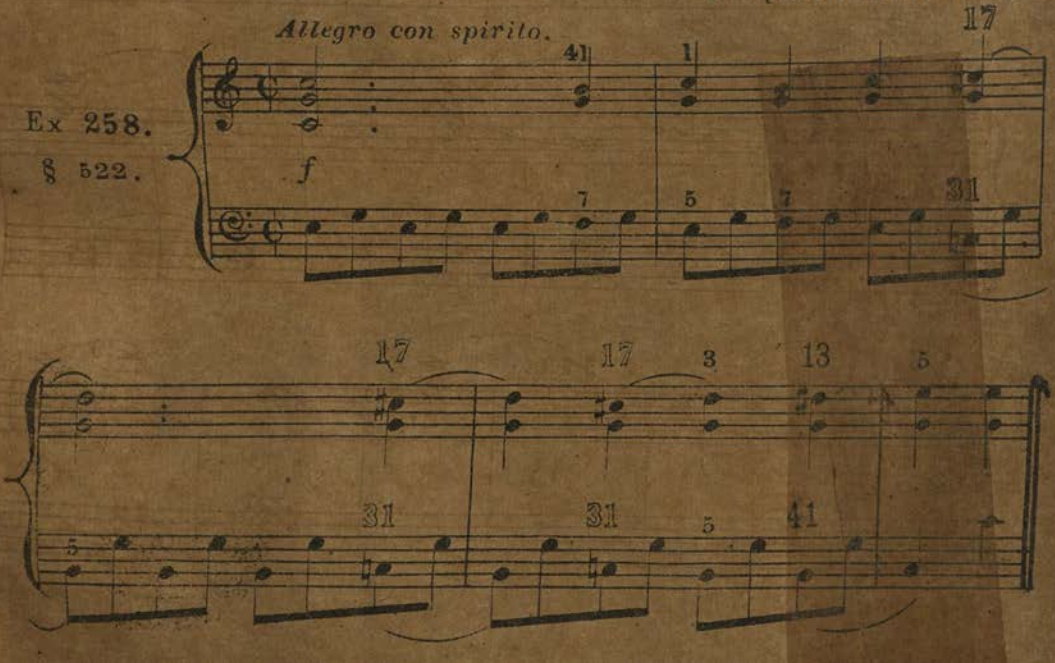


HAYDN'S SYMPHONIES No 8.

Transposed from ♭E to C.

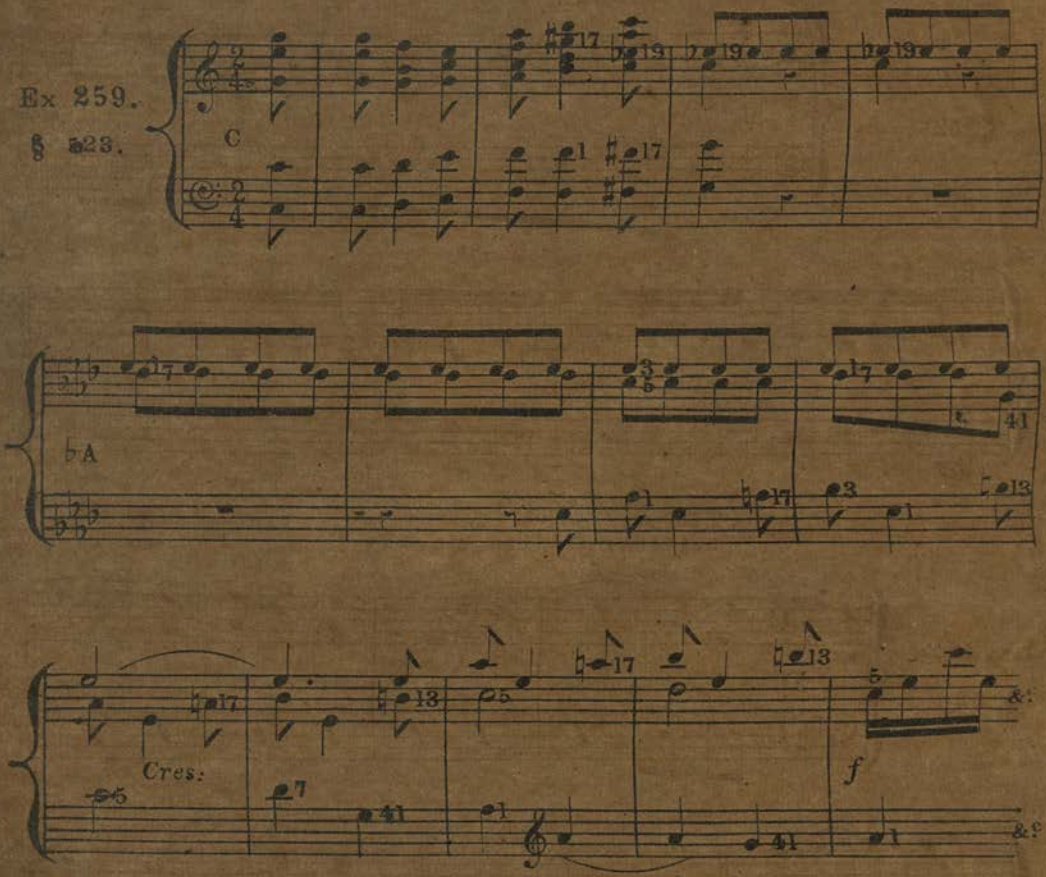
Ex 258.

§ 522.



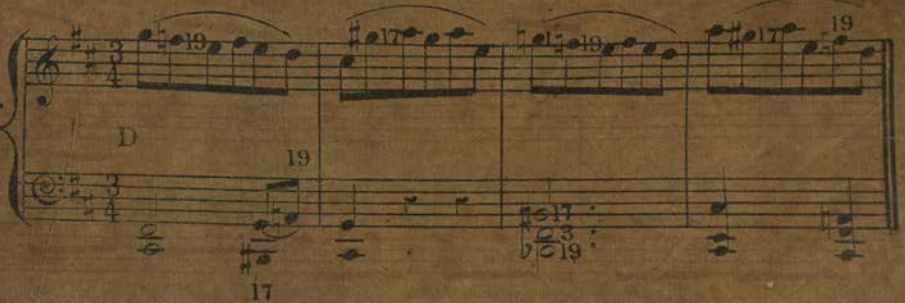
MOZART'S AIR WITH VARIATIONS Nº 10.

Ex 259.
§ 523.



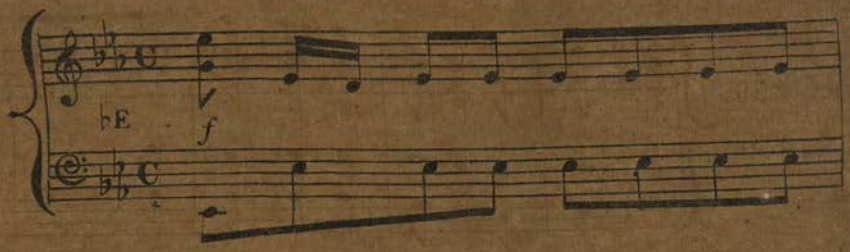
MOZART'S SONATA OP: 19.

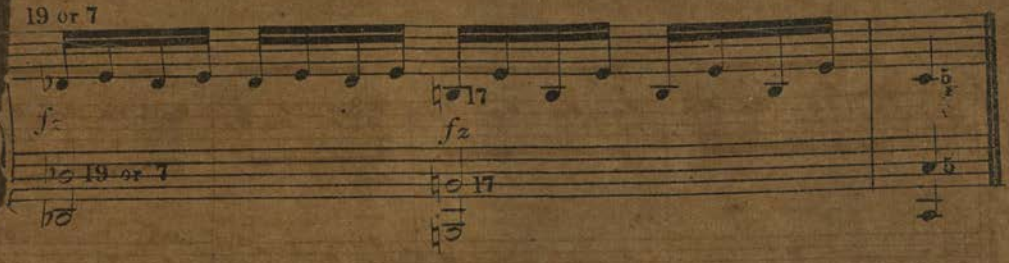
Ex: 260.
§ 524.



OVERTURE TO THE ZAUBERFLOTE. MOZART.

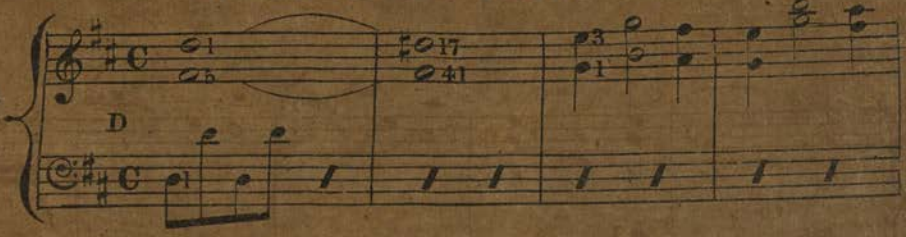
261.
525.

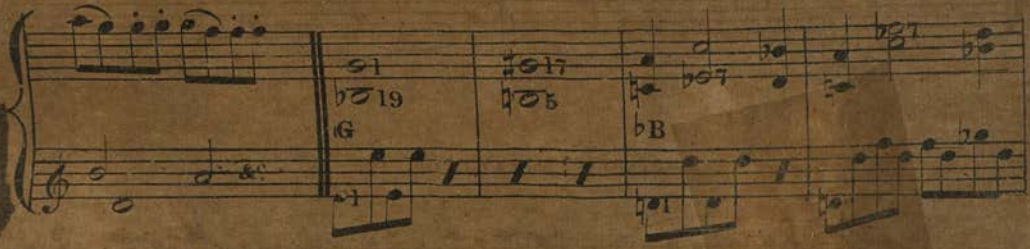


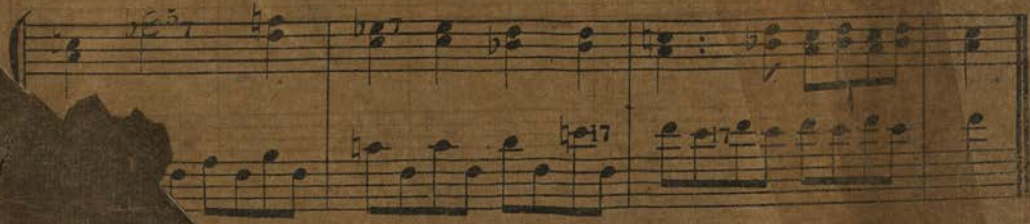


OVERTURE TO DON GIOVANNI. MOZART.

Ex: 262.
§ 526.



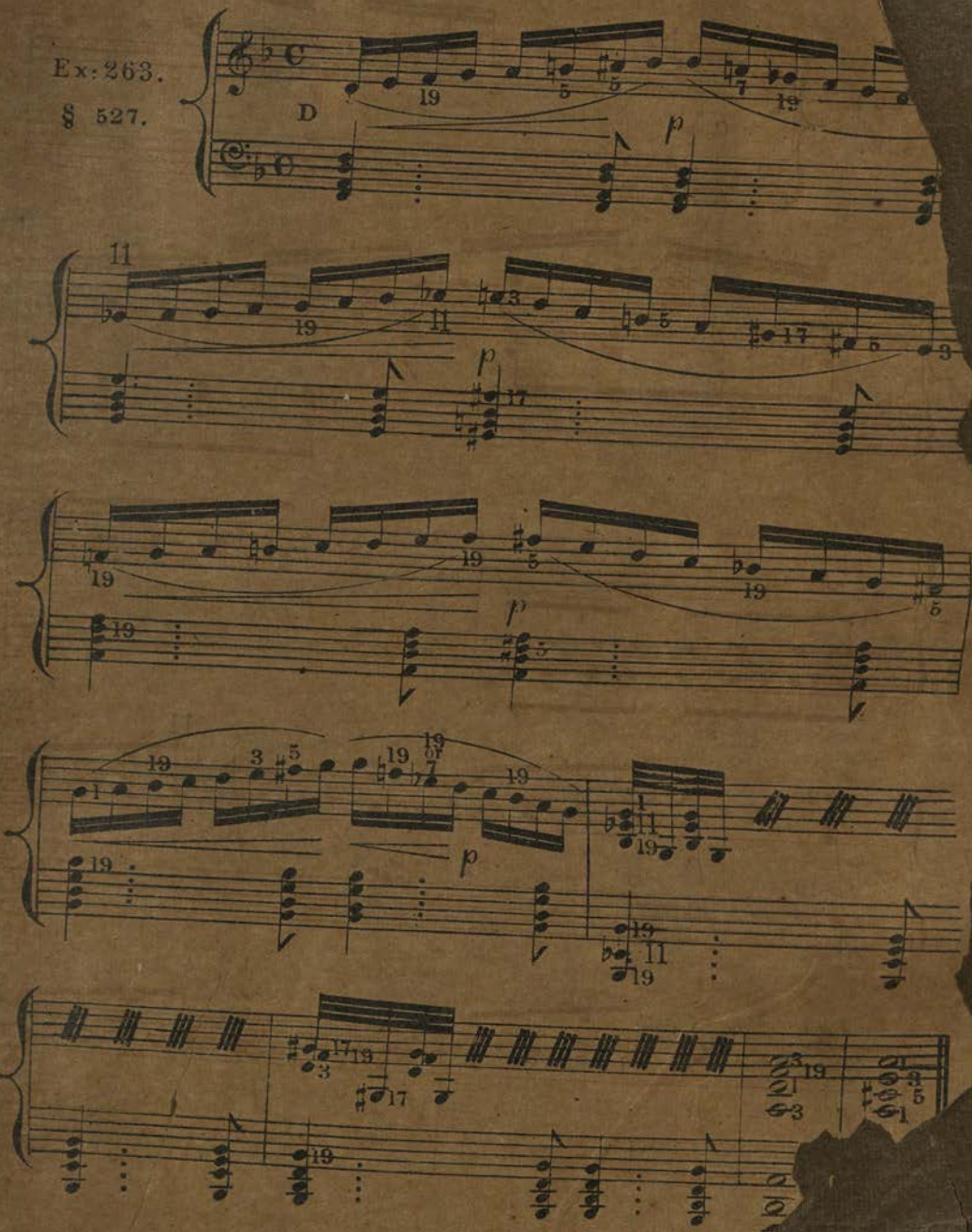




ANOTHER PART OF THE SAME.

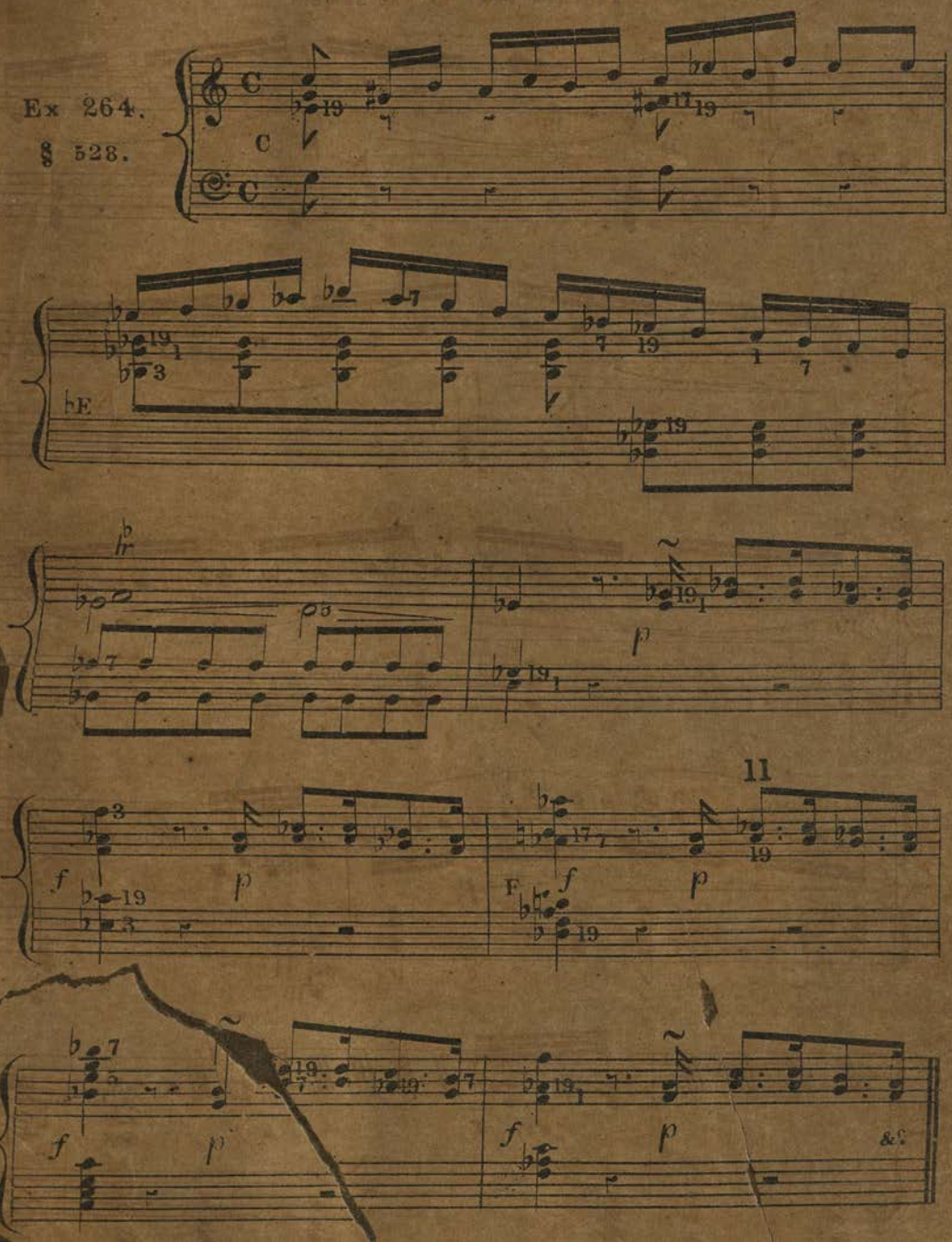
Ex: 263.

§ 527.



FANTASIA BY MOZART.

Ex 264.
§ 528.

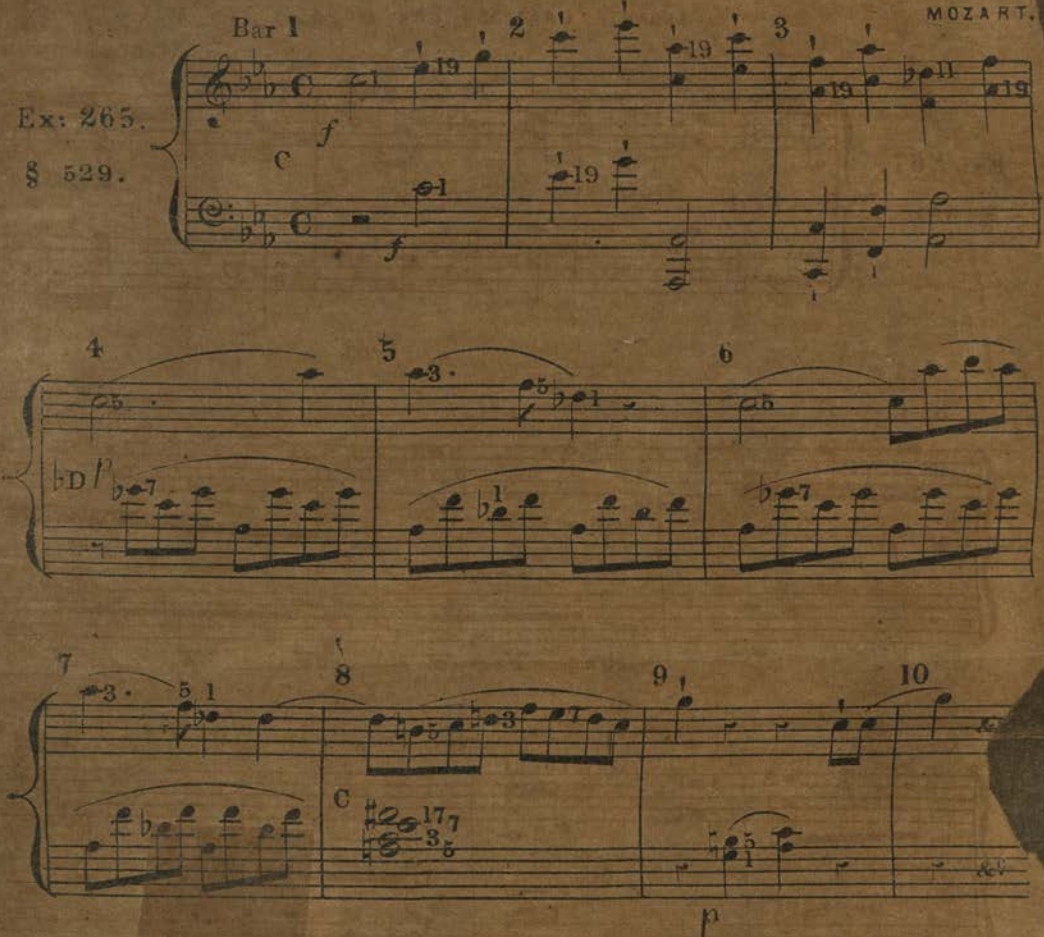


FANTASIA & SONATA.

MOZART.

Ex: 265.

§ 529.

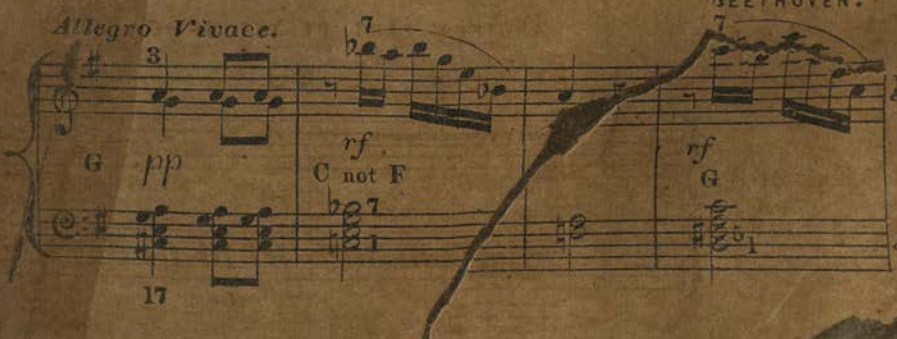


TRIO FOR THE PIANO, VIOLIN & VIOLONCELLO.

BEETHOVEN.

Ex: 266.

§ 530.

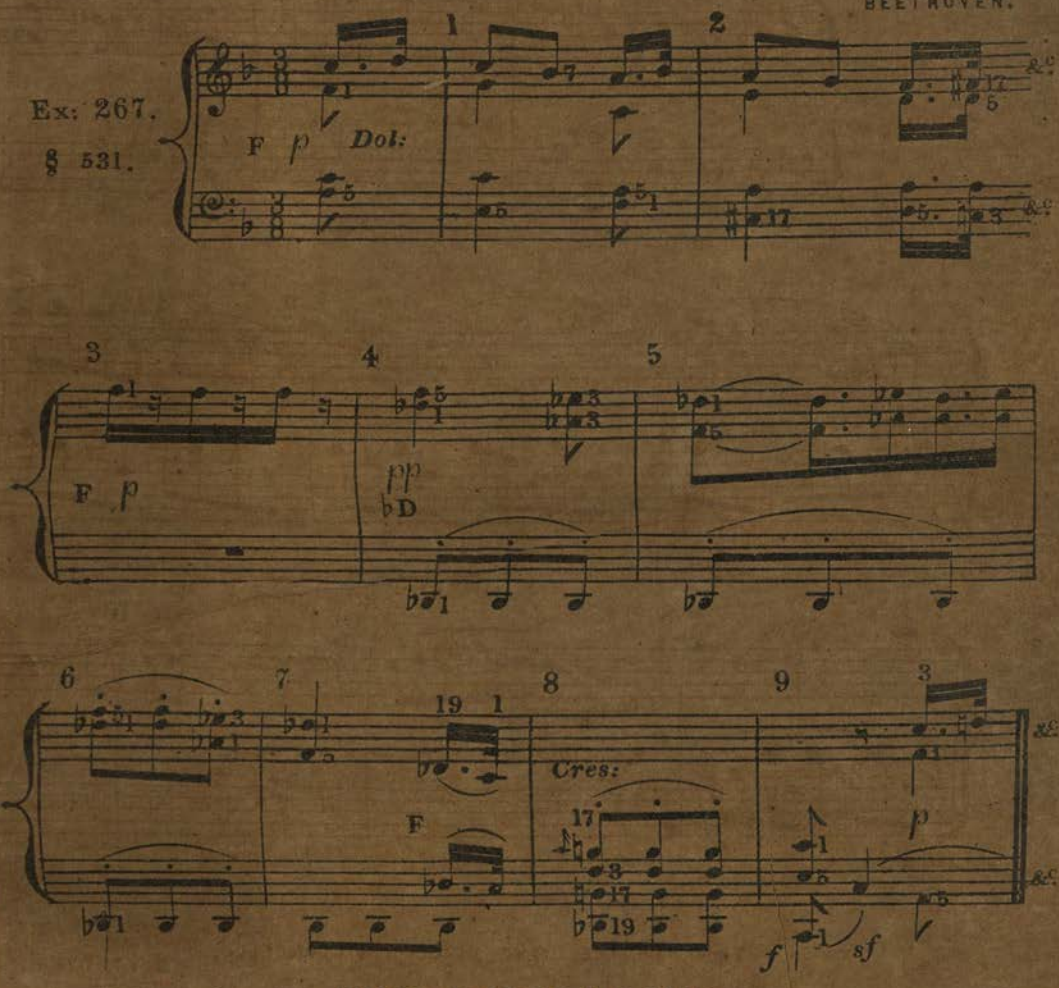


ANDANTE GRAZIOSO CON MOTO.

BEETHOVEN.

Ex: 267.

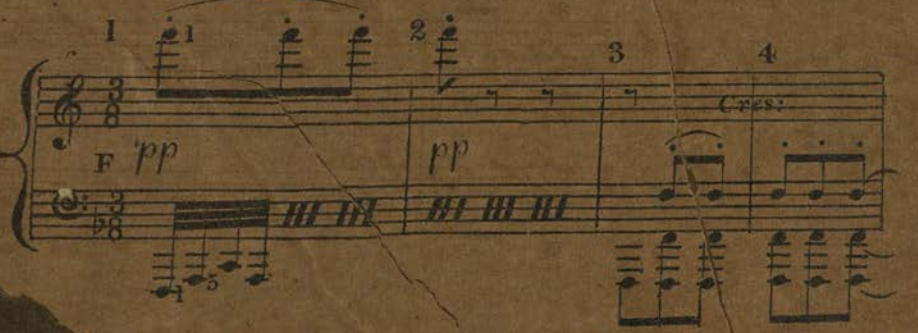
§ 531.

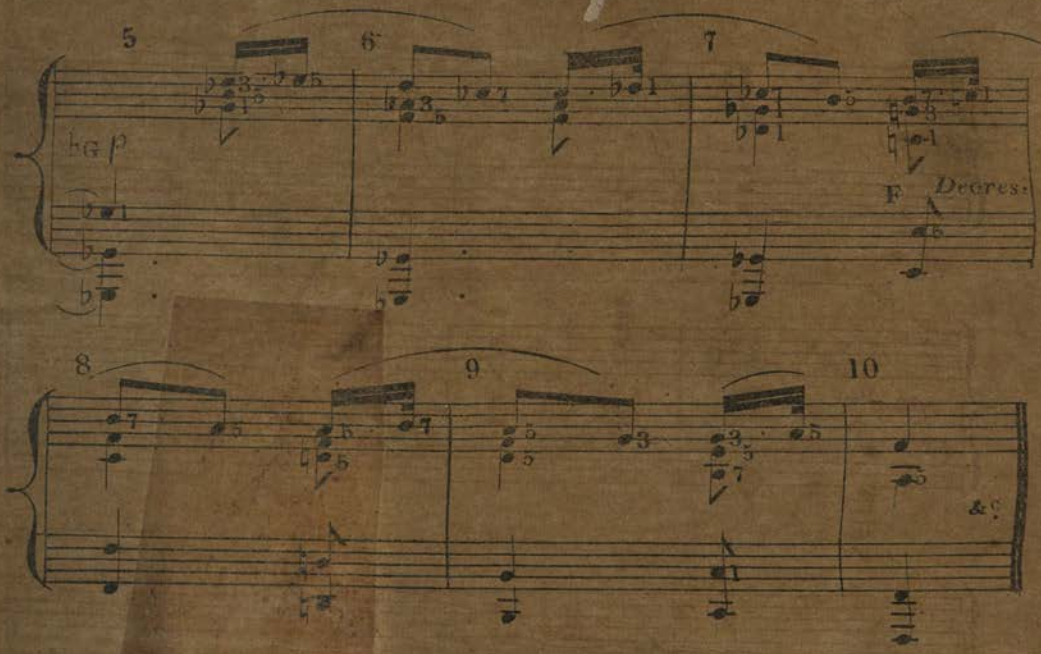


ANOTHER PART OF THE SAME.

Ex: 268.

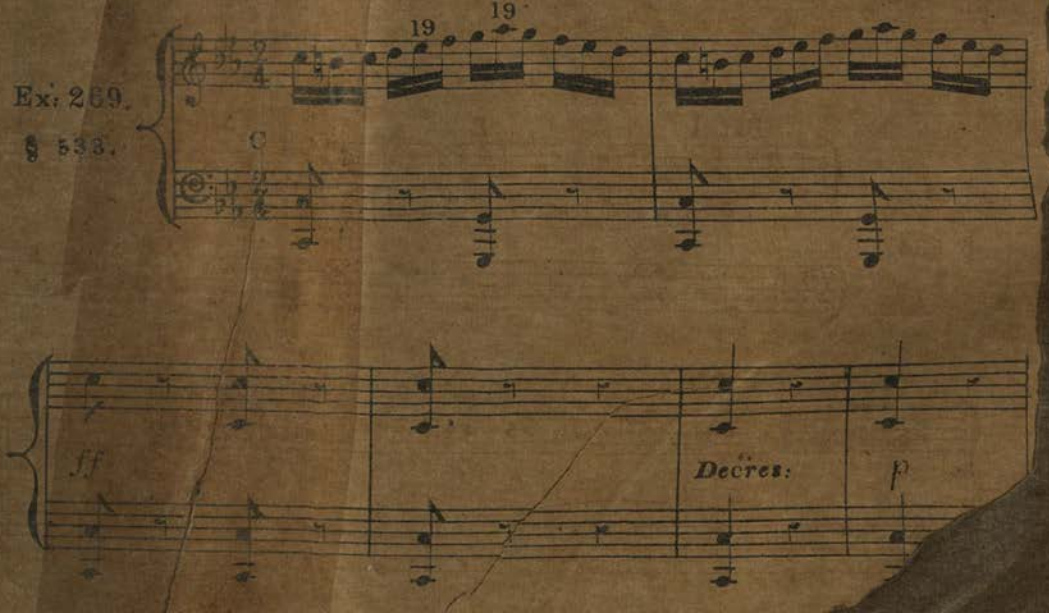
§ 532.

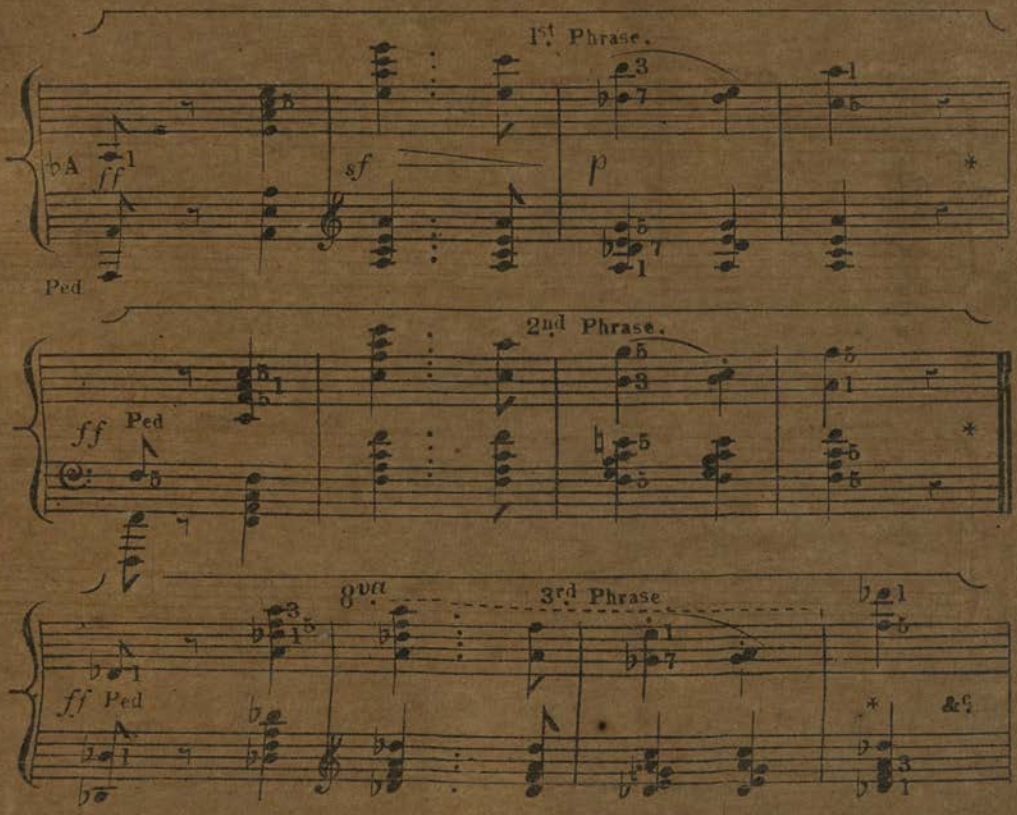




BEETHOVENS GRAND SONATA Op: 53.

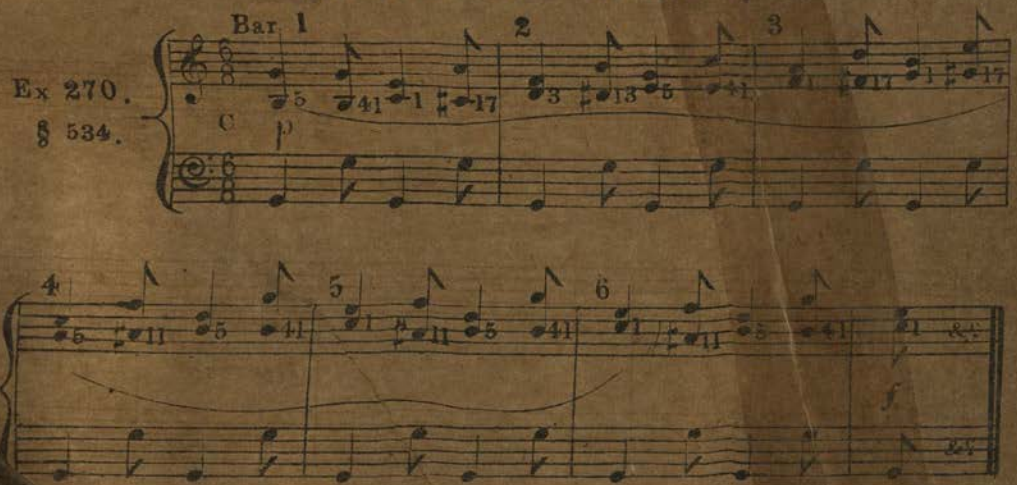
Ex: 269.
§ 533.





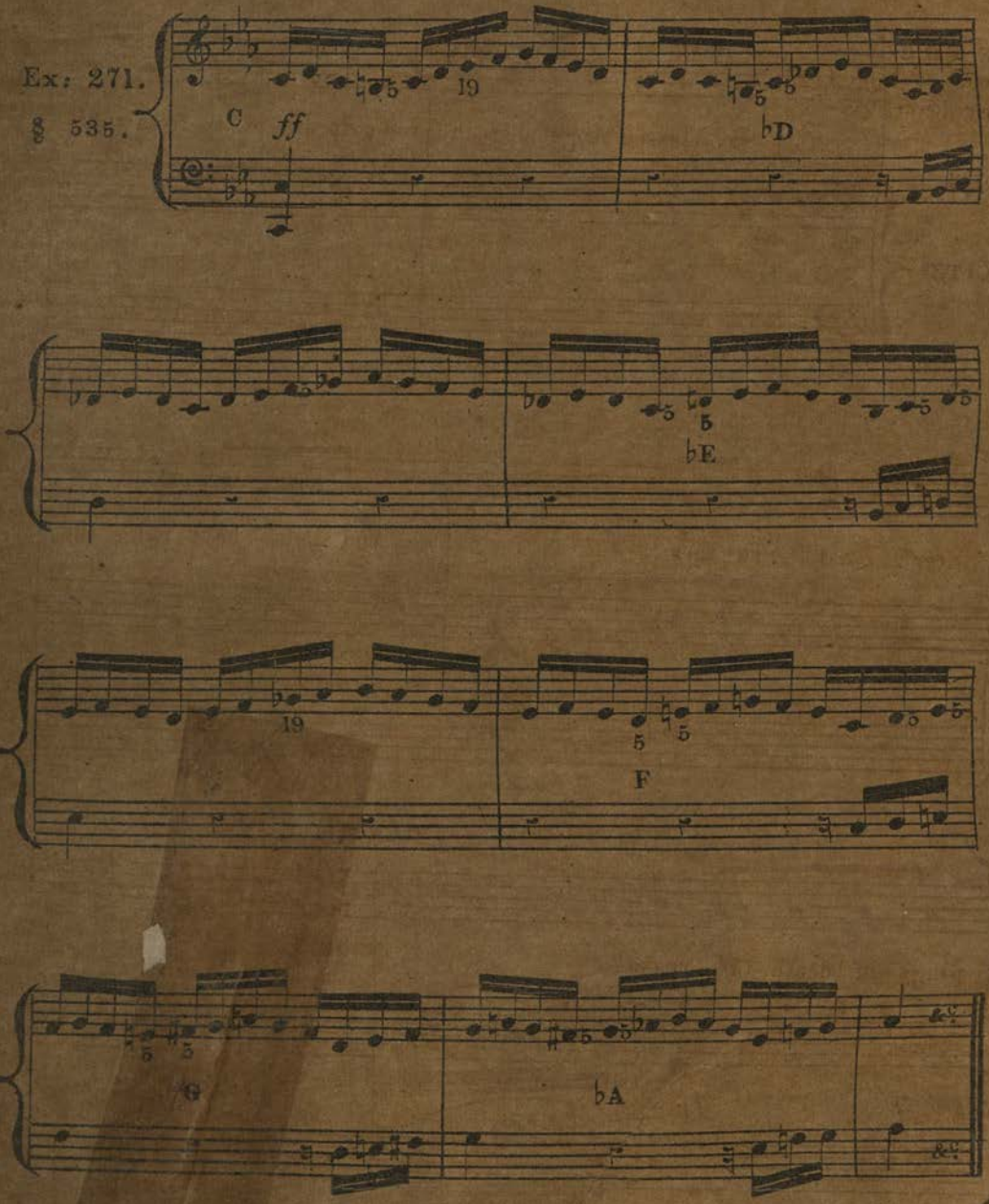
BEETHOVEN'S SONATA PASTORALE Op. 28.

Ex 270.
§ 534.



TRIO BY BEETHOVEN Nº 13.

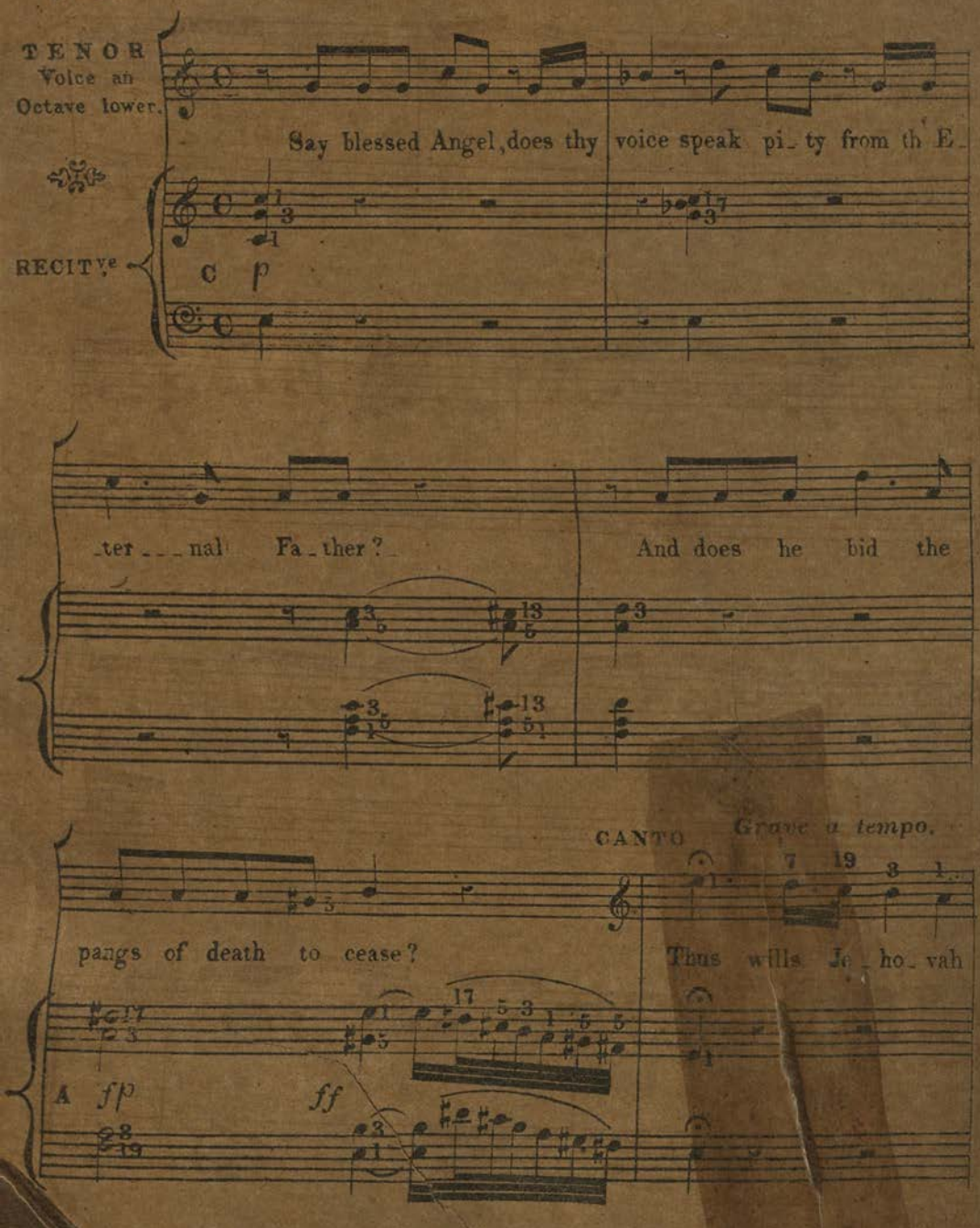
Ex: 271.
§ 535.

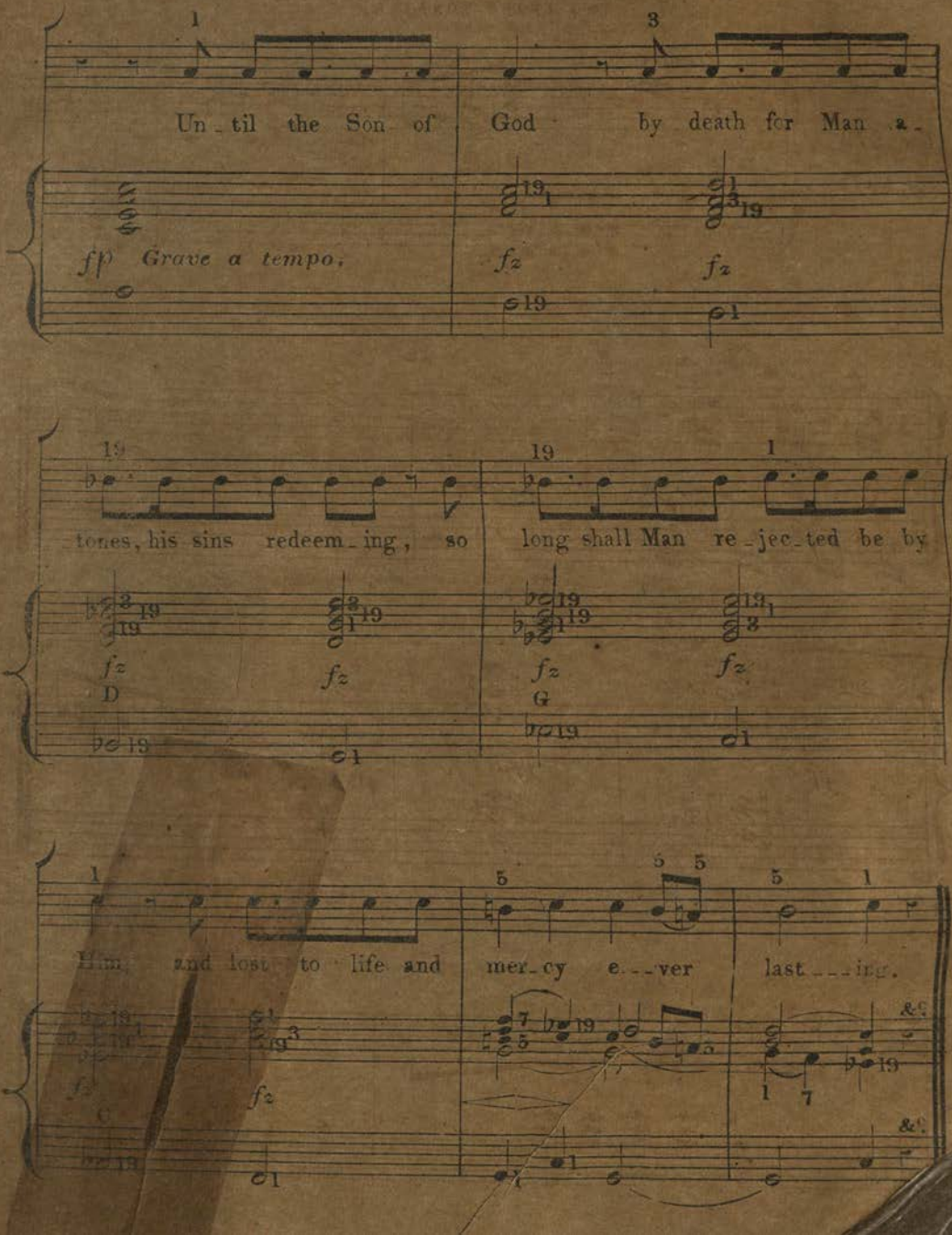


Ex: 272.

§ 536.

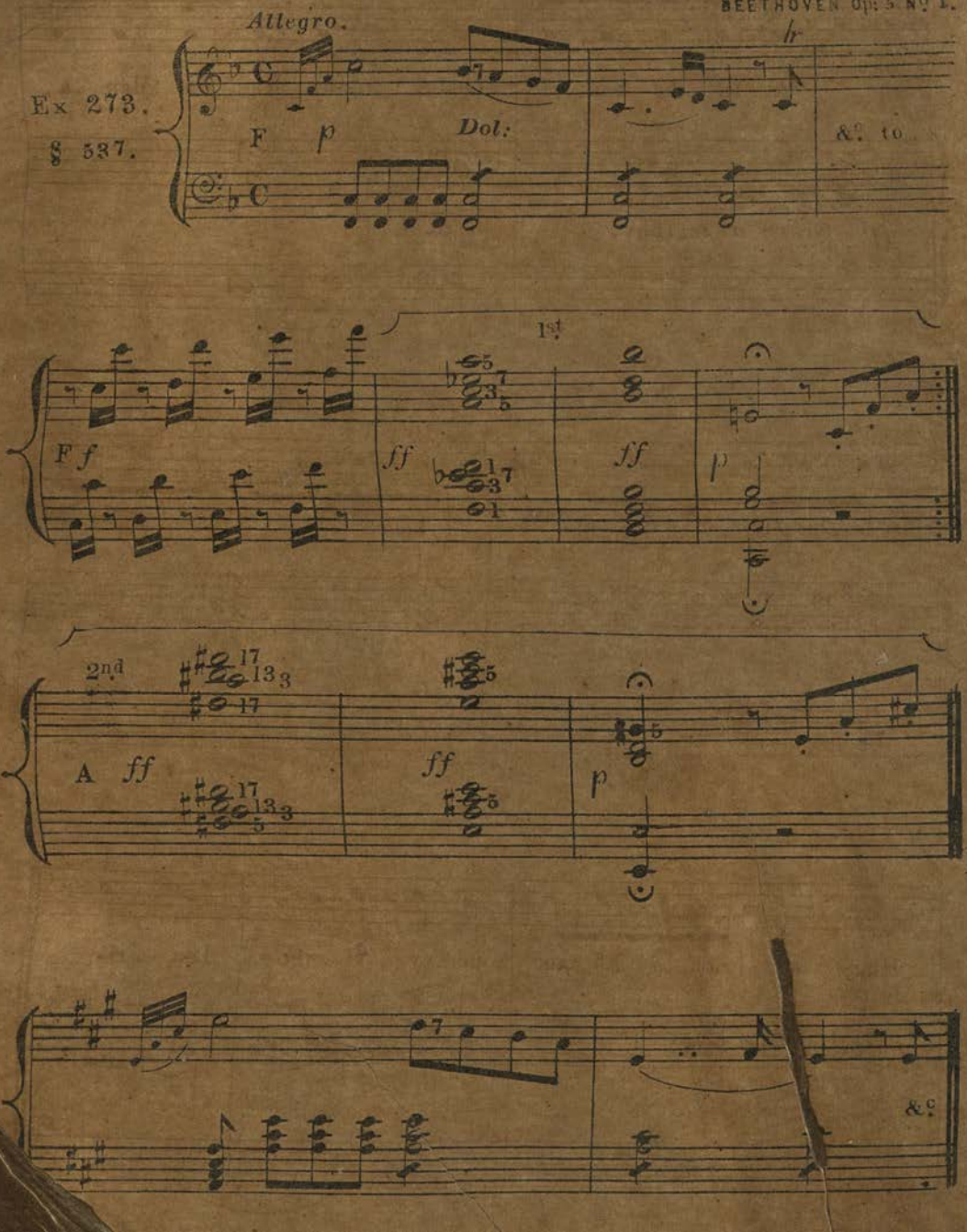
BEETHOVEN'S ORATORIO, THE MOUNT OF OLIVES.




Un_til the Son of God by death for Man a_
ff Grave a tempo,
fz fz
tones, his sins redeem_ing, so long shall Man re_jec_ted be by
fz fz fz fz
D G
Him, and lost to life and mer_cy e_ver last_ing.
fz fz
C

TWO GRAND SONATAS.

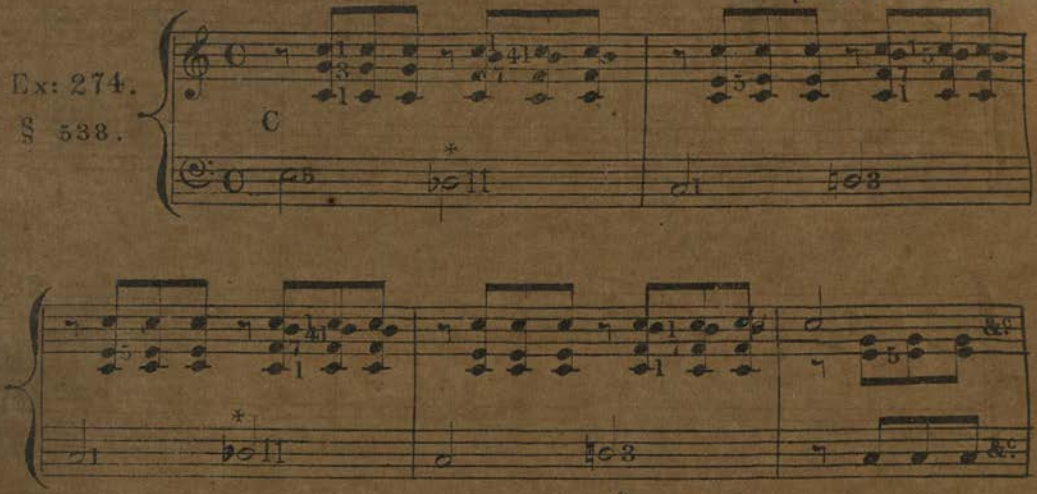
BEETHOVEN Op: 5 Nº 1.



BEETHOVEN'S SONATA No 1 Op:14.

Transposed from E to C.

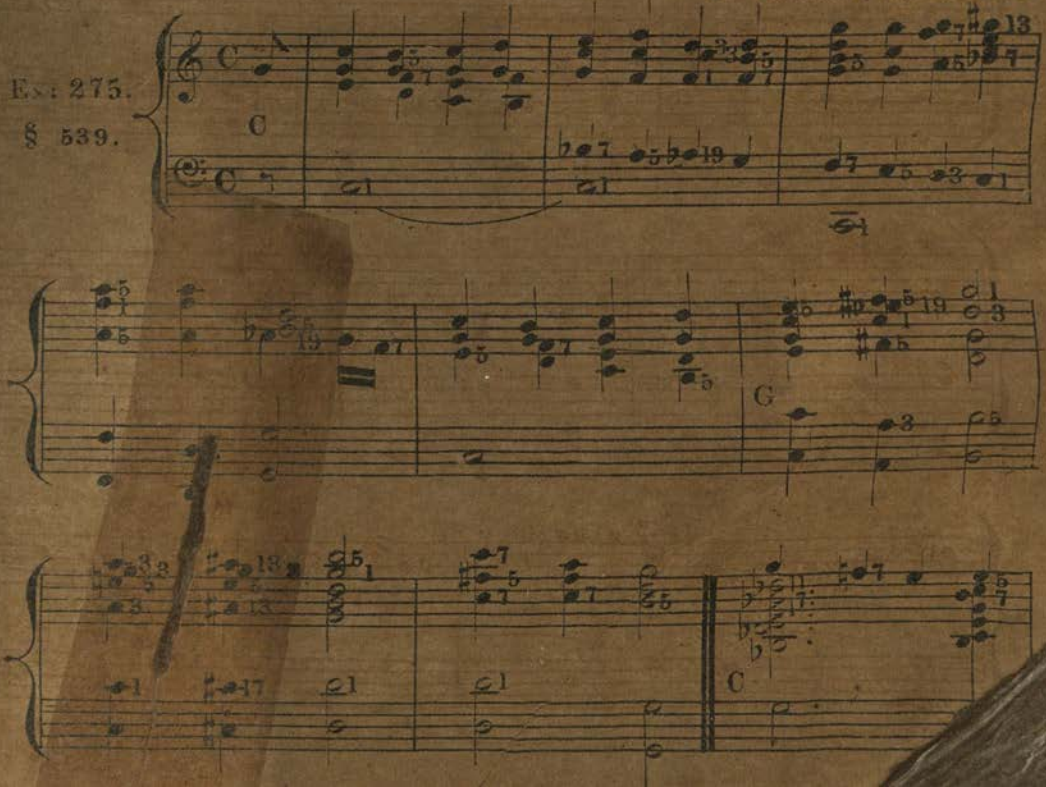
Ex: 274.
§ 538.

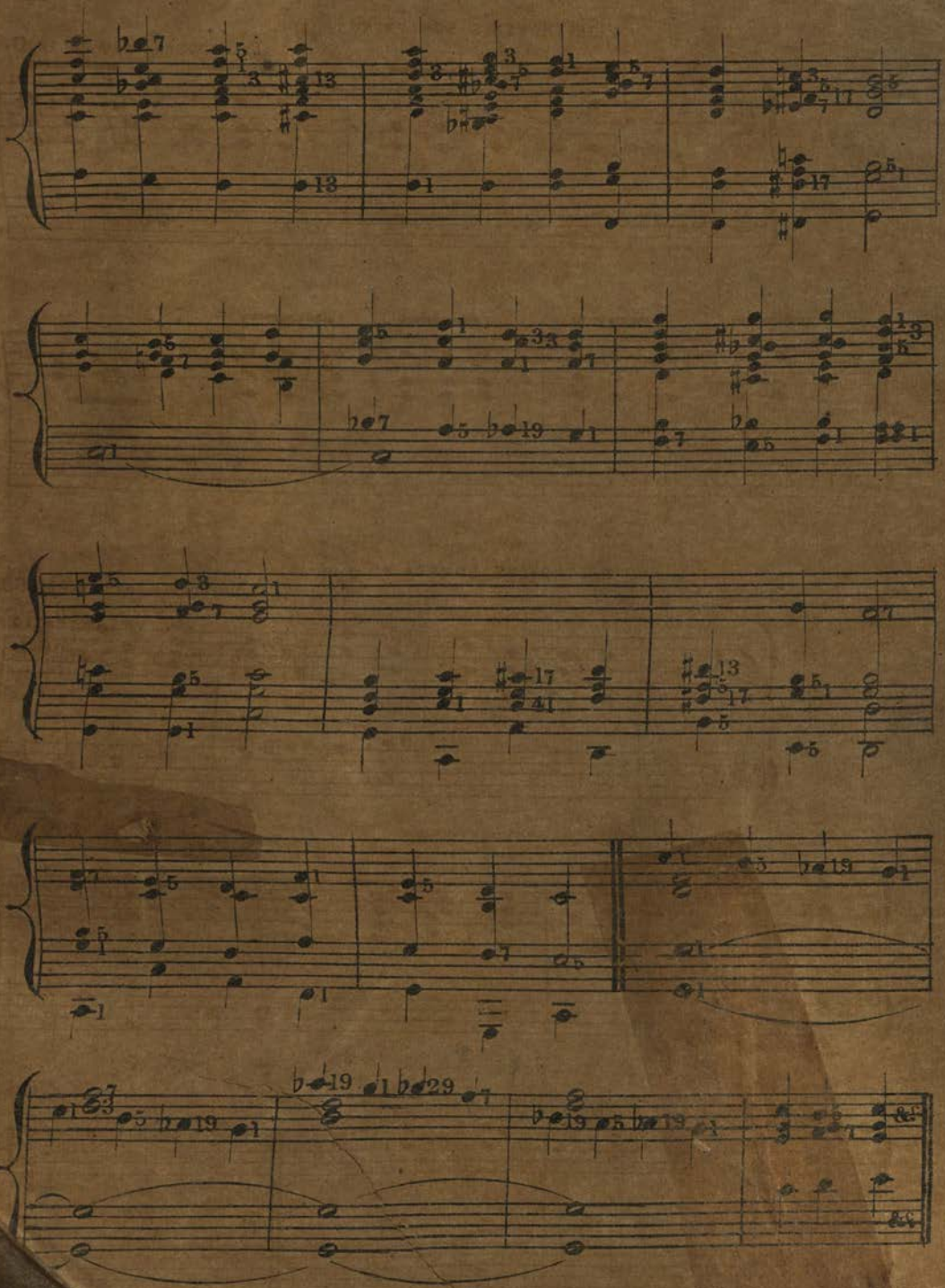


BEETHOVEN'S SONATA No. 2 Op:14.

2nd Movement bars 43 to 68

Ex: 275.
§ 539.



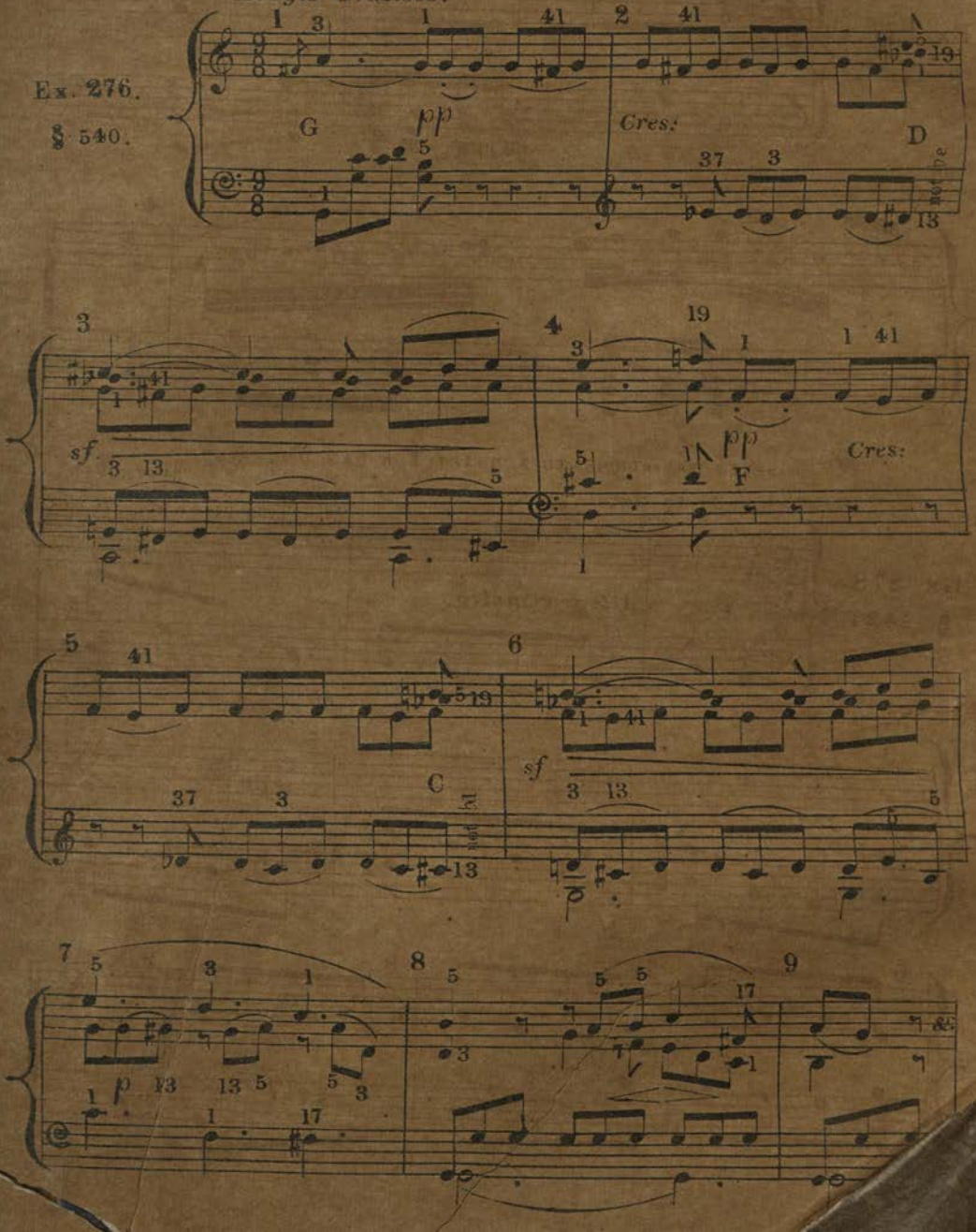


BEETHOVEN'S SONATA IN C Op: 29.

Adagio Grazioso.

Ex. 276.

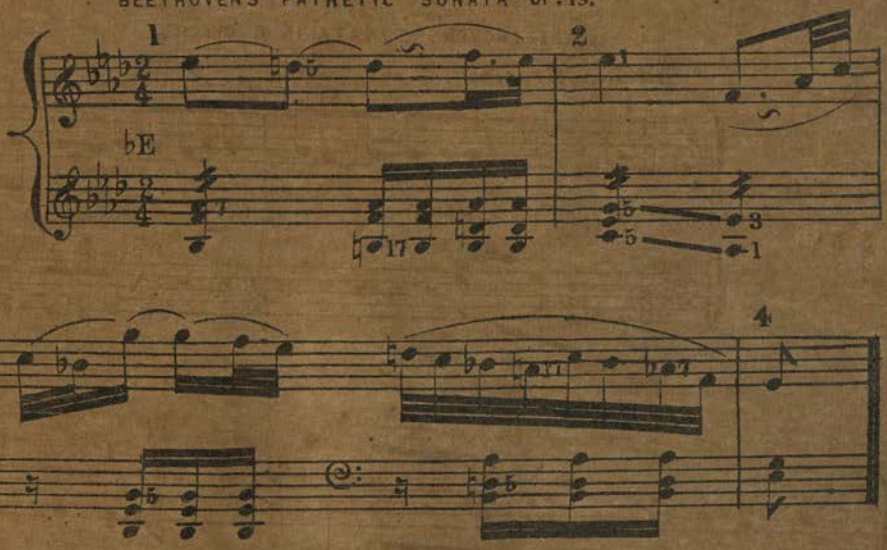
§ 540.



BEETHOVEN'S PATHETIC SONATA OP:13.

Ex. 277.

§ 541.



SEVEN VARIATIONS TO A QUARTET IN DAS OPFERFEST.

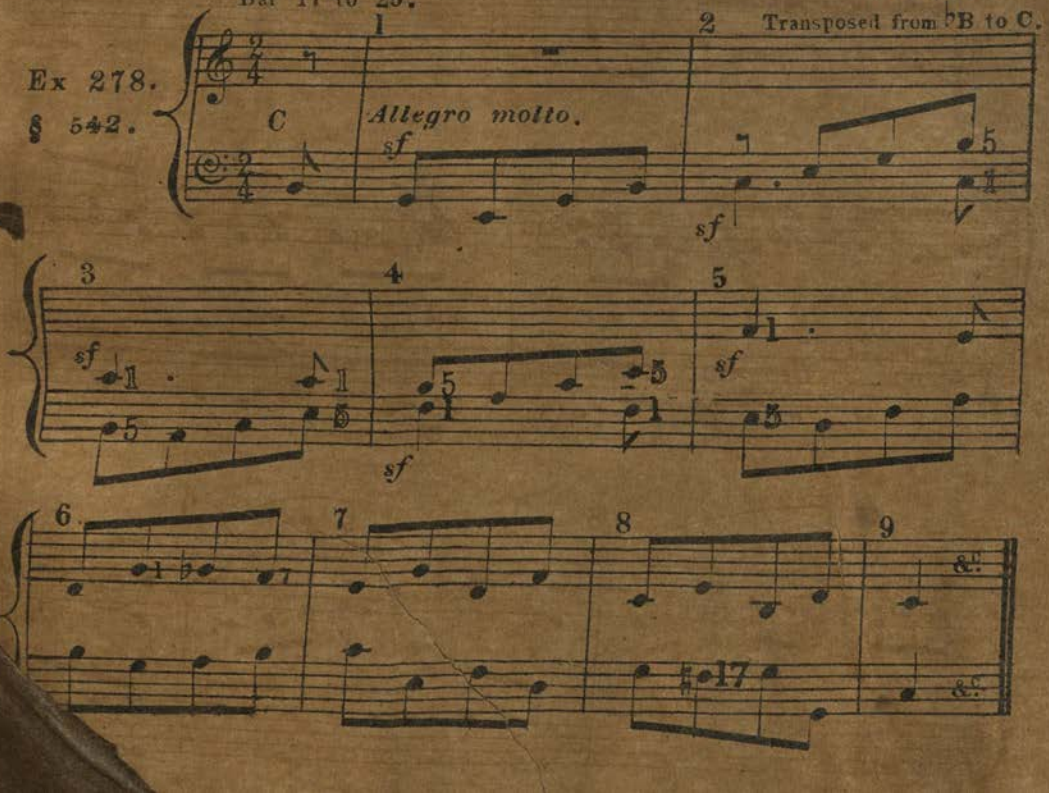
BEETHOVEN.

Bar 17 to 25.

Transposed from bB to C.

Ex 278.

§ 542.

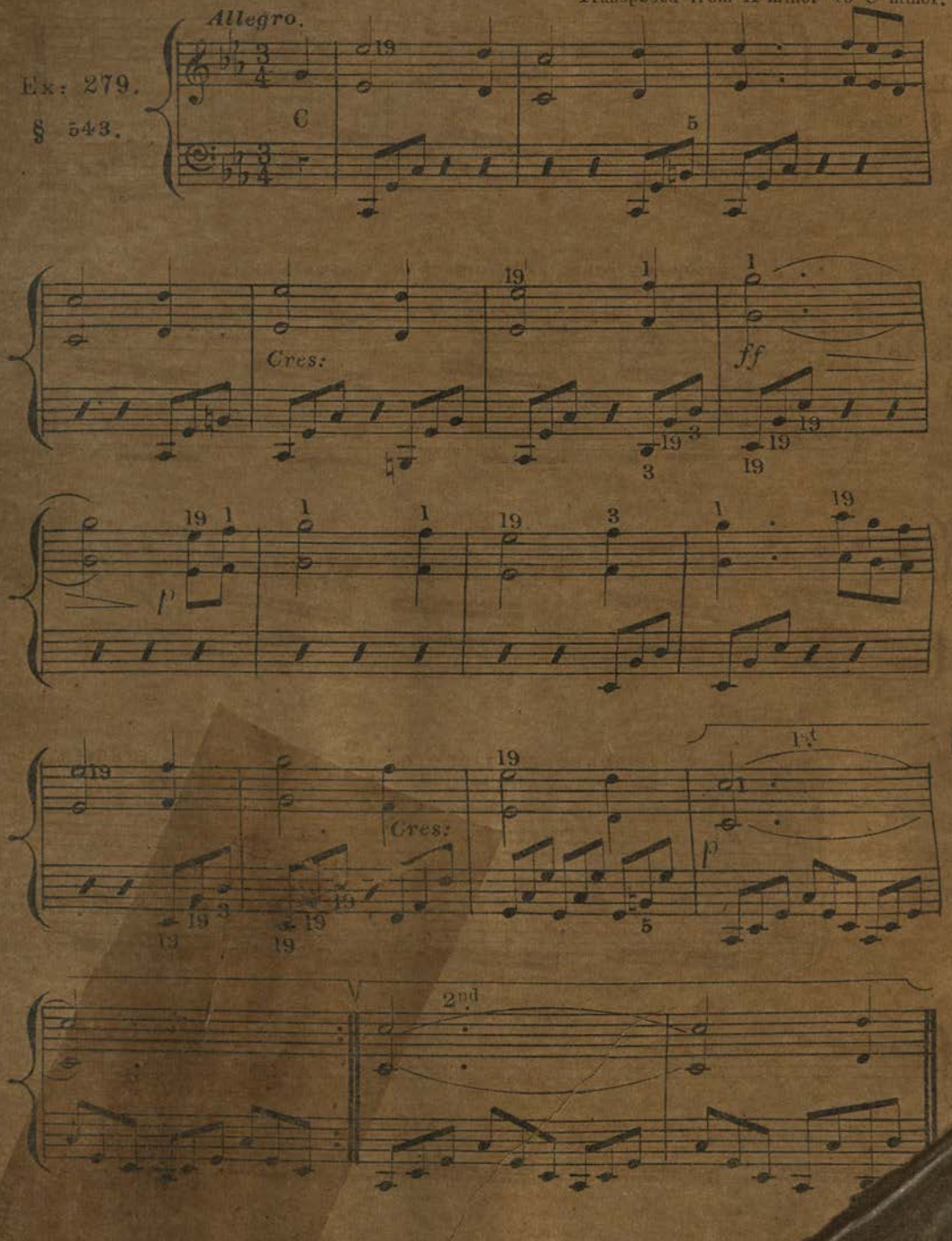


BEETHOVEN'S AIRS OU BAGATELLES Nº 2.

Transposed from A minor to C minor.

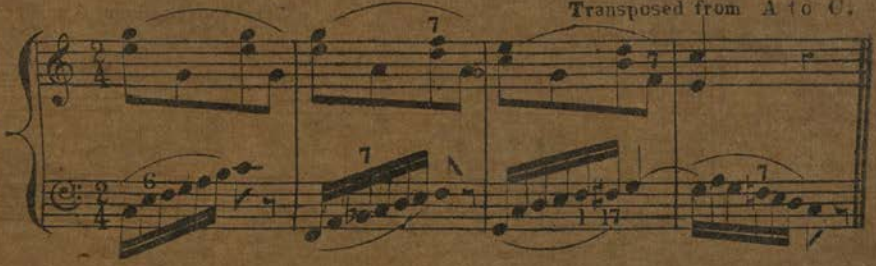
Ex: 279.

§ 543.



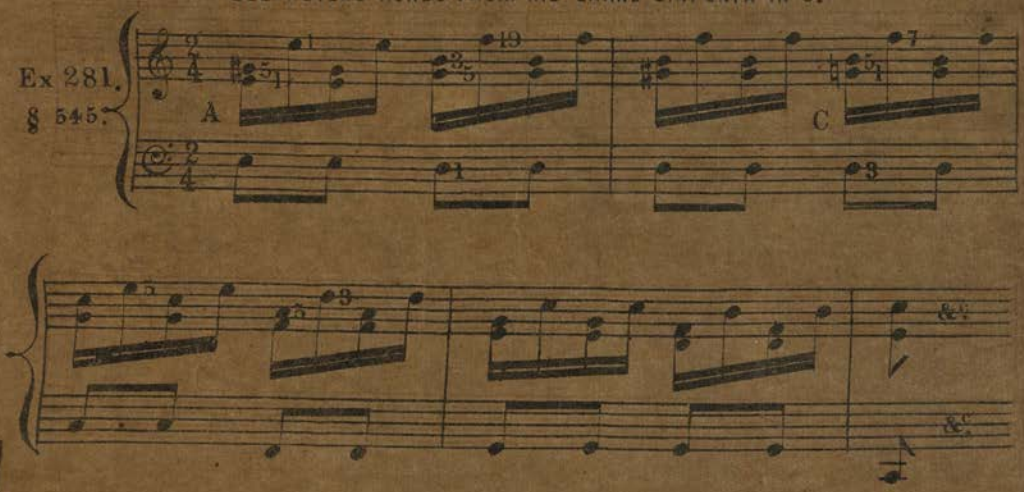
QUANT'E PIU BELLA. VARIS BY BEETHOVEN
Transposed from A to C.

Ex 280.
§ 544.



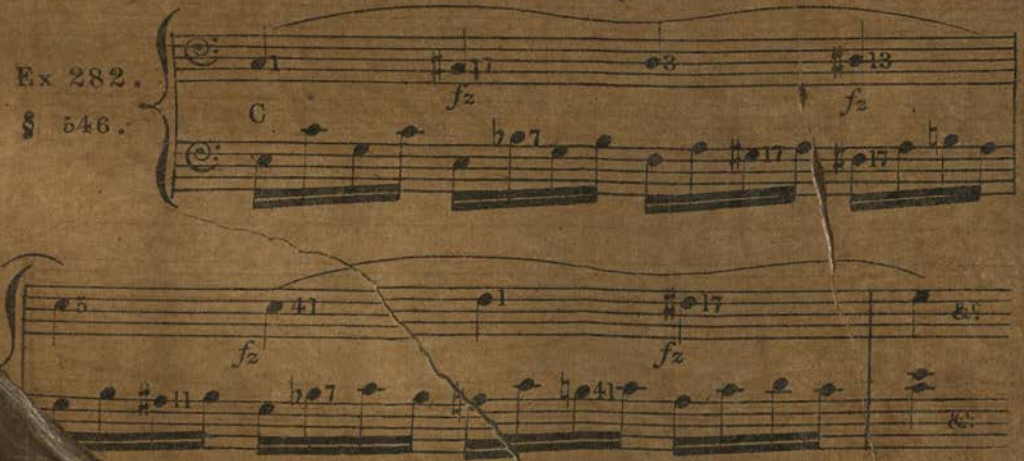
BEETHOVEN'S RONDO FROM HIS GRAND SINFONIA IN C.

Ex 281.
§ 545.



RONDO GRAZIOSO Op: 51. BEETHOVEN.

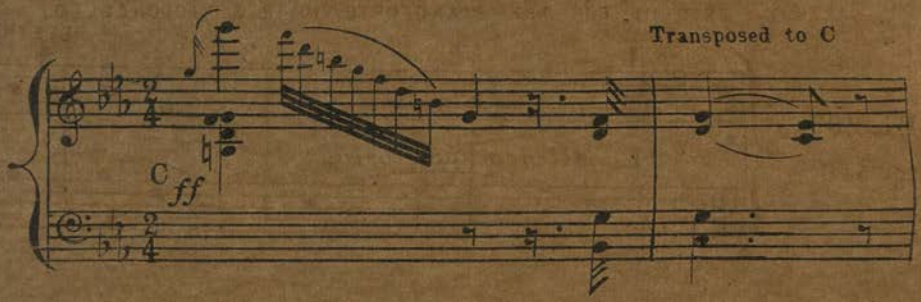
Ex 282.
§ 546.

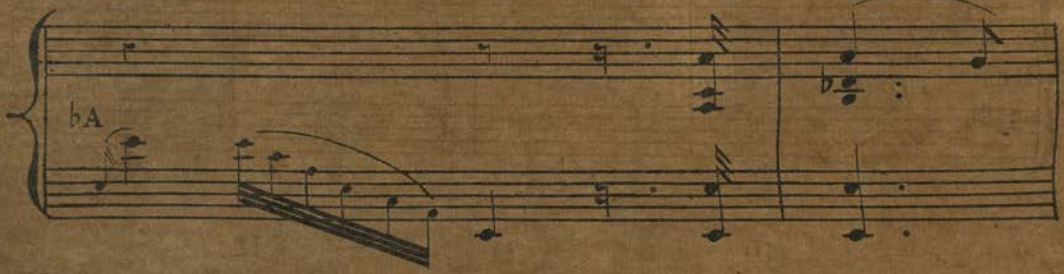


BEETHOVEN'S GRAND SONATA. LETTER C

Transposed to C

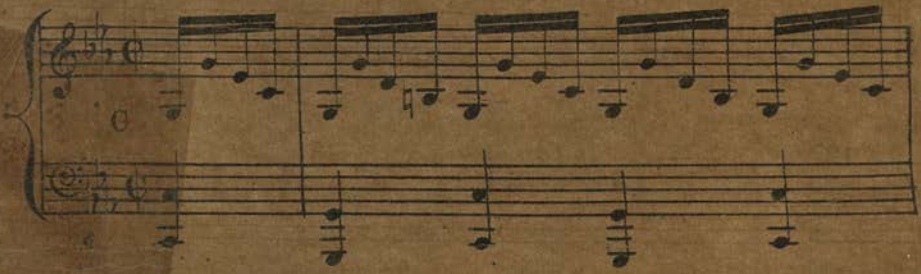
Ex. 283.
§ 547.

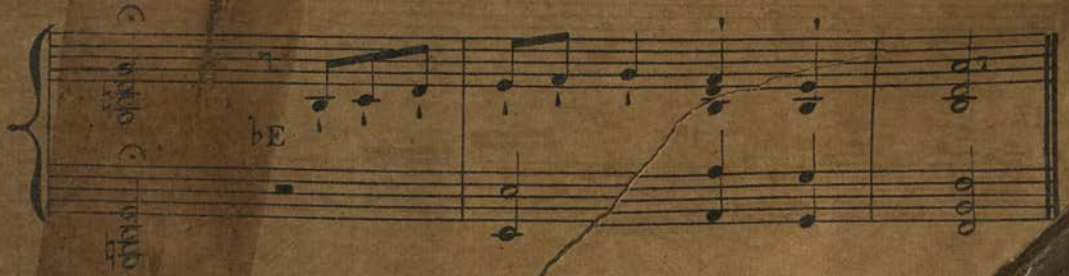




The same.

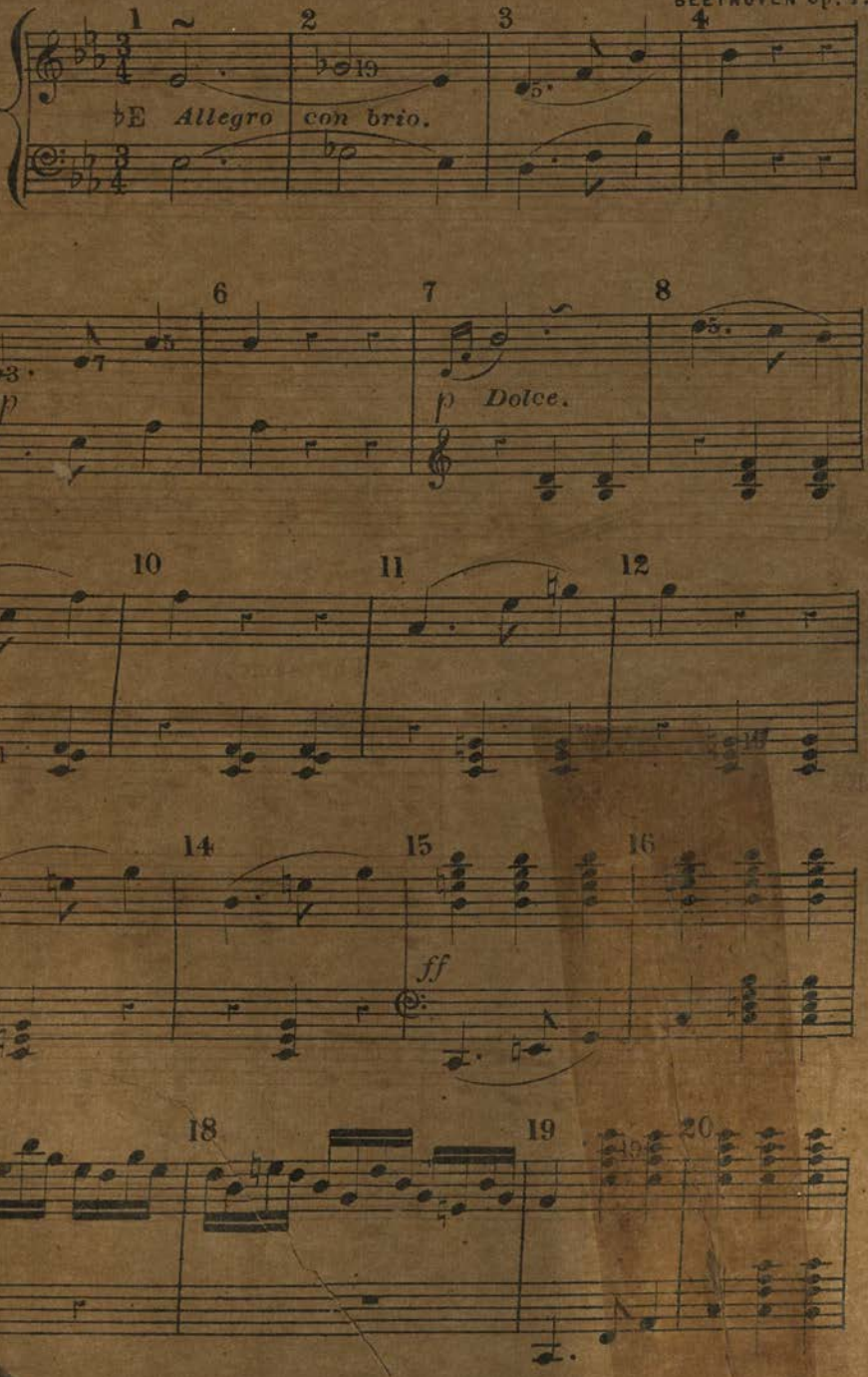
Ex. 284
§ 547

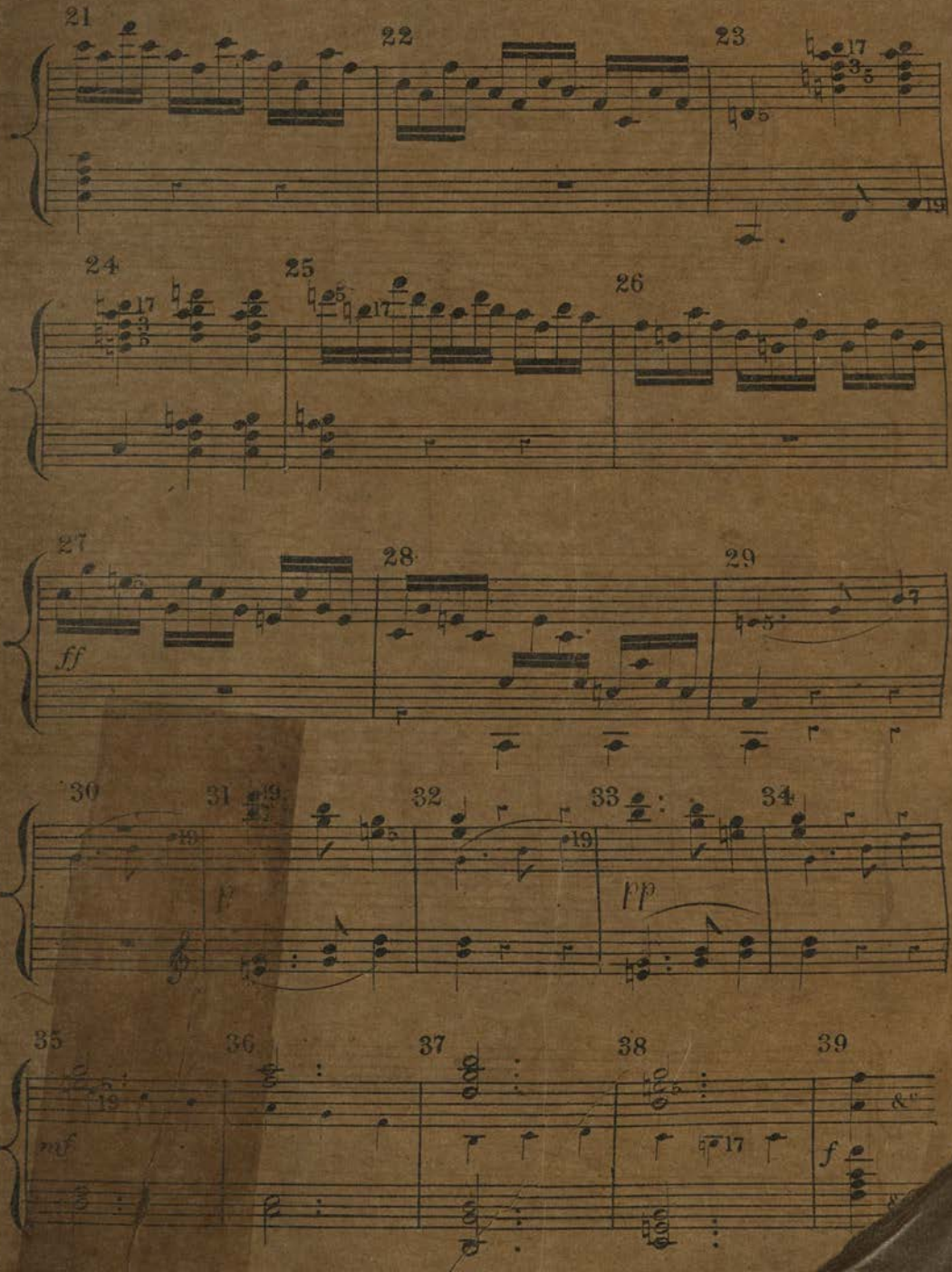




TRIO III FOR THE PIANOFORTE VIOLIN & VIOLONCELLO. BEETHOVEN Op: 1.

Ex: 285.
§ 548.




21
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39
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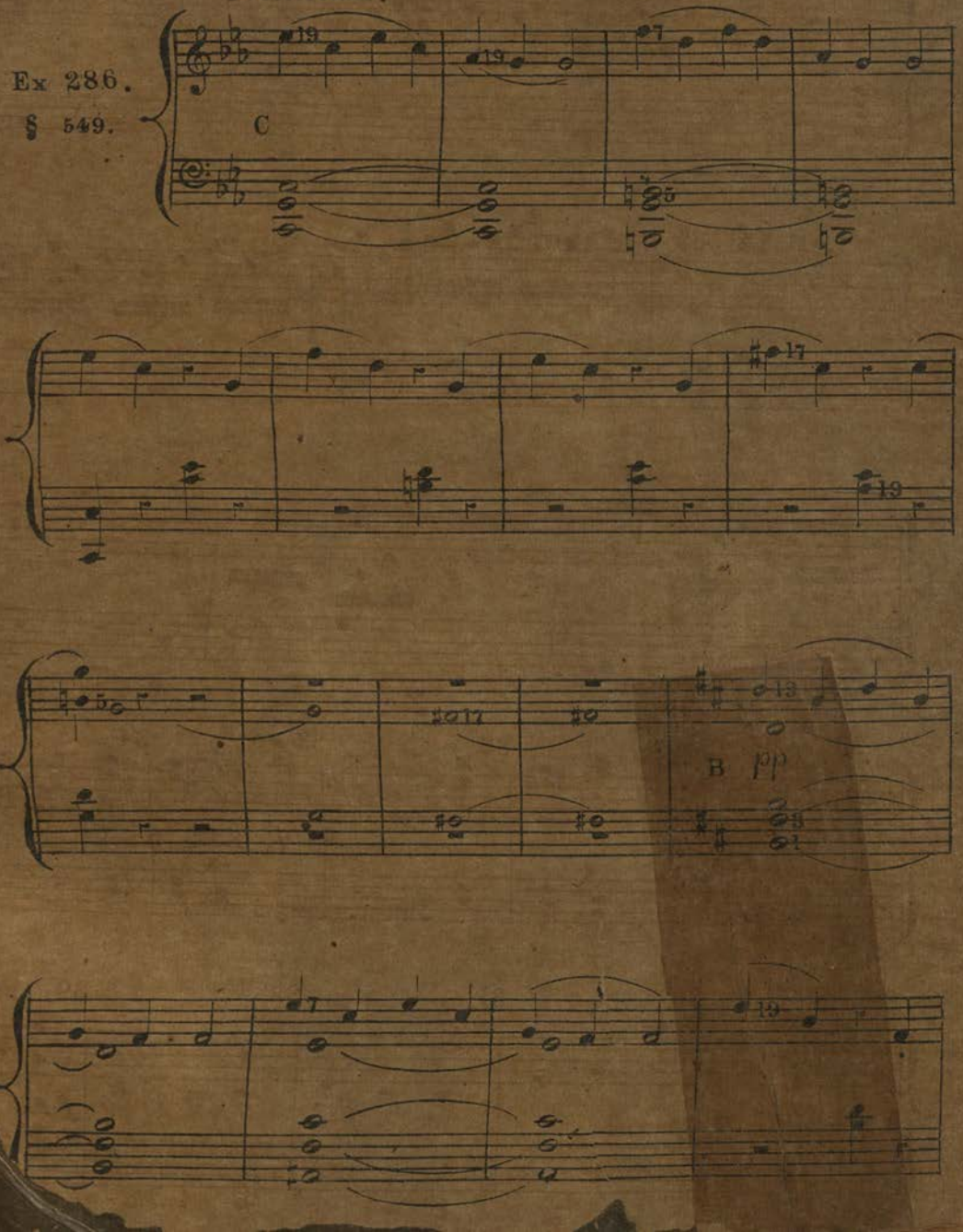
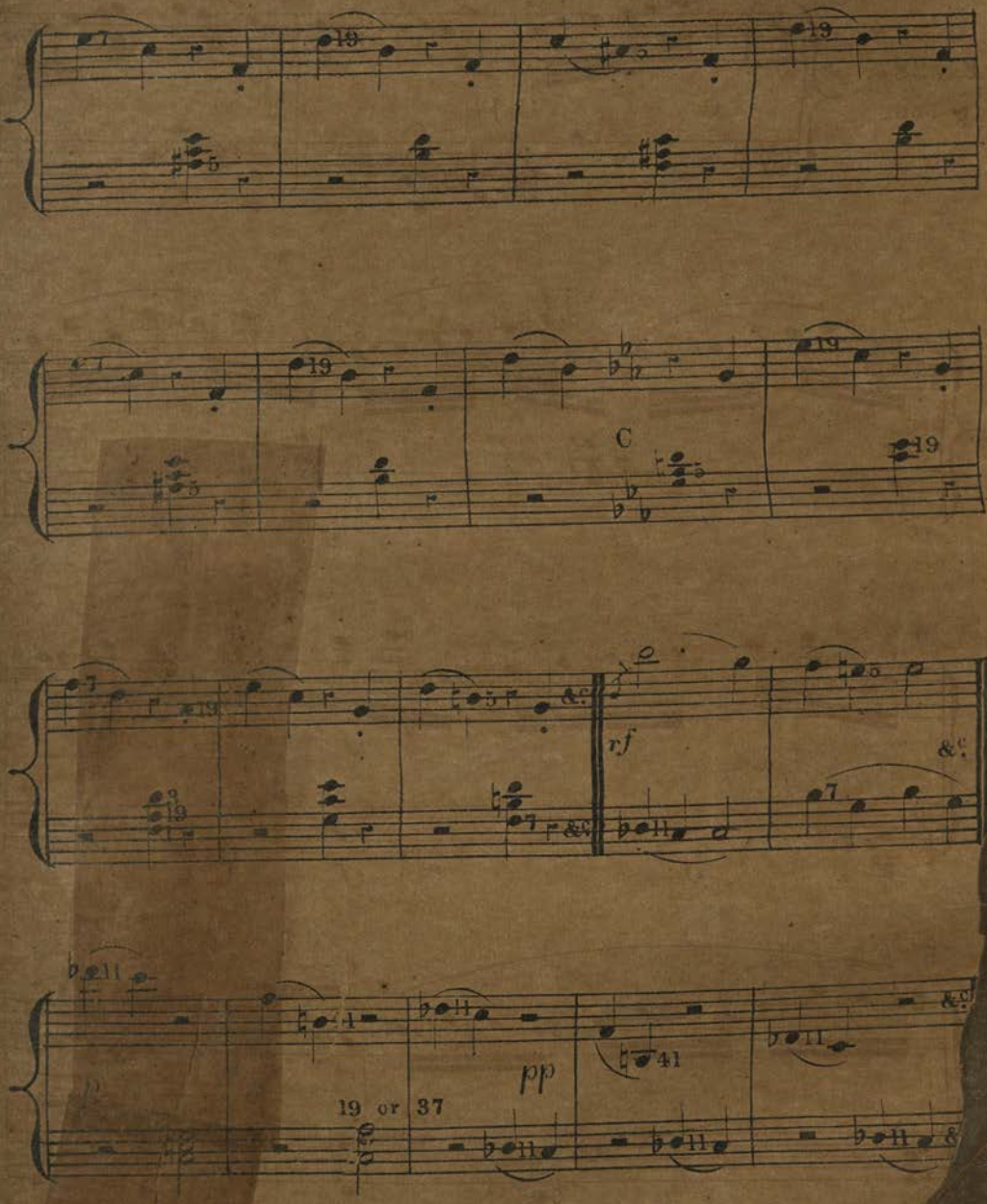
p
pp
mf
f

FINALE TO THE SAME.

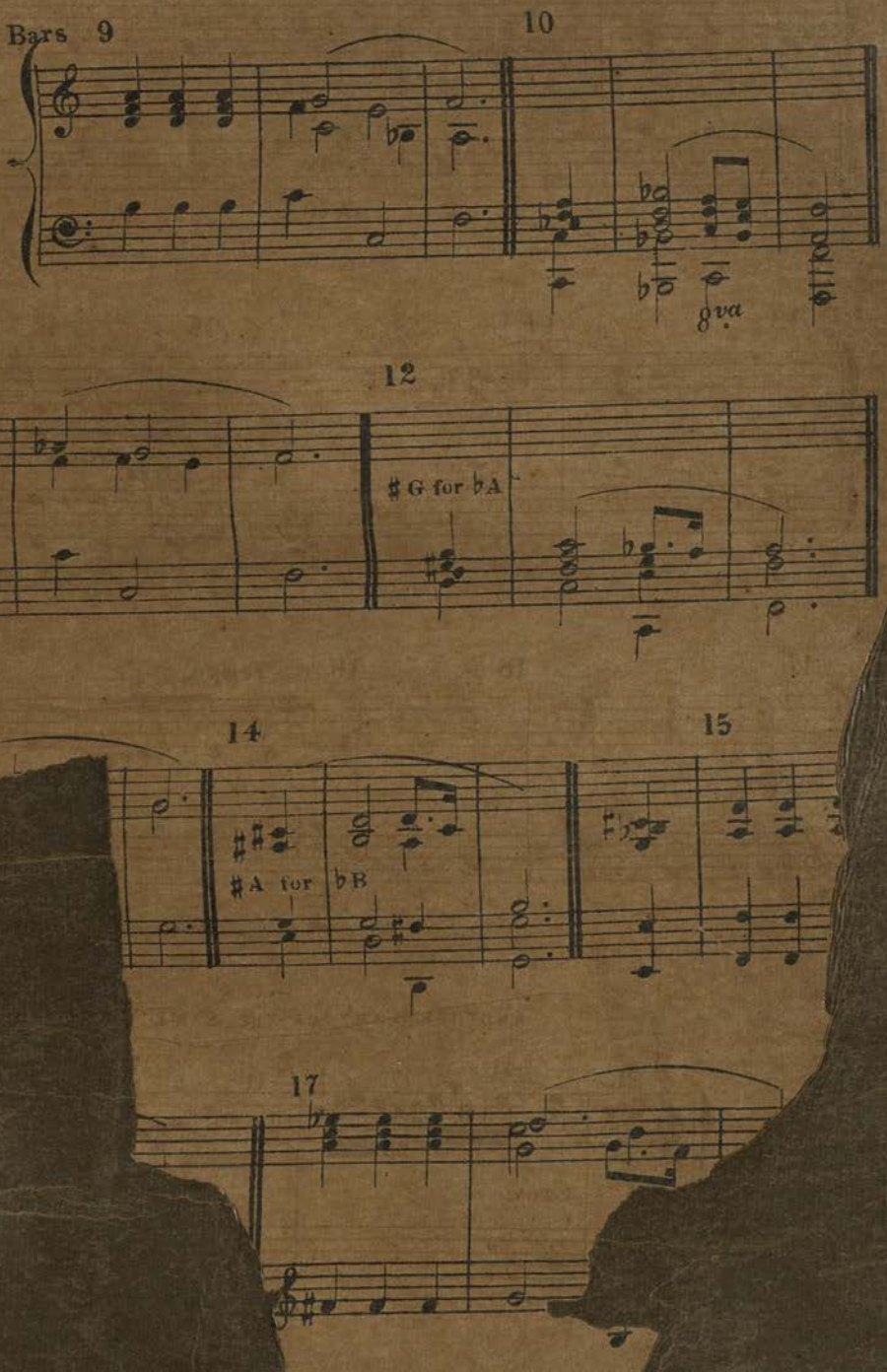
Prestissimo.

Ex 286.
§ 549.



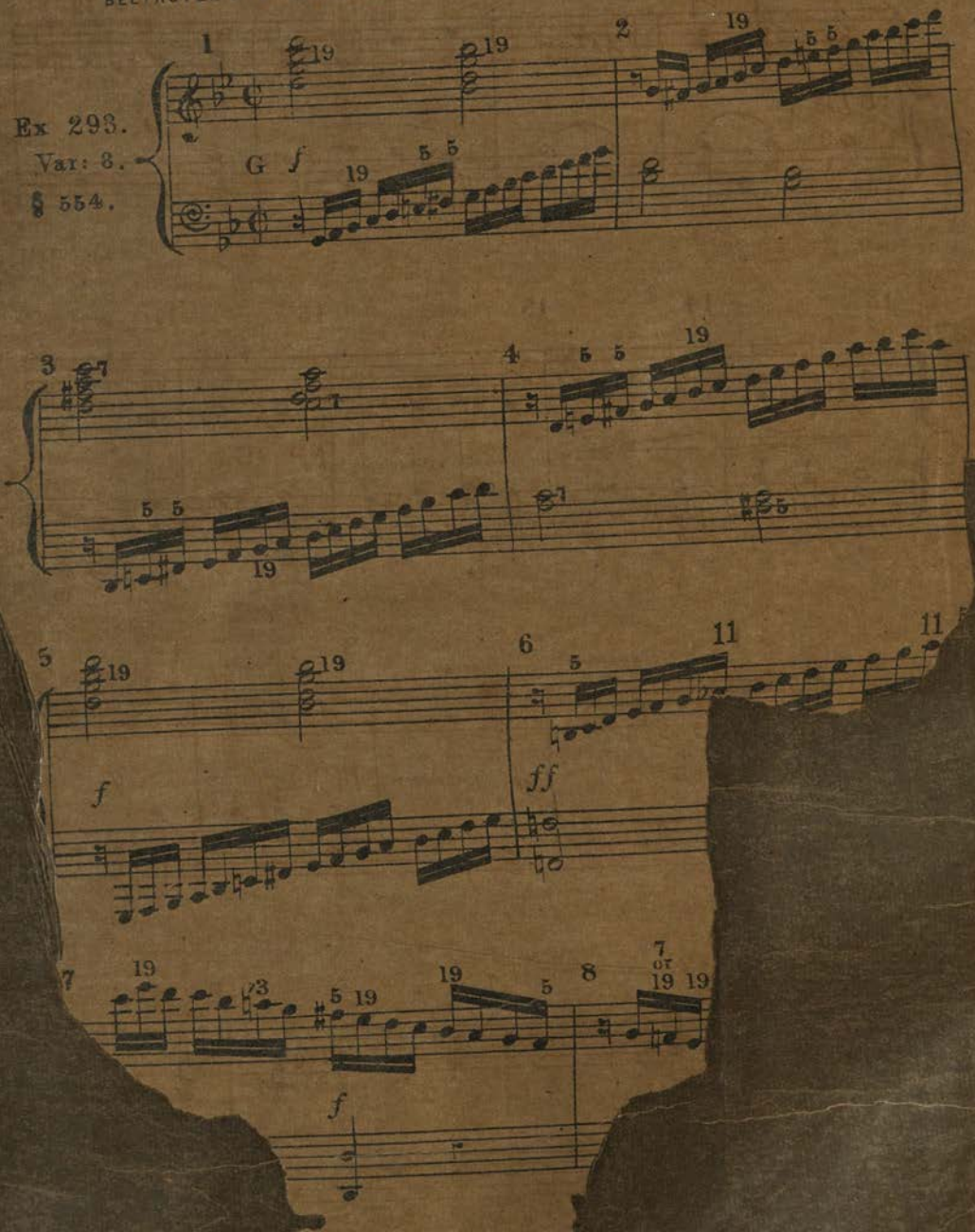

C
rf
pp
19 or 37

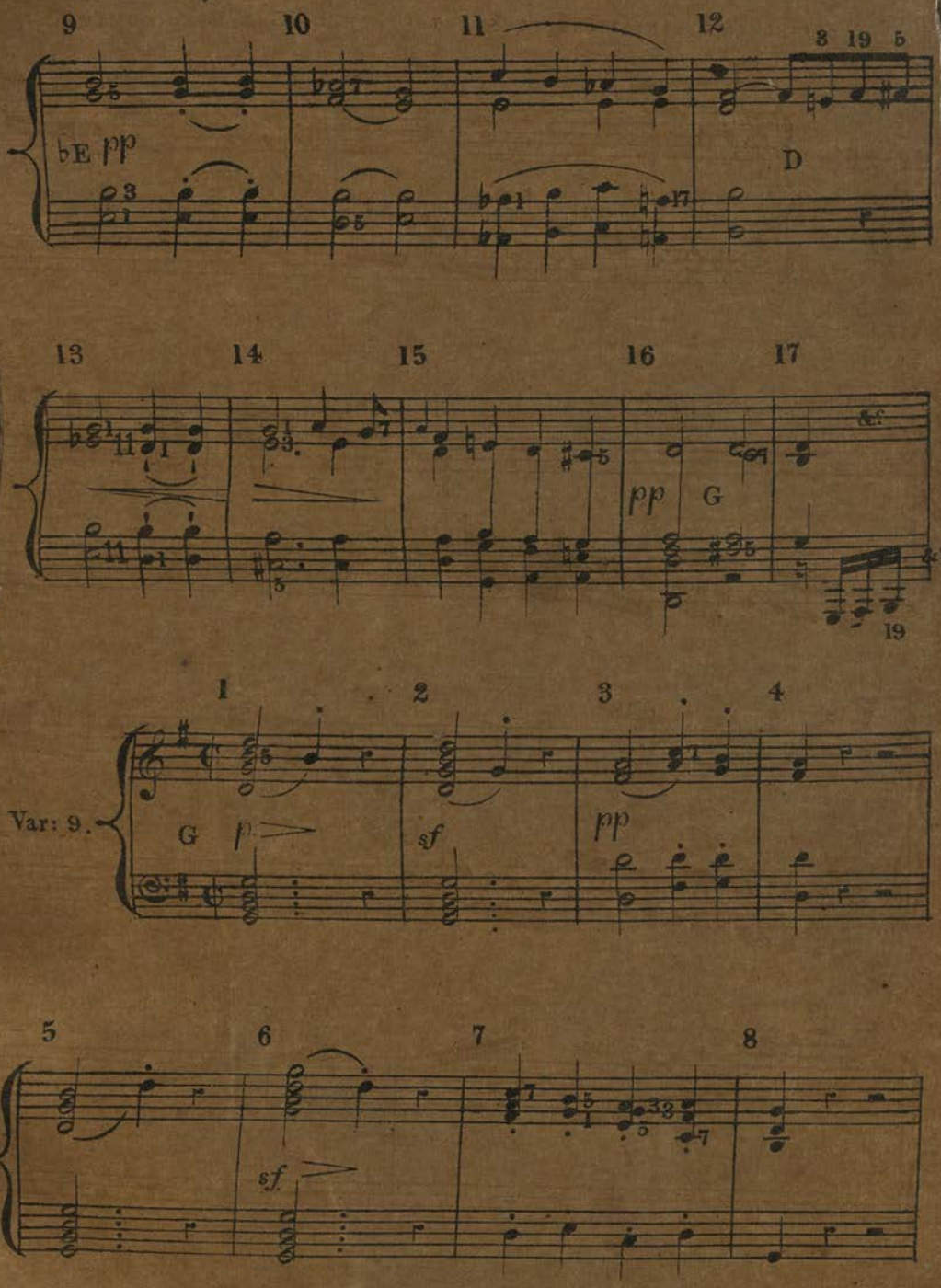
Ex 292.
§ 552.



BEETHOVEN'S VARIATIONS TO "SEE THE CONQUERING HERO COMES."

Ex 293.
Var: 8.
§ 554.




9
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G
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Var: 9.
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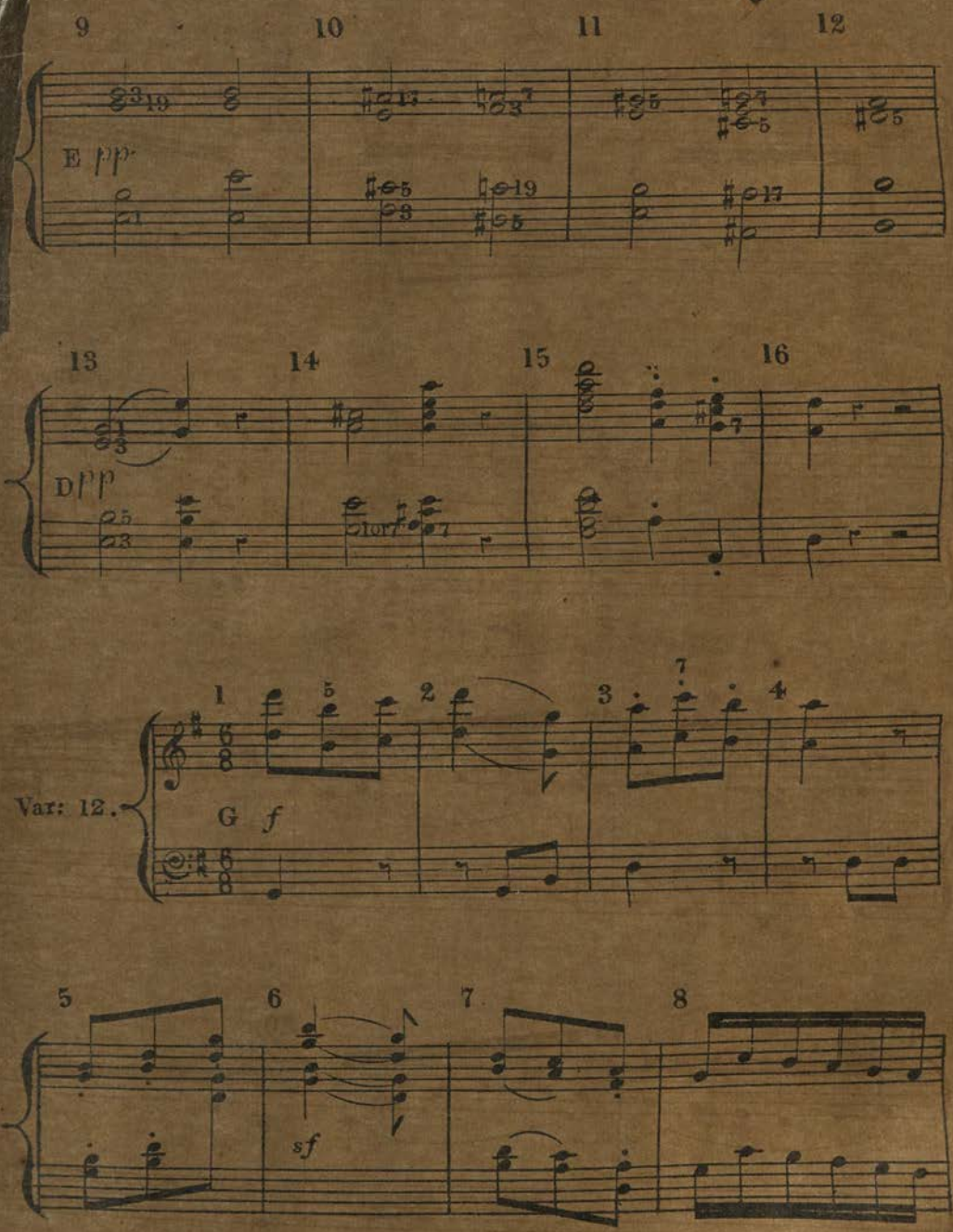
8
sf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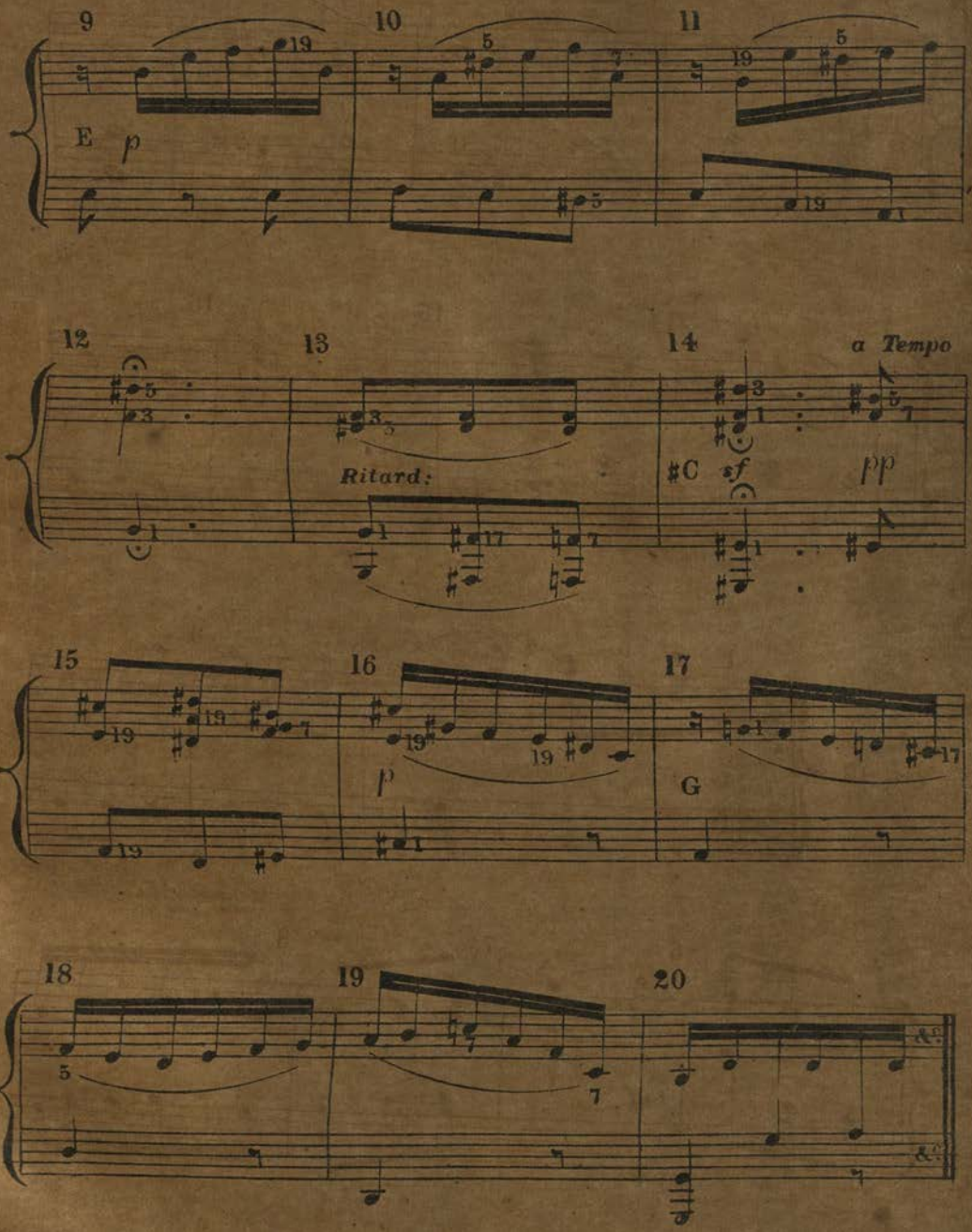
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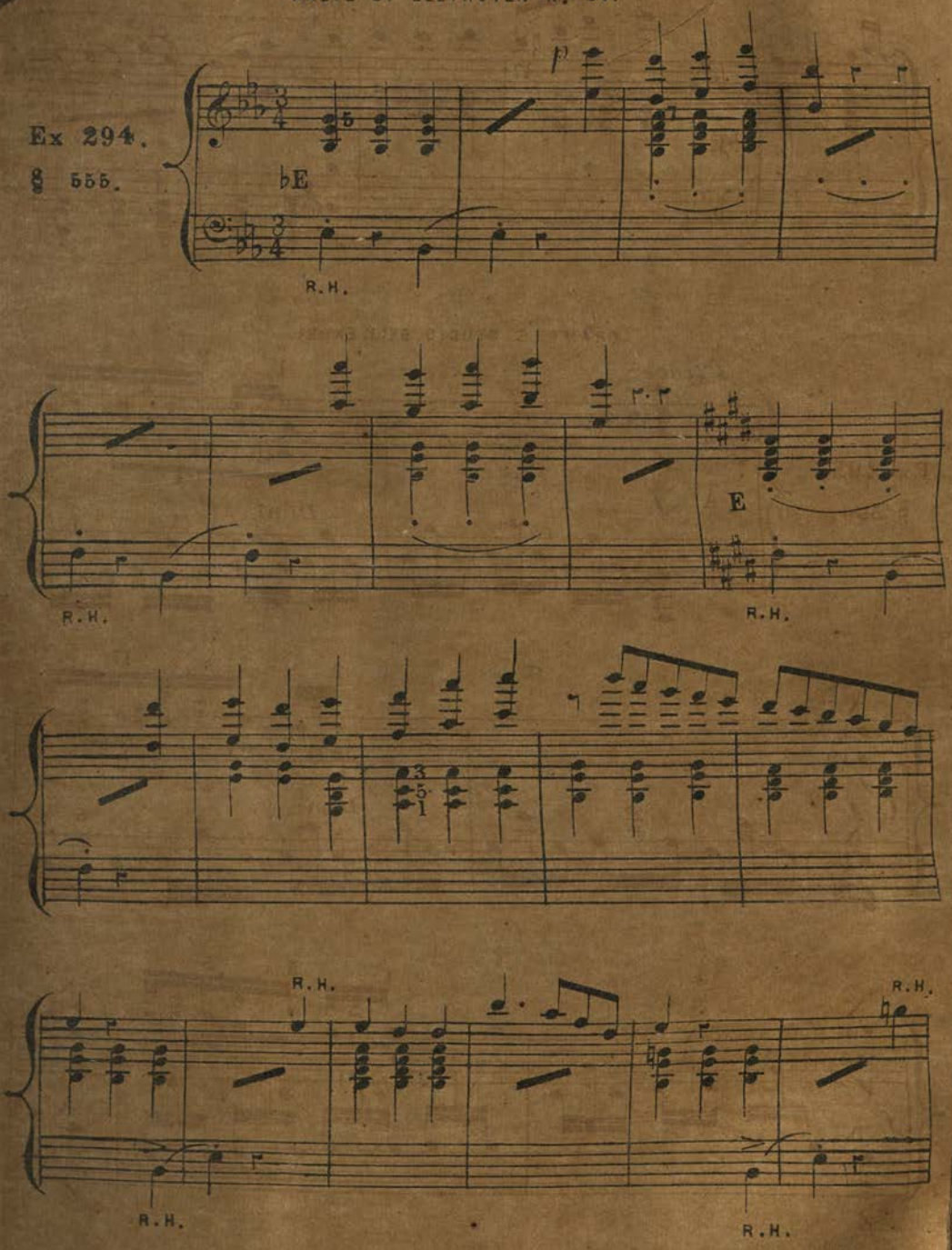
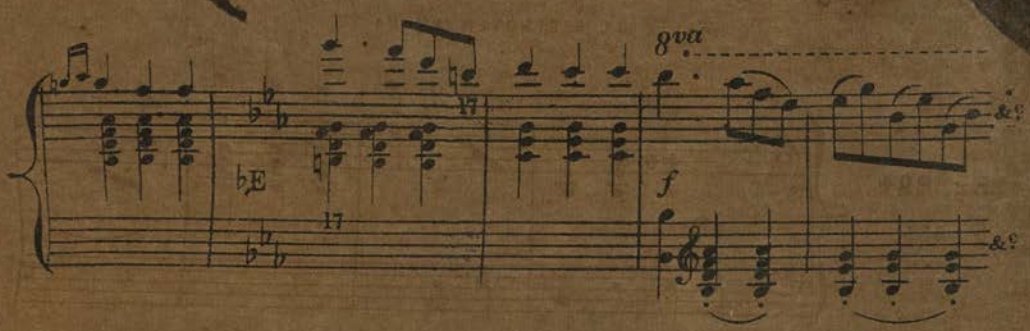
WALTZ BY BEETHOVEN Nº 24.

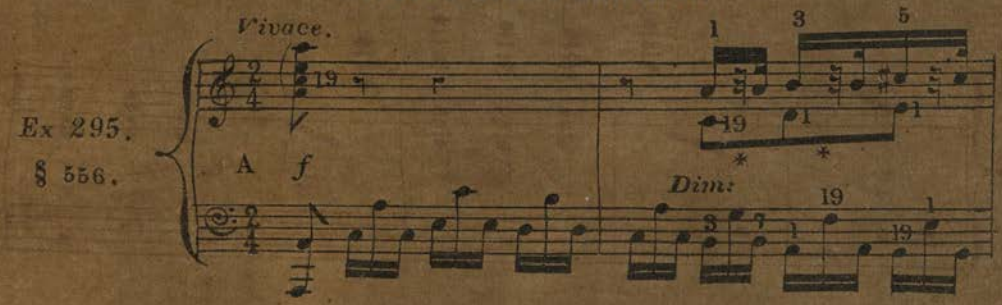
Ex 294.

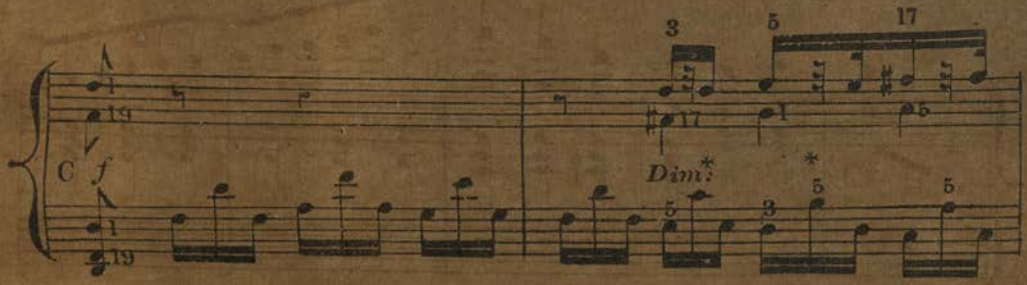
§ 555.

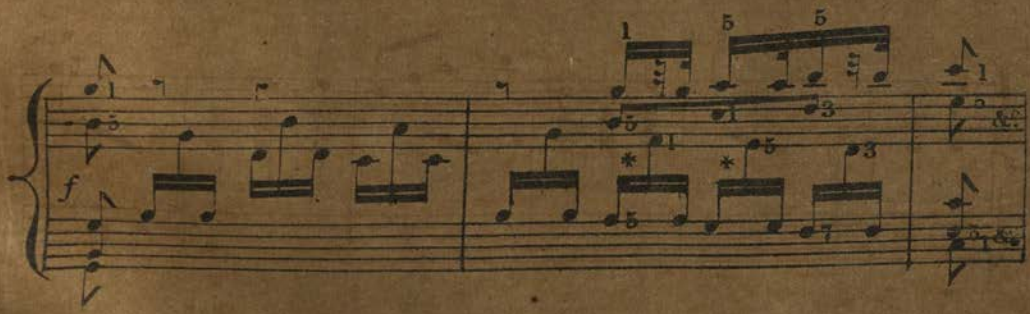




CRAMER'S STUDIO BK: I. EX: VI.







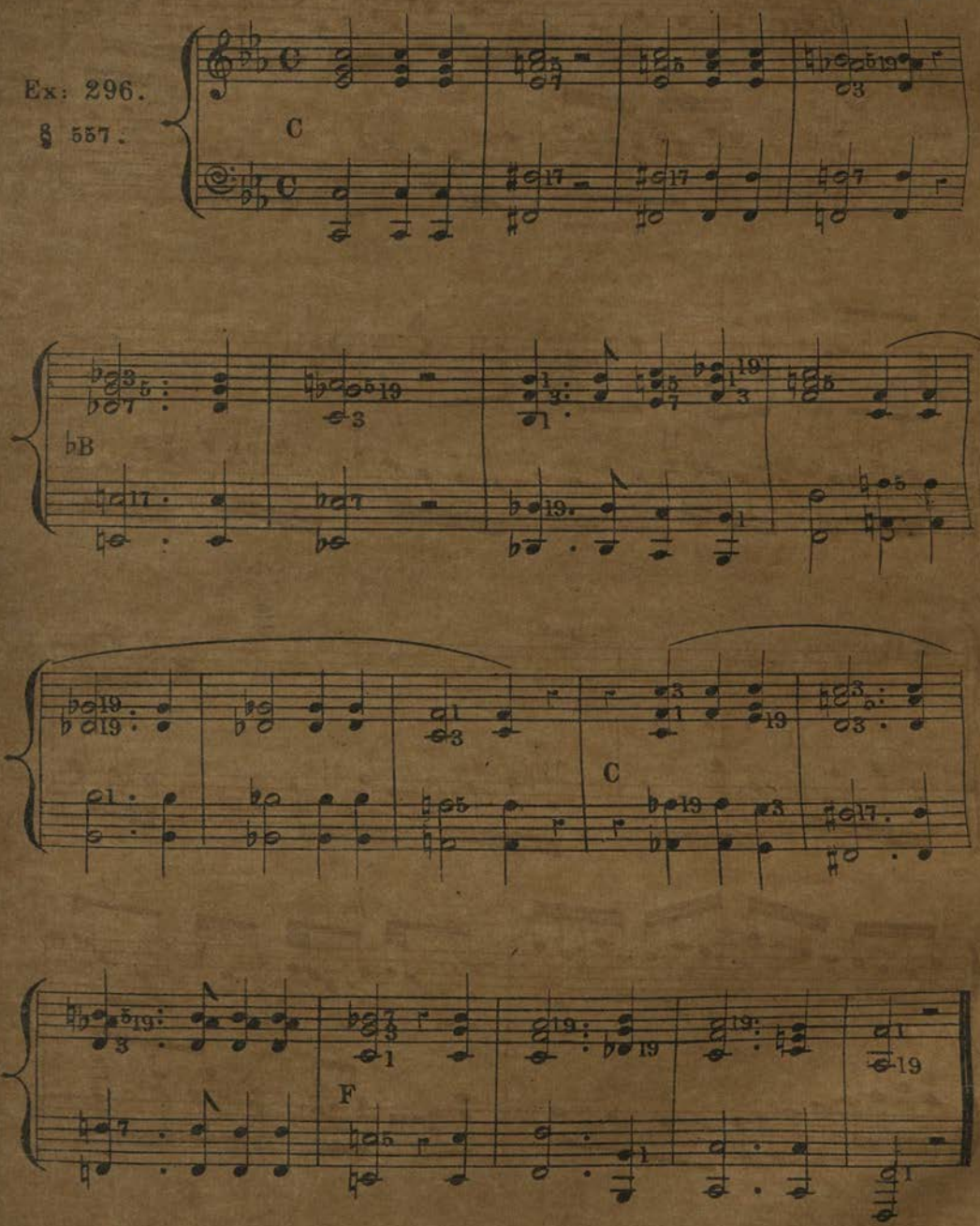
O DREADFUL DREAM. IPHIGENIA IN TAURIDE.

Transposed

GLUCK.

Ex: 296.

§ 557.

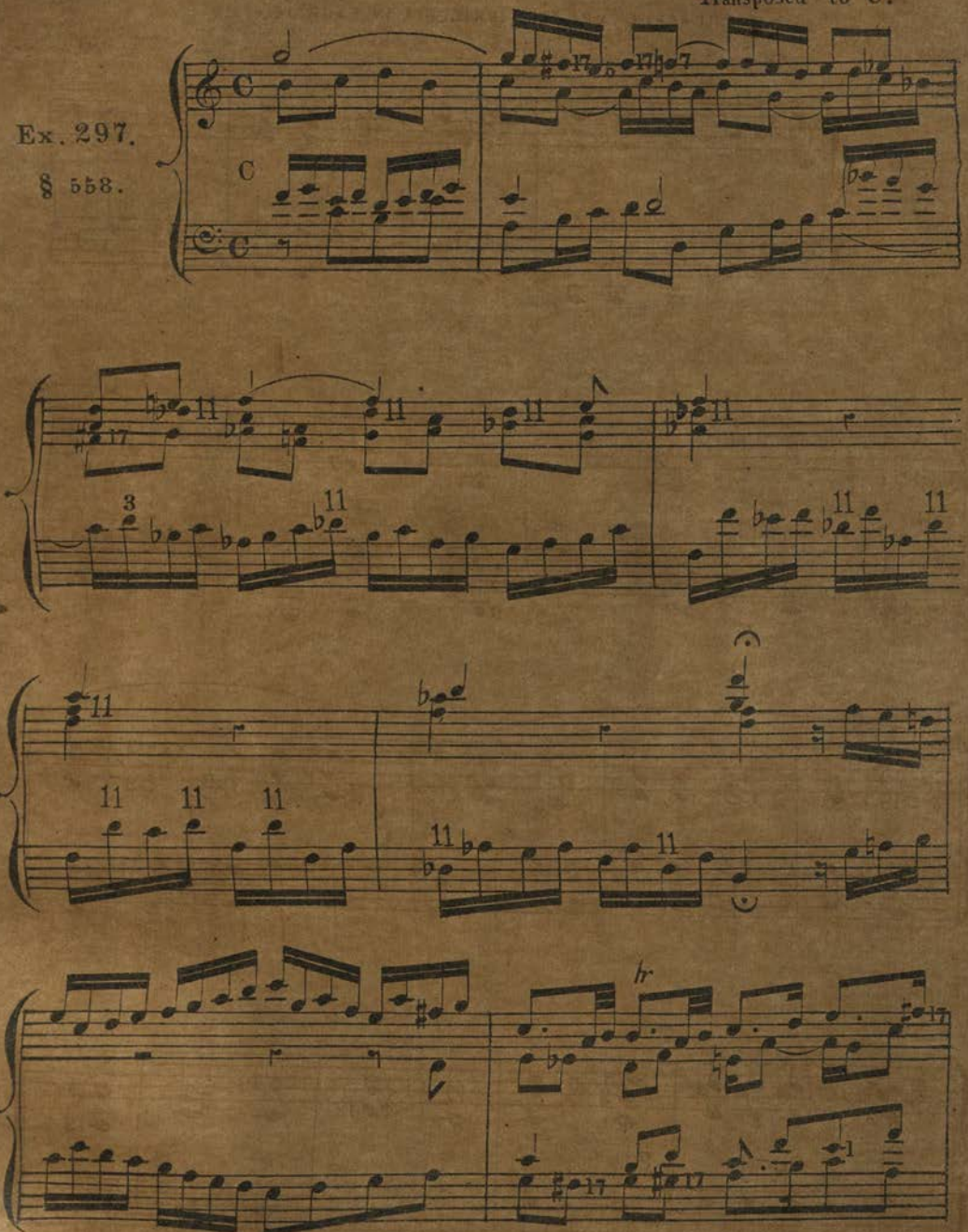


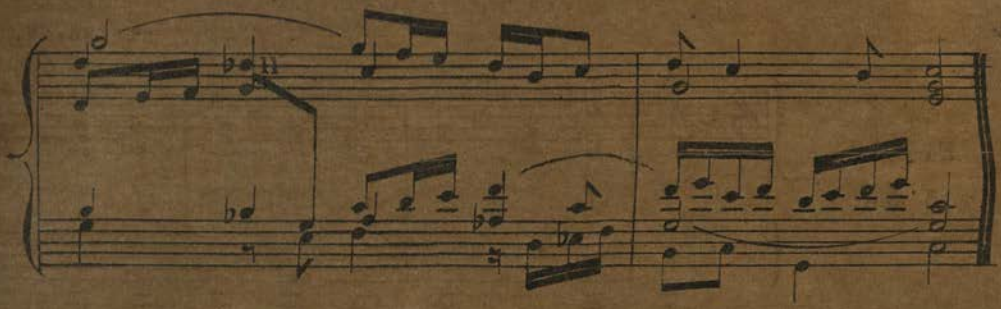
SEB: BACH'S FUGUE IN bA Nº 41.

Transposed to C.

Ex. 297.

§ 558.



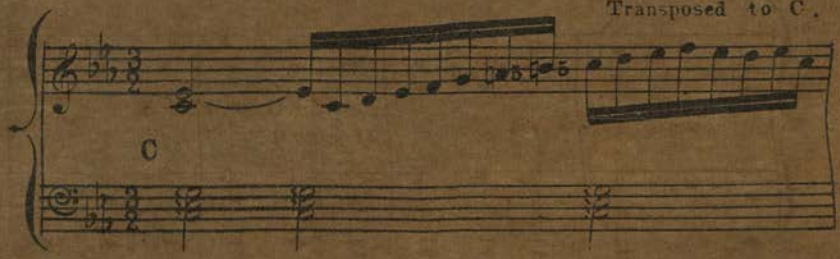


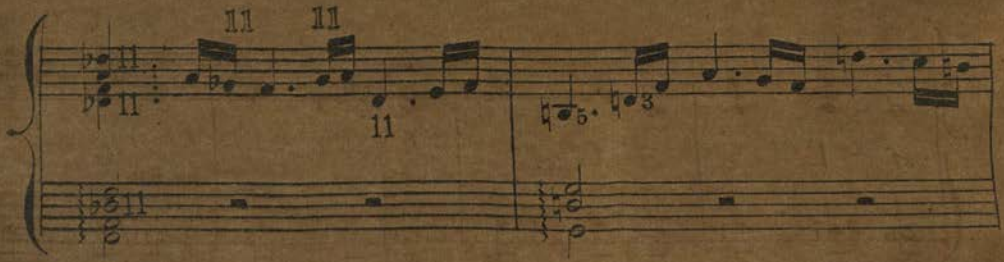
SEB: BACH'S PRELUDE IN DE. Nº 8.

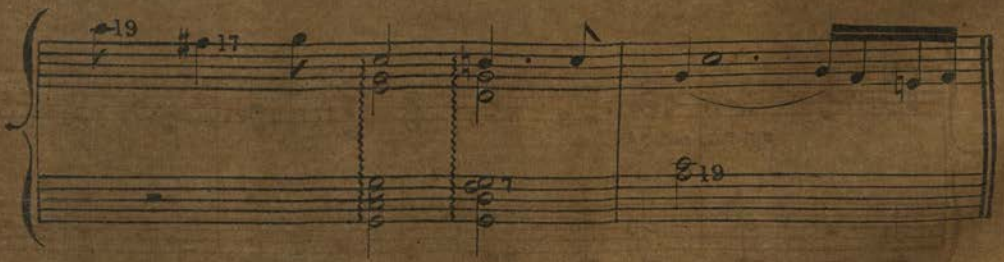
Transposed to C.

Ex. 298.

§ 558.





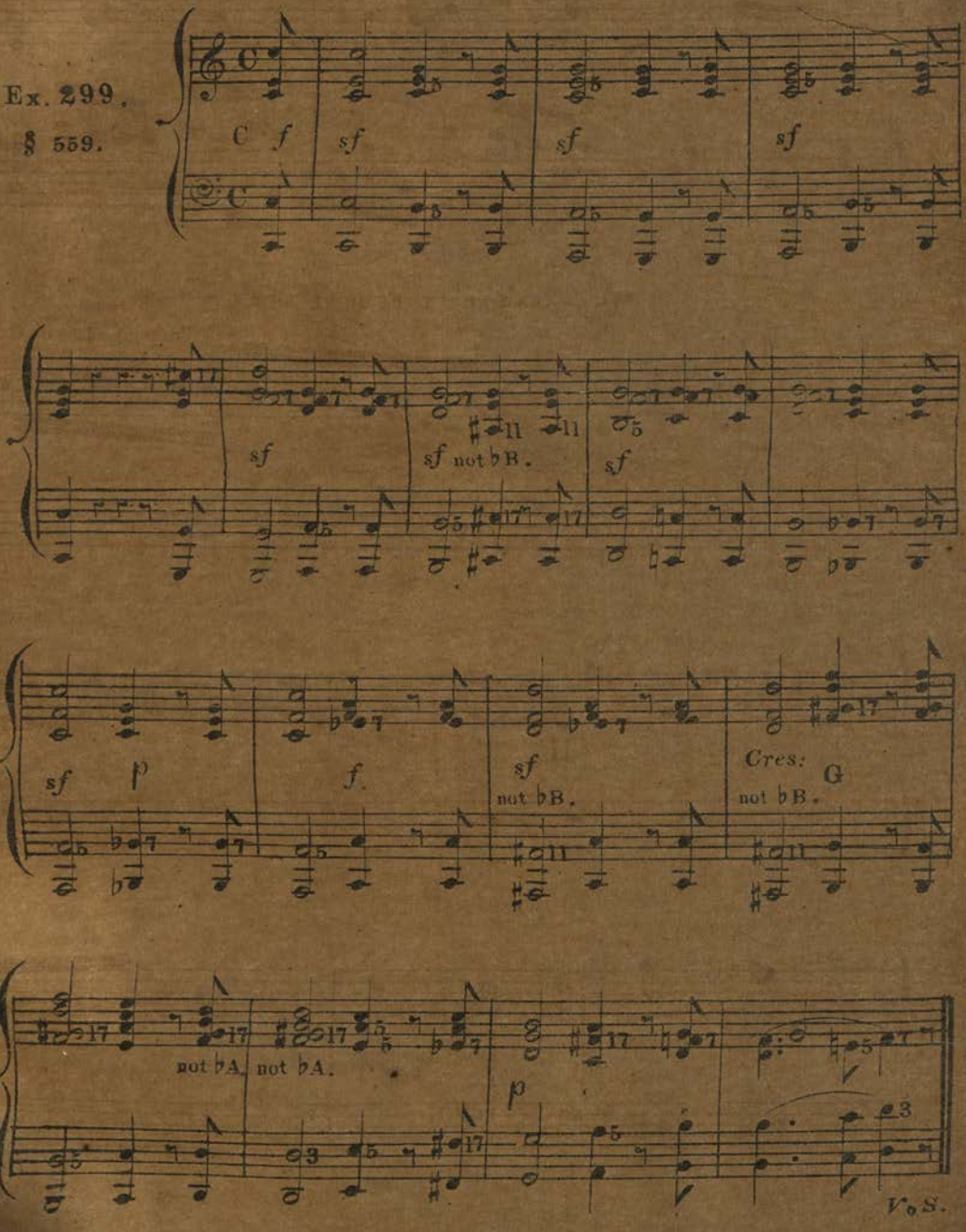


VARIATIO EX 183.

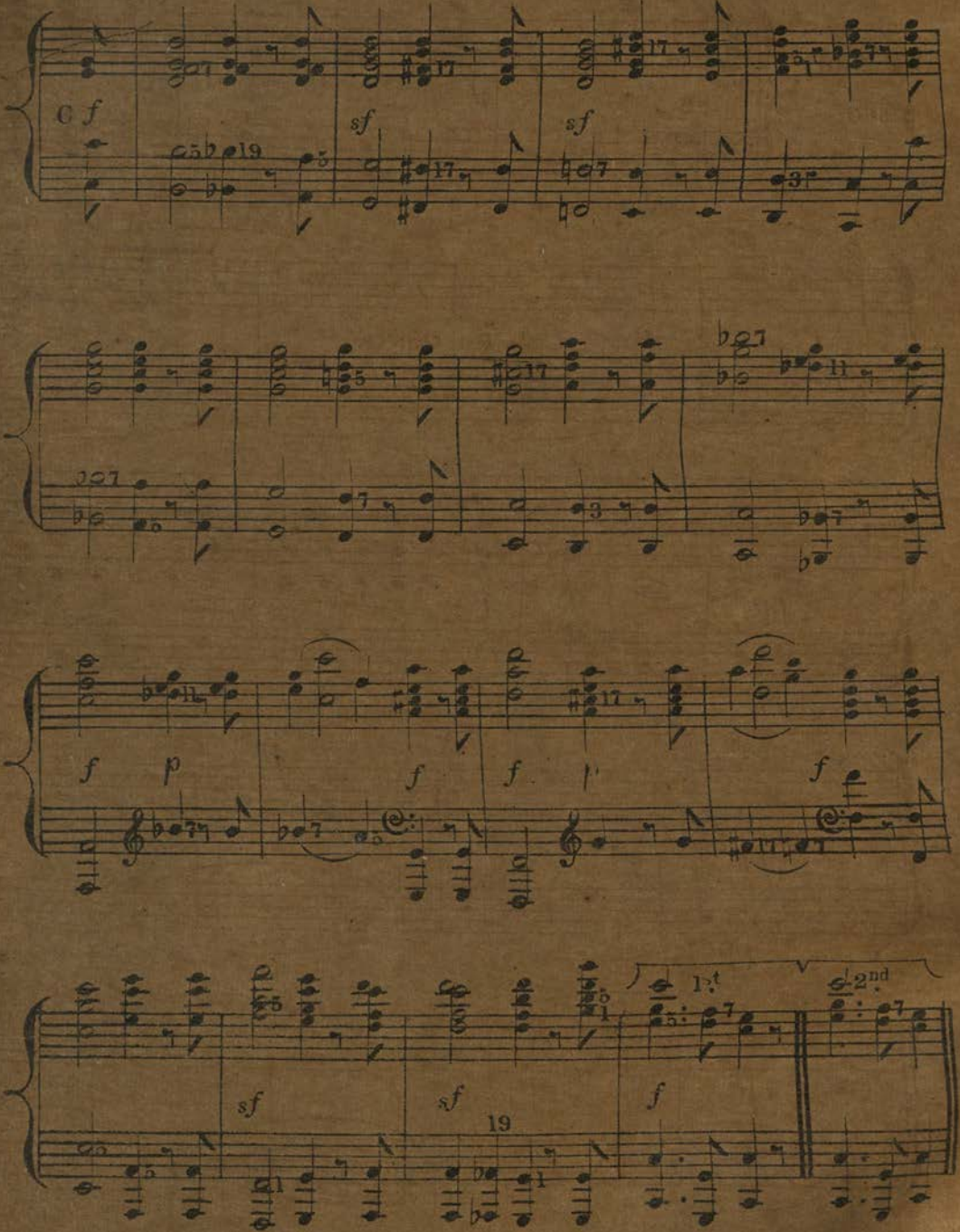
BEETHOVEN.

Ex. 299.

§ 559.



V. S.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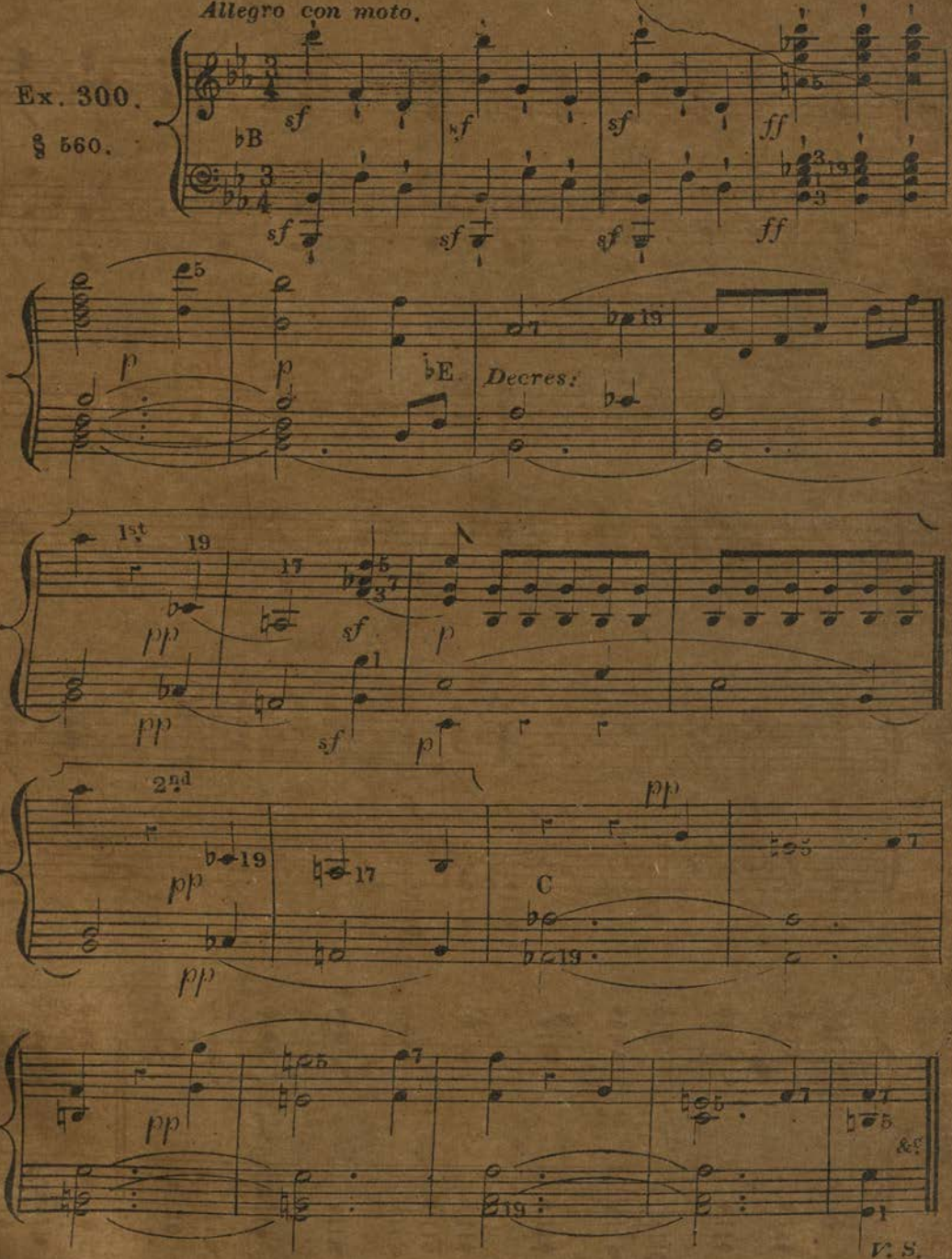
2nd

BEETHOVEN'S GRAND SYMPHONY EROICA.

Allegro con moto.

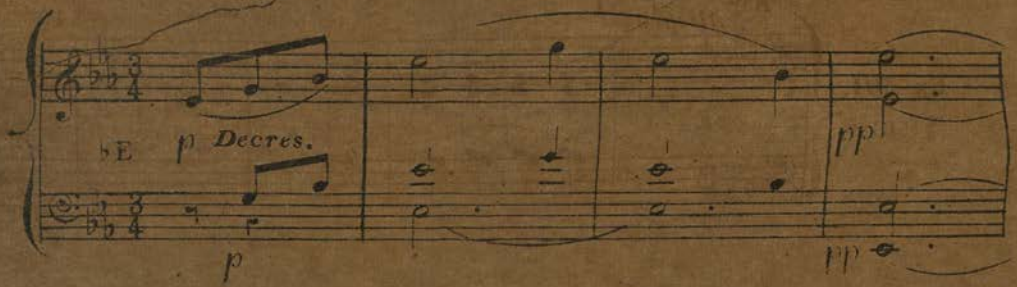
Ex. 300.

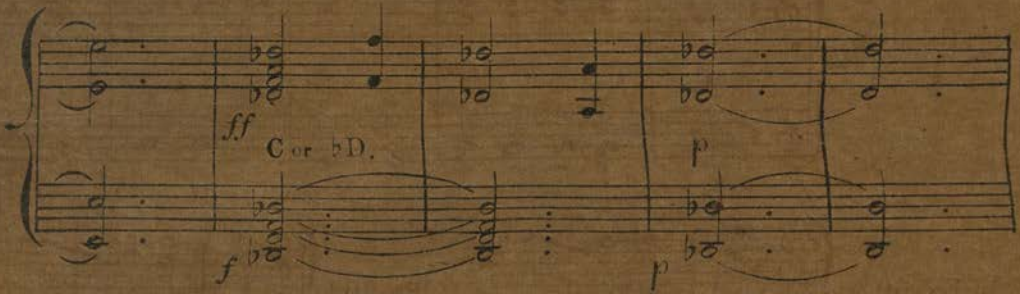
§ 560.

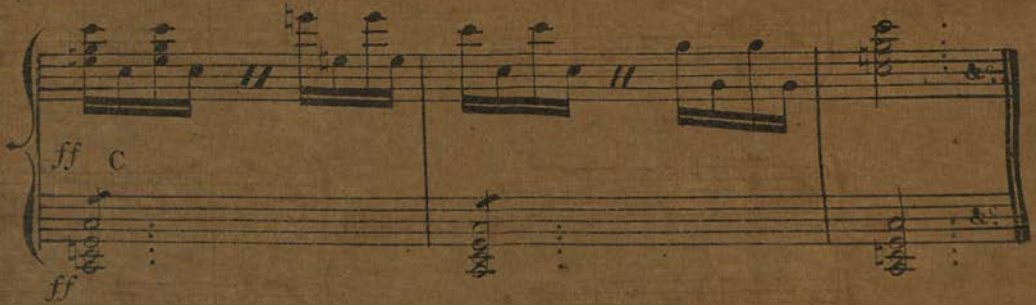


V. S.

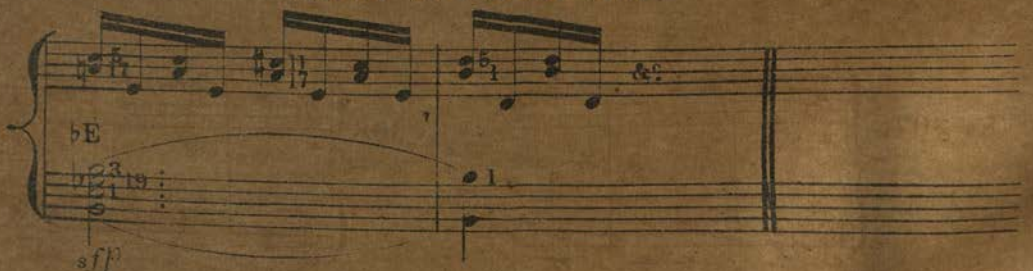
ANOTHER PART OF THE SAME







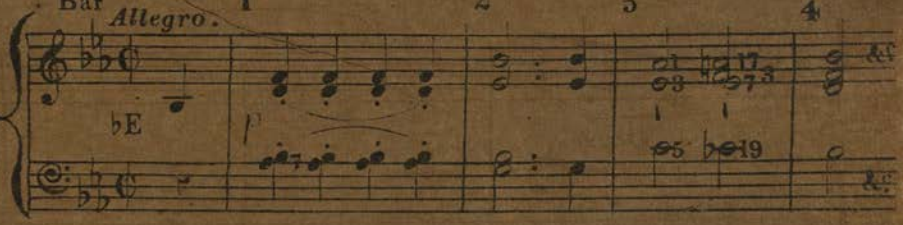
ANOTHER PART OF THE SAME.



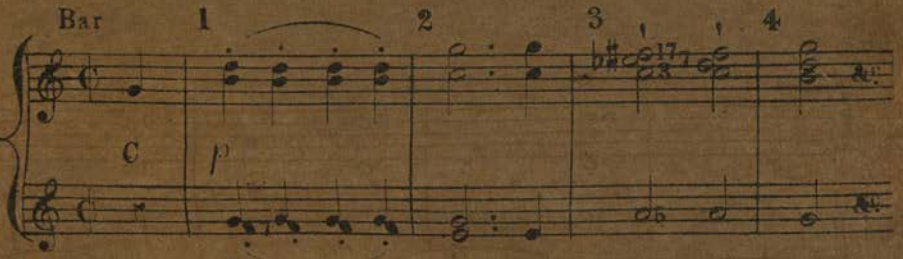
GRAND SONATA PATHETIQUE.

BEETHOVEN

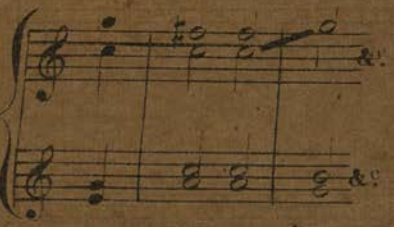
Ex 301. § 561.



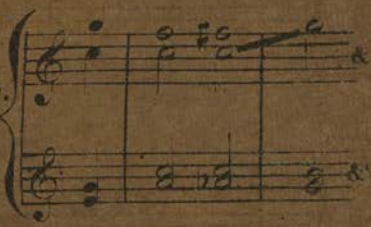
Ex 302. § 561.



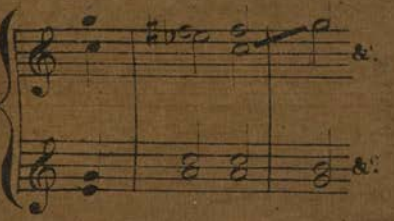
Ex 303. § 561.



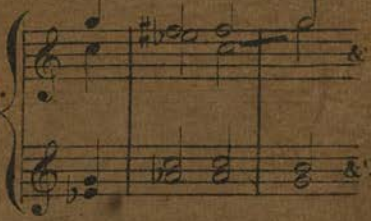
Ex 304.



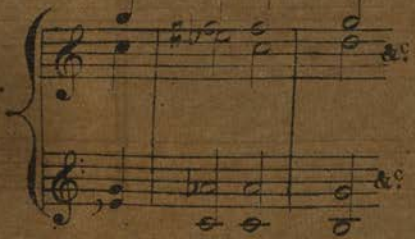
Ex 305.



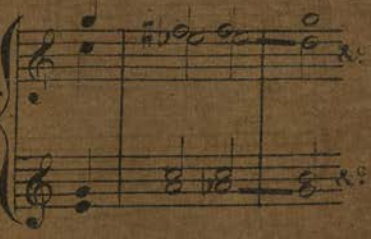
Ex 306.



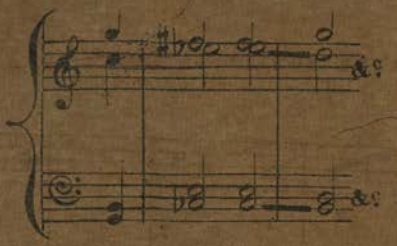
Ex 307.



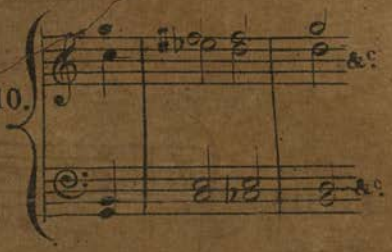
Ex 308.



Ex 309.

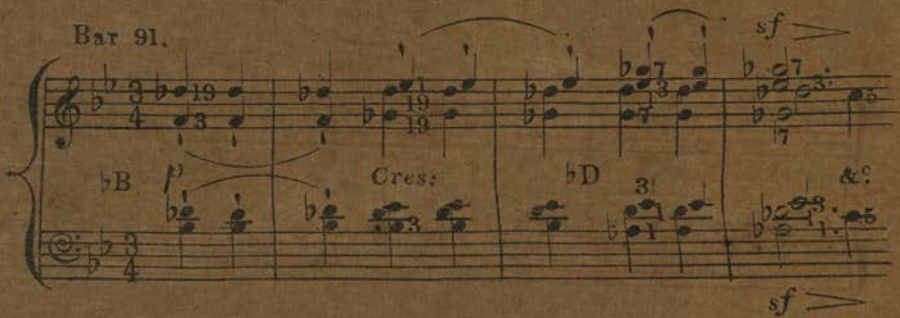


Ex 310.

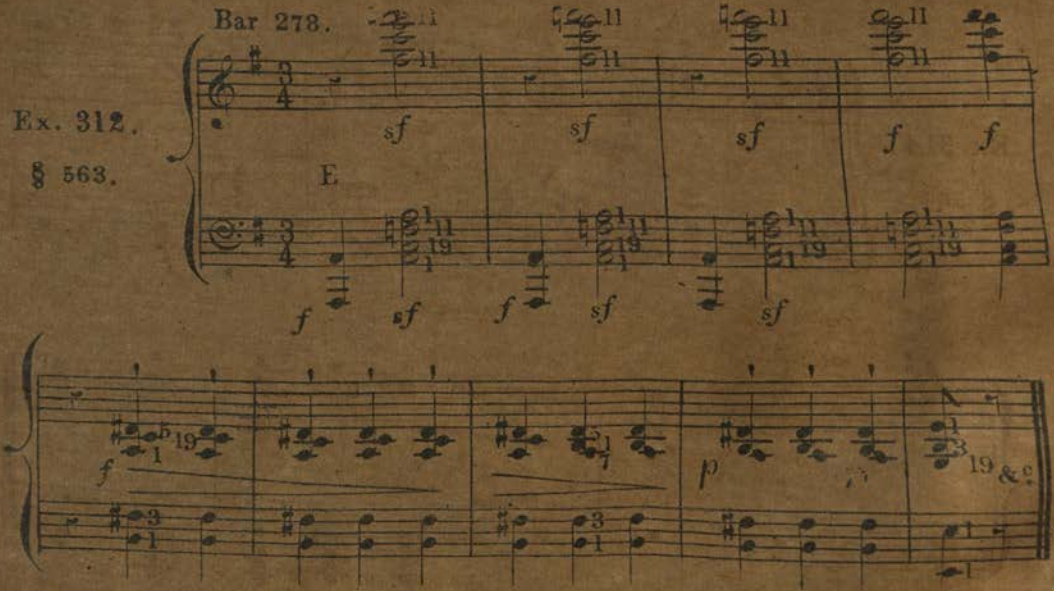


BEETHOVEN'S GRAND SYMPHONY EROICA.

Ex. 311.
§ 562.

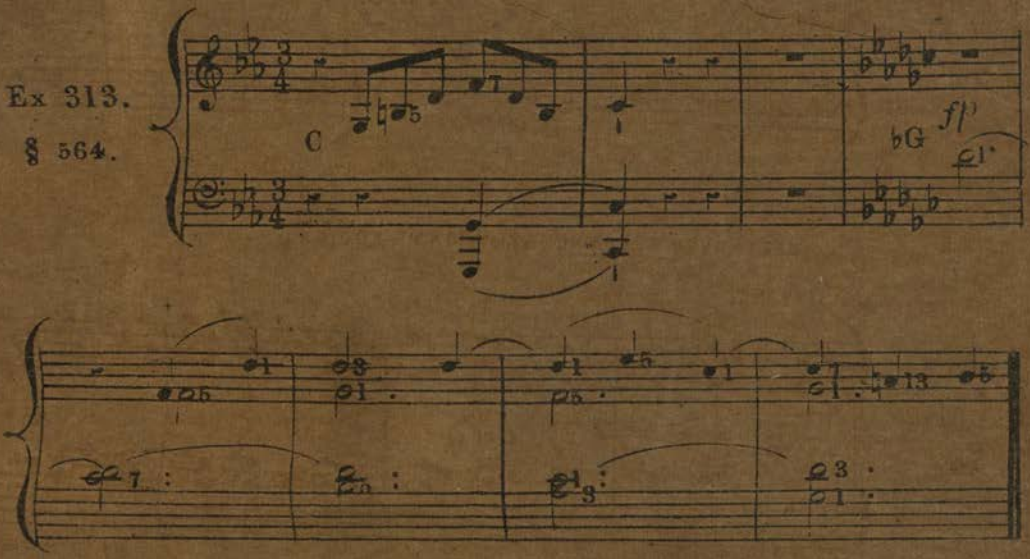


Ex. 312.
§ 563.



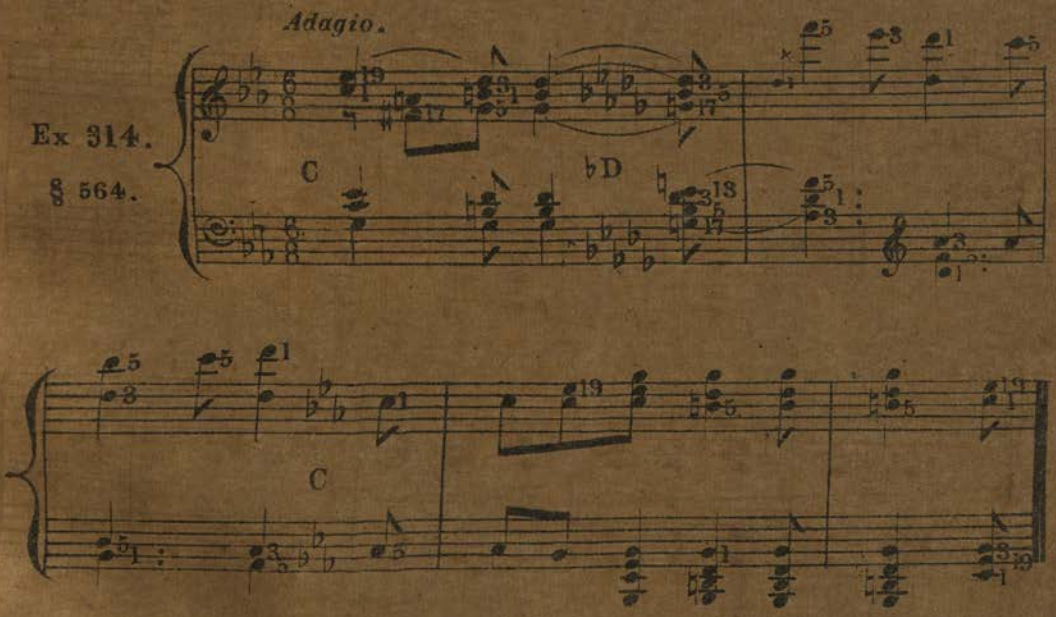
BEETHOVEN'S GRAND SONATA. LETTER C

Ex 313.
§ 564.



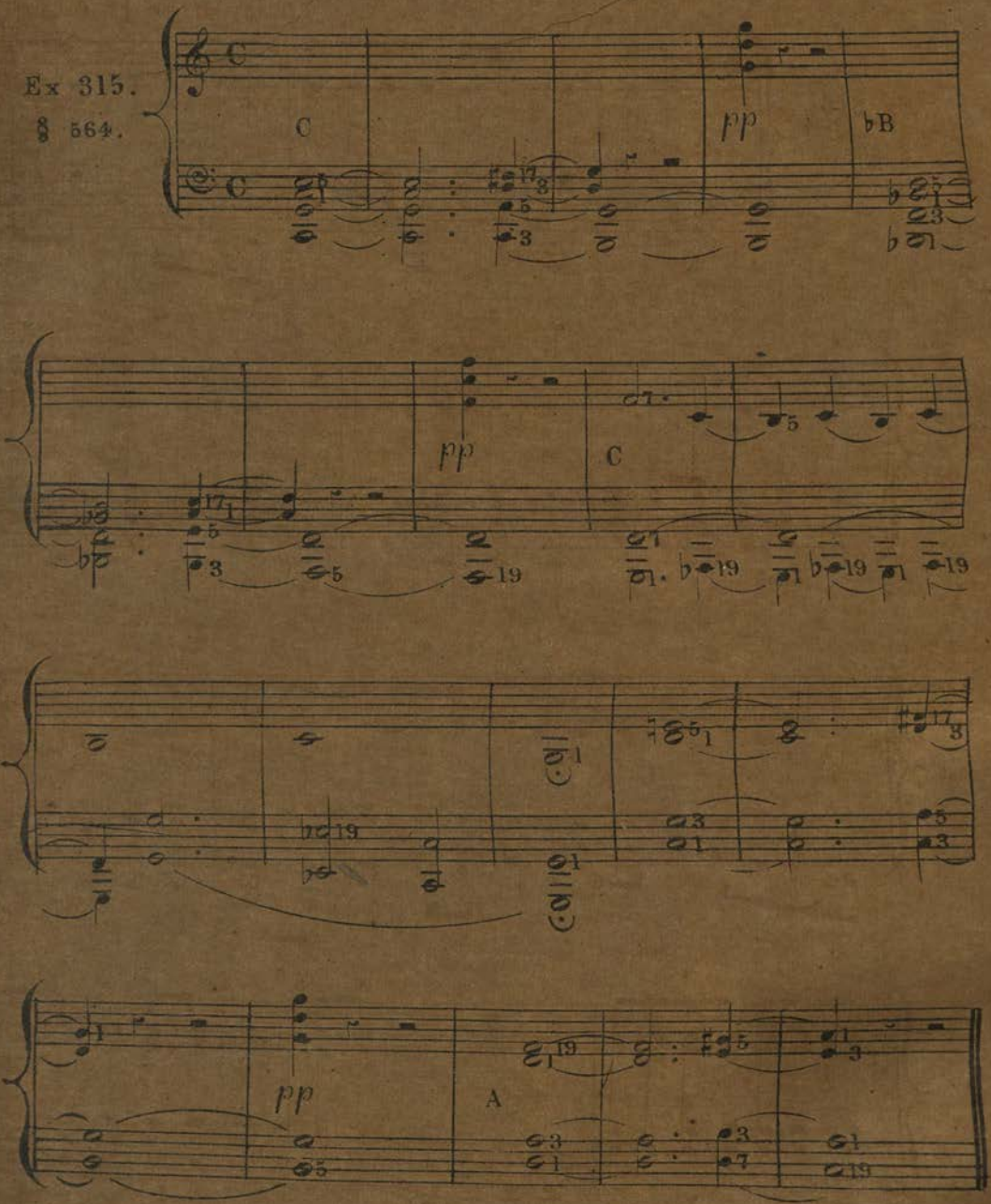
BEETHOVEN'S GRAND SONATA Op: 106.

Ex 314.
§ 564.



PASSAGES SIMPLIFIED FROM BEETHOVEN'S SONATA Op: 53.

Ex 315.
§ 564.



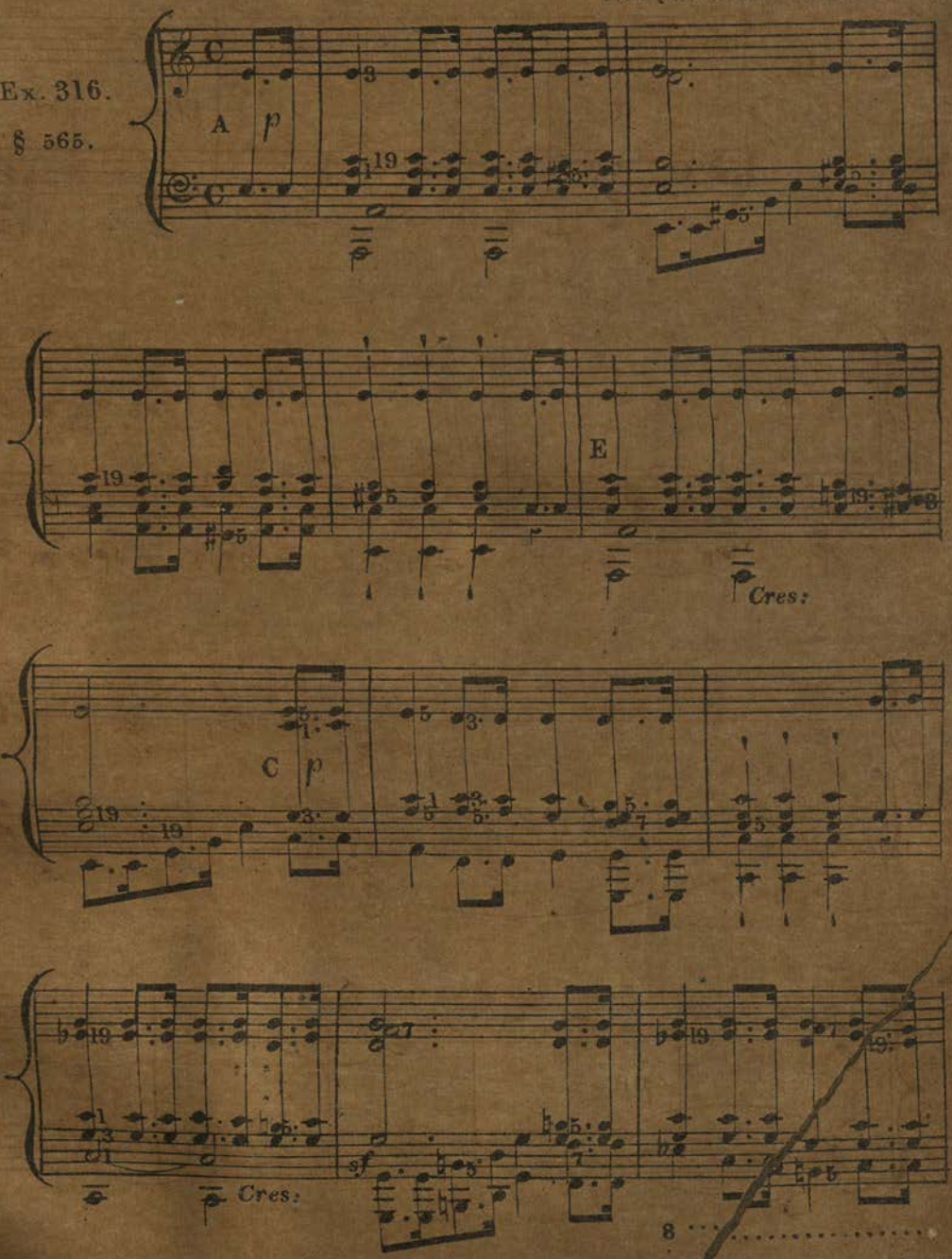
MARCIA FUNEBRE SULLA MORTE D'UN EROE.

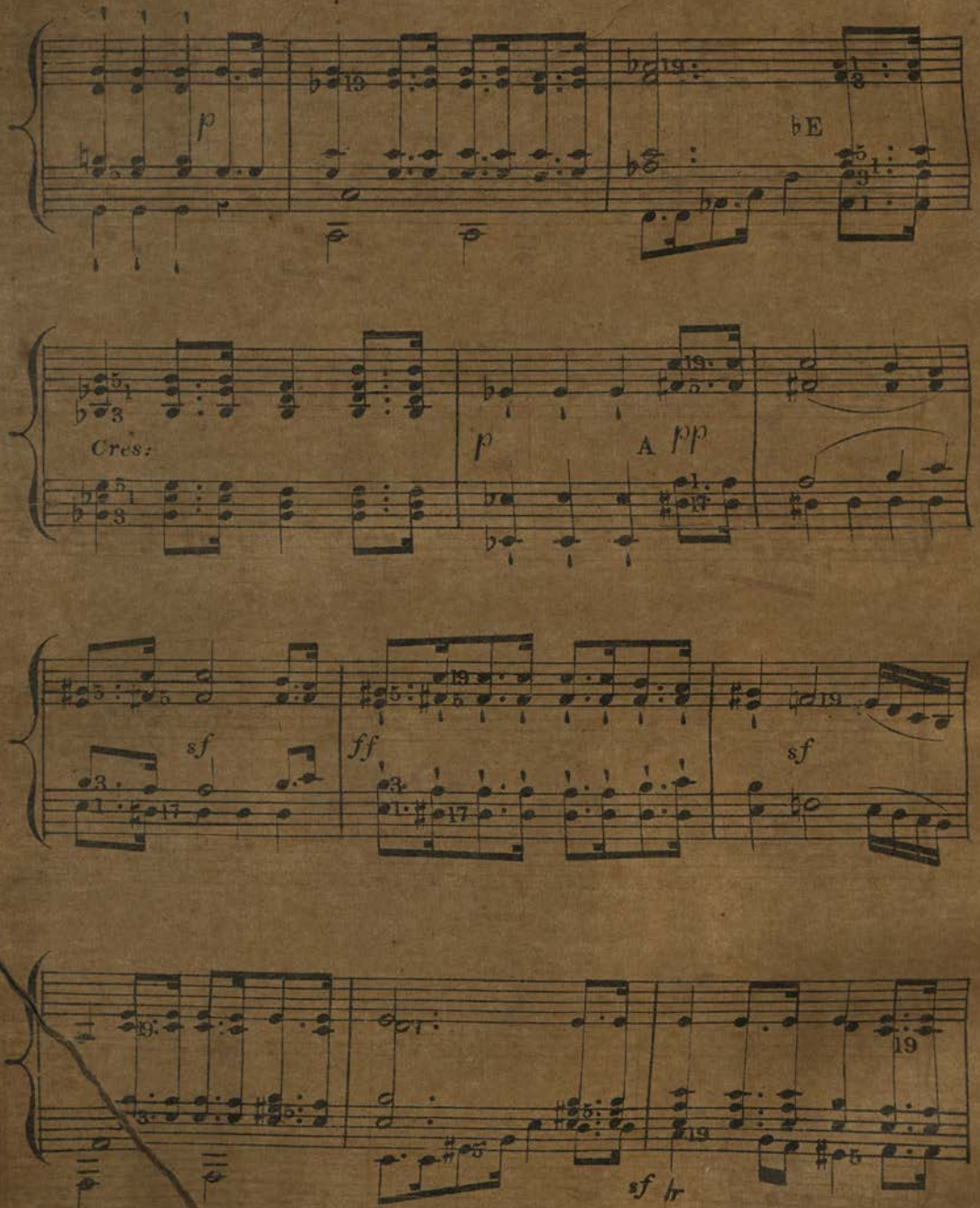
BEETHOVEN.

Transposed from ♭A to A.

Ex. 316.

§ 565.




p
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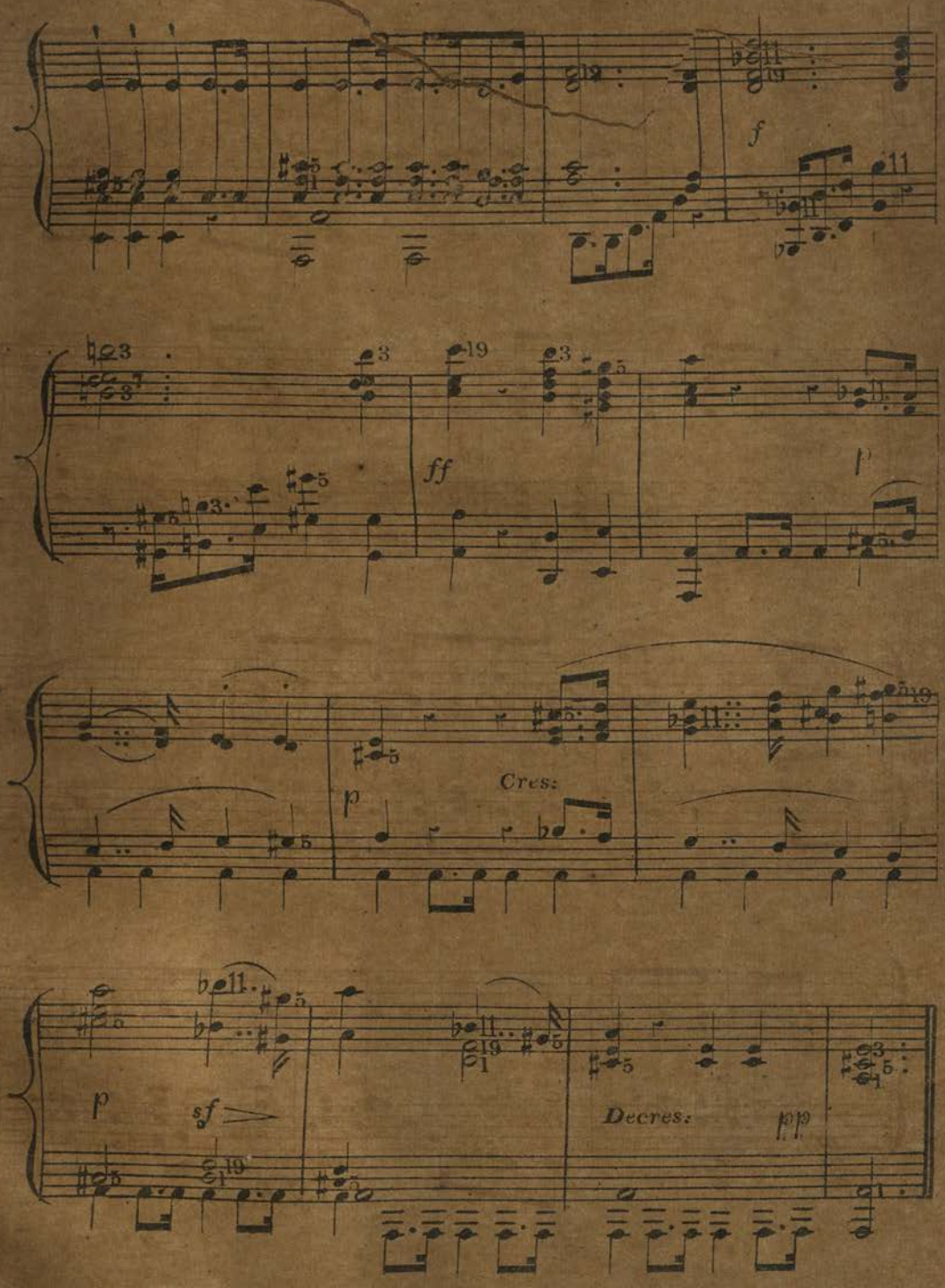

f
ff
p
Cres:
p
sf
Decres:
pp

SANCTUS.

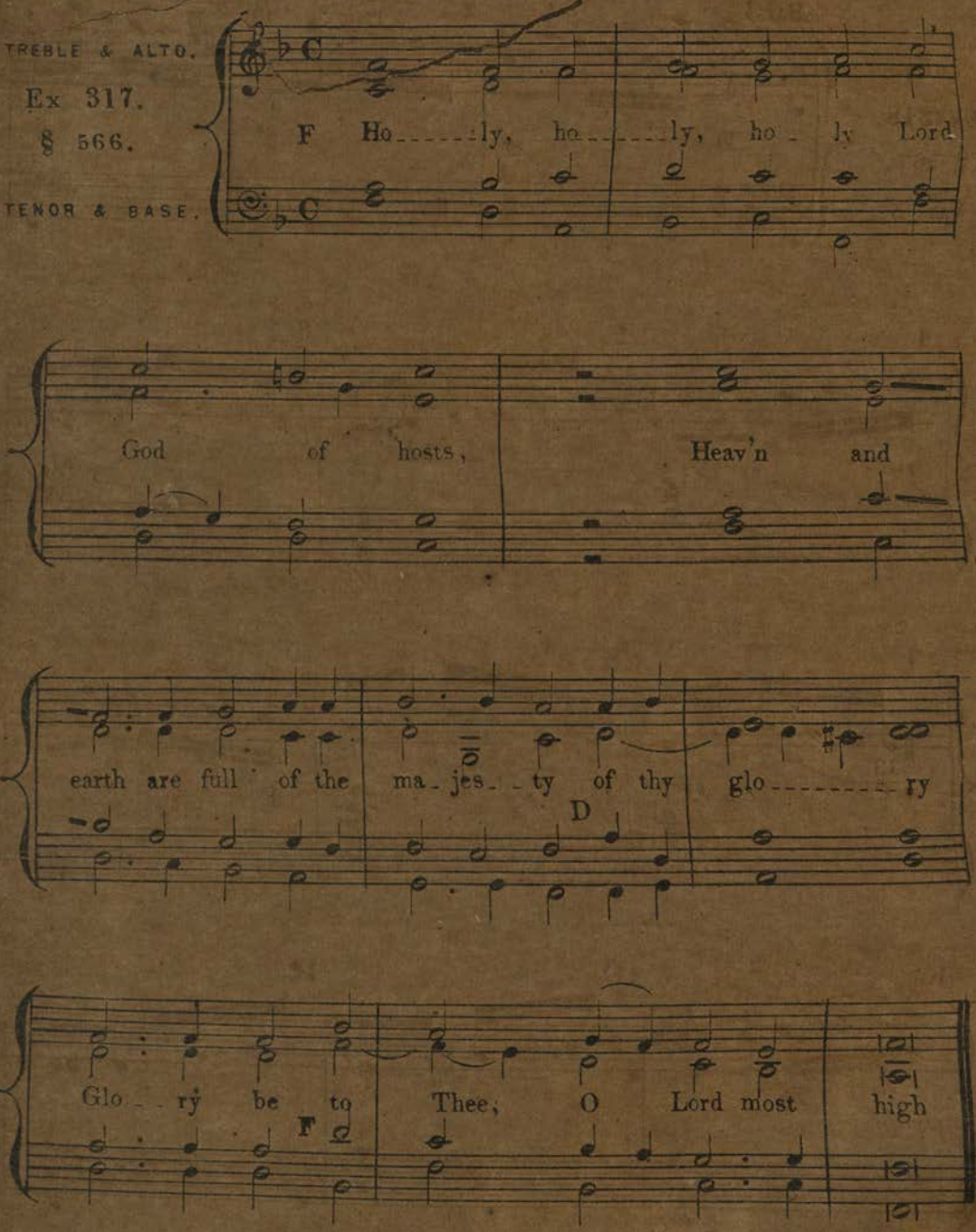
ORLANDO GIBBONS.

TREBLE & ALTO.

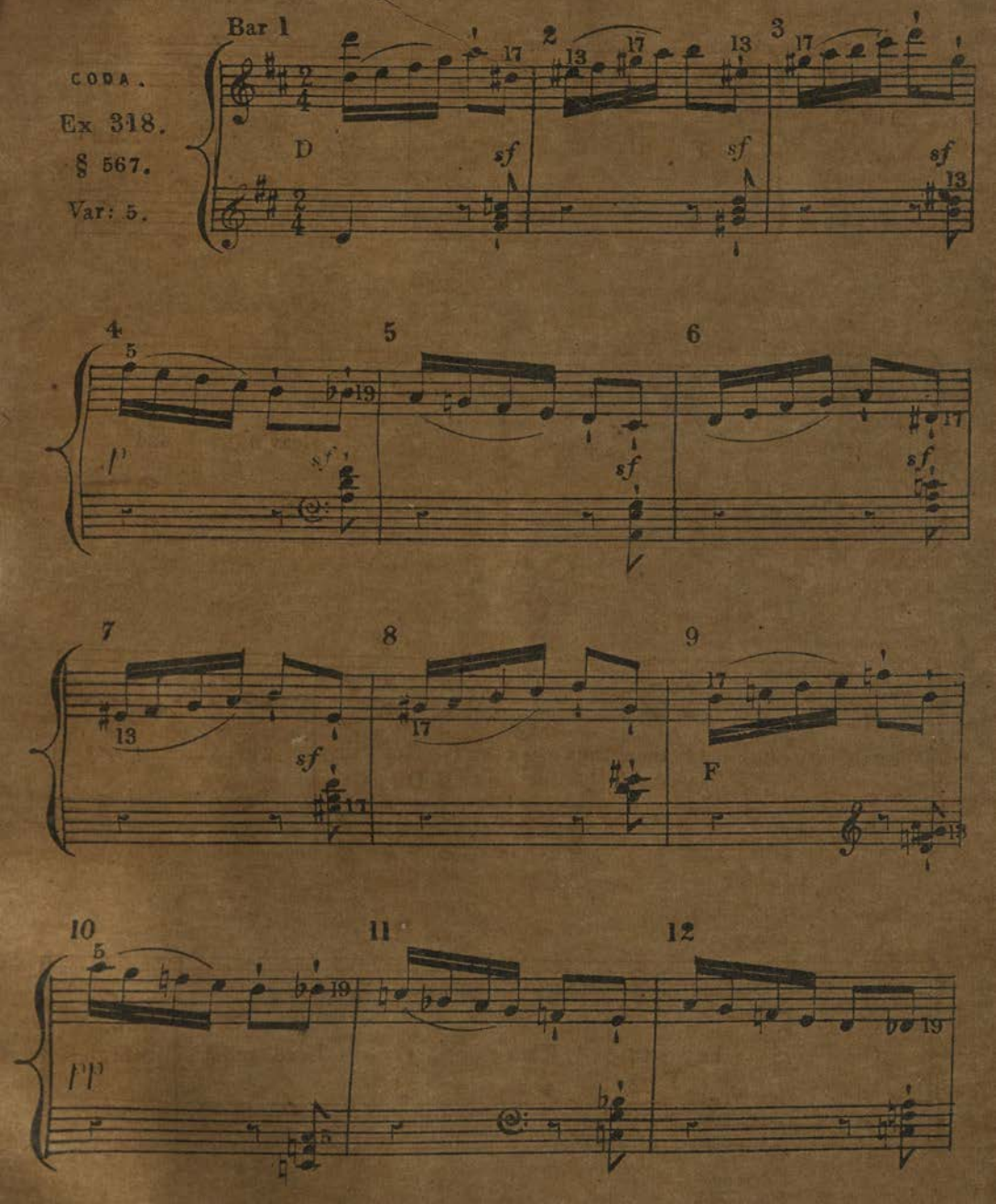
Ex 317.

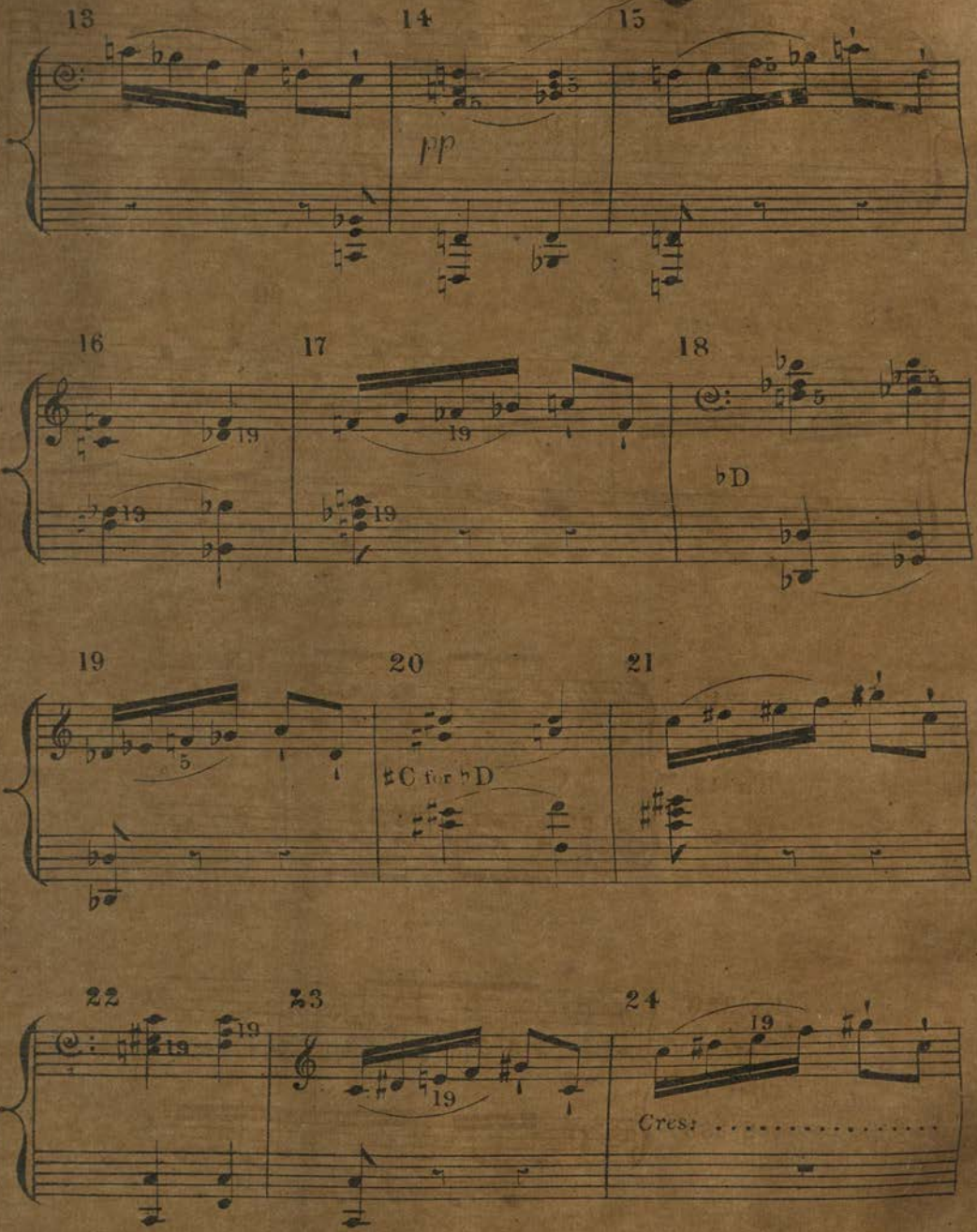
§ 566.

TENOR & BASE.



BEE[redacted]EN'S VARIATIONS TO RULE BRITANNIA.




13
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♭D
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♯C for ♭D
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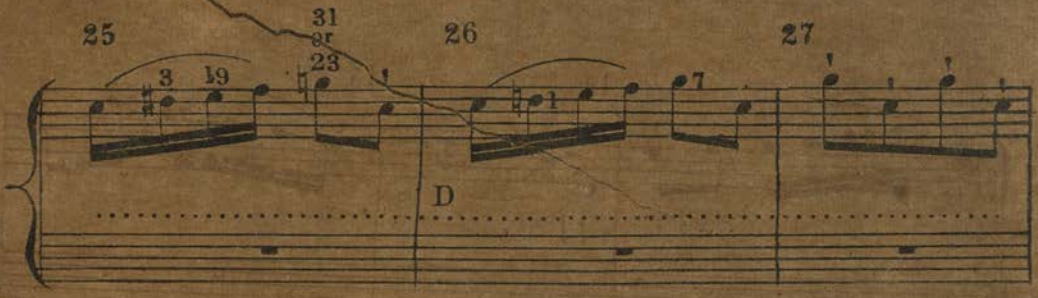
25
31
or
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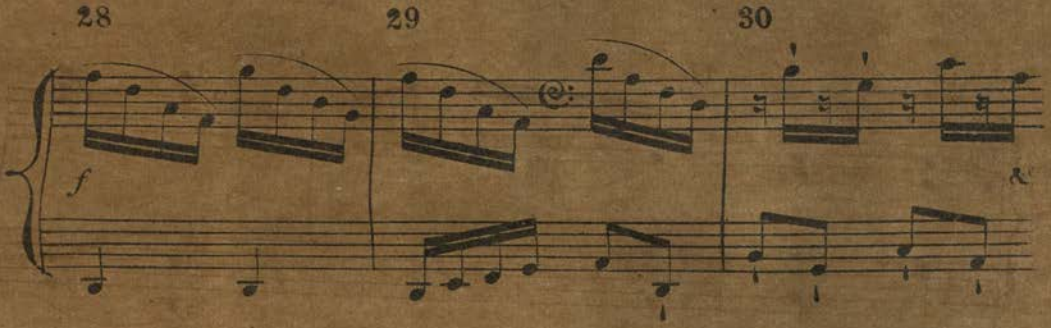
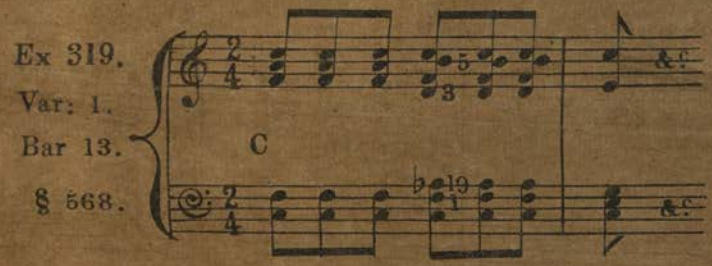
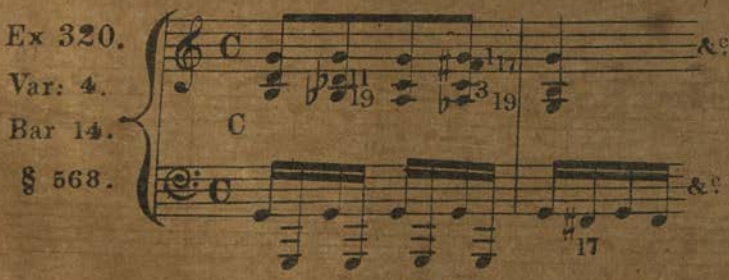
27
D

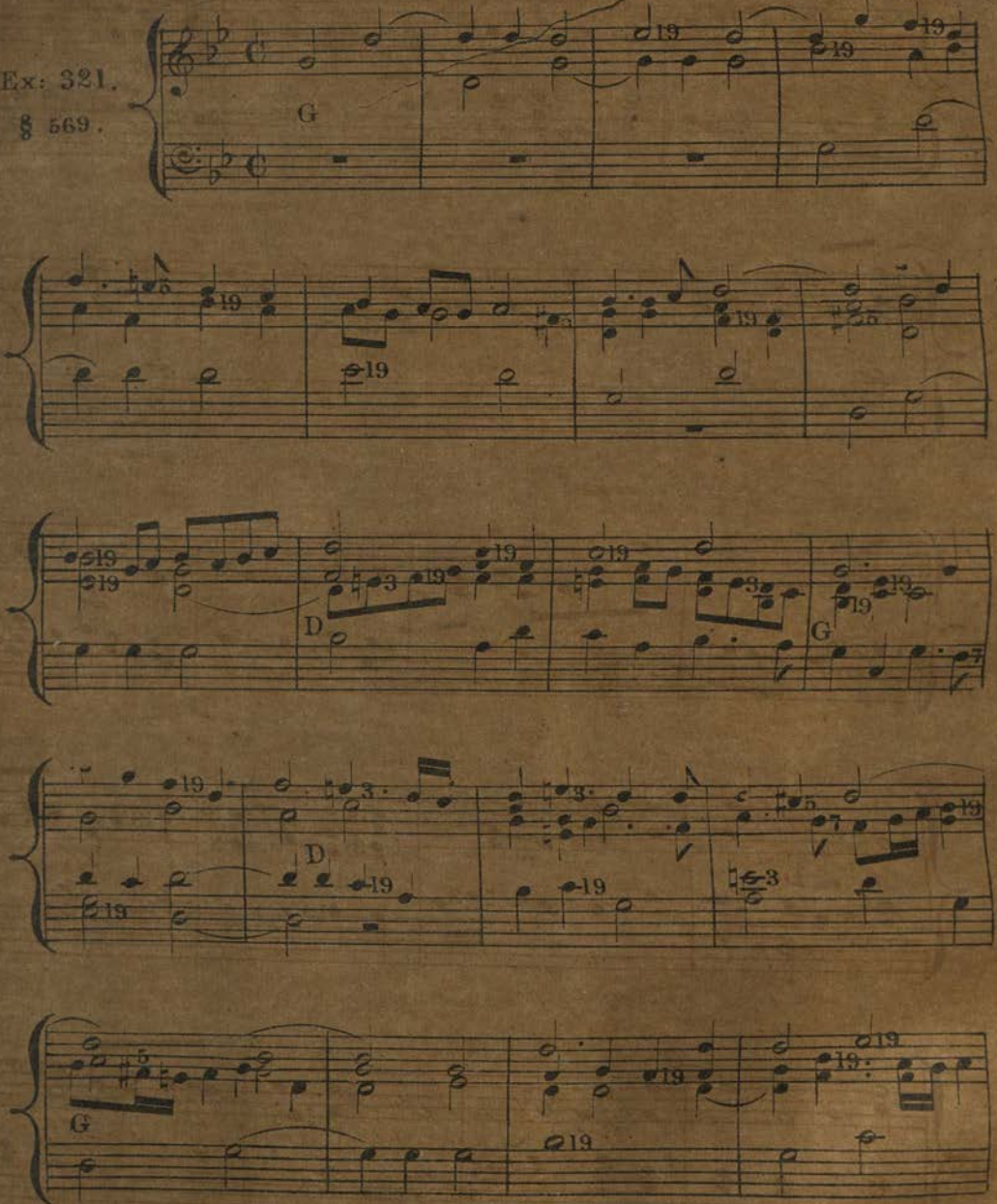

28
29
30
f

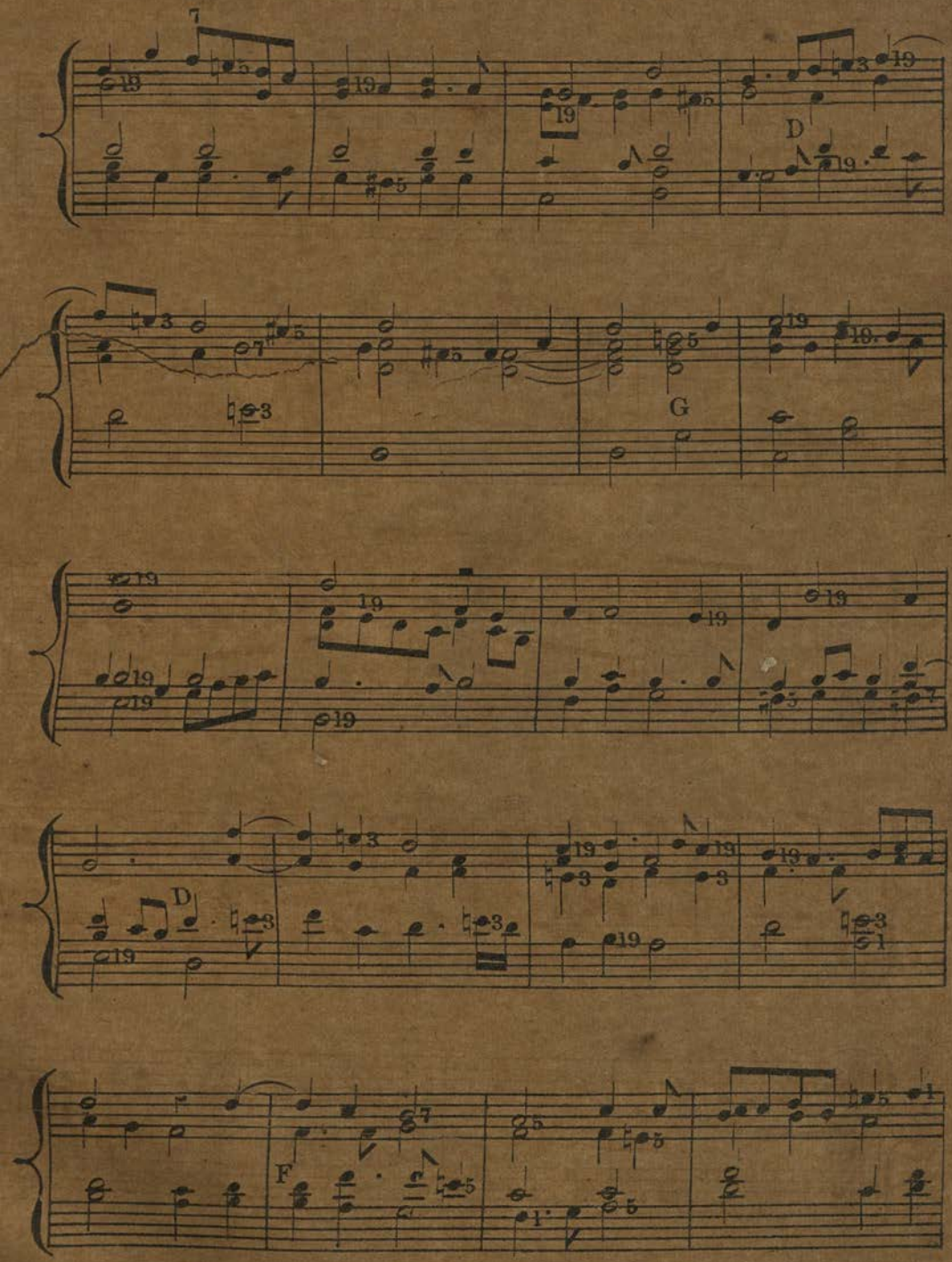

Ex 319.
Var: 1.
Bar 13.
§ 568.
C

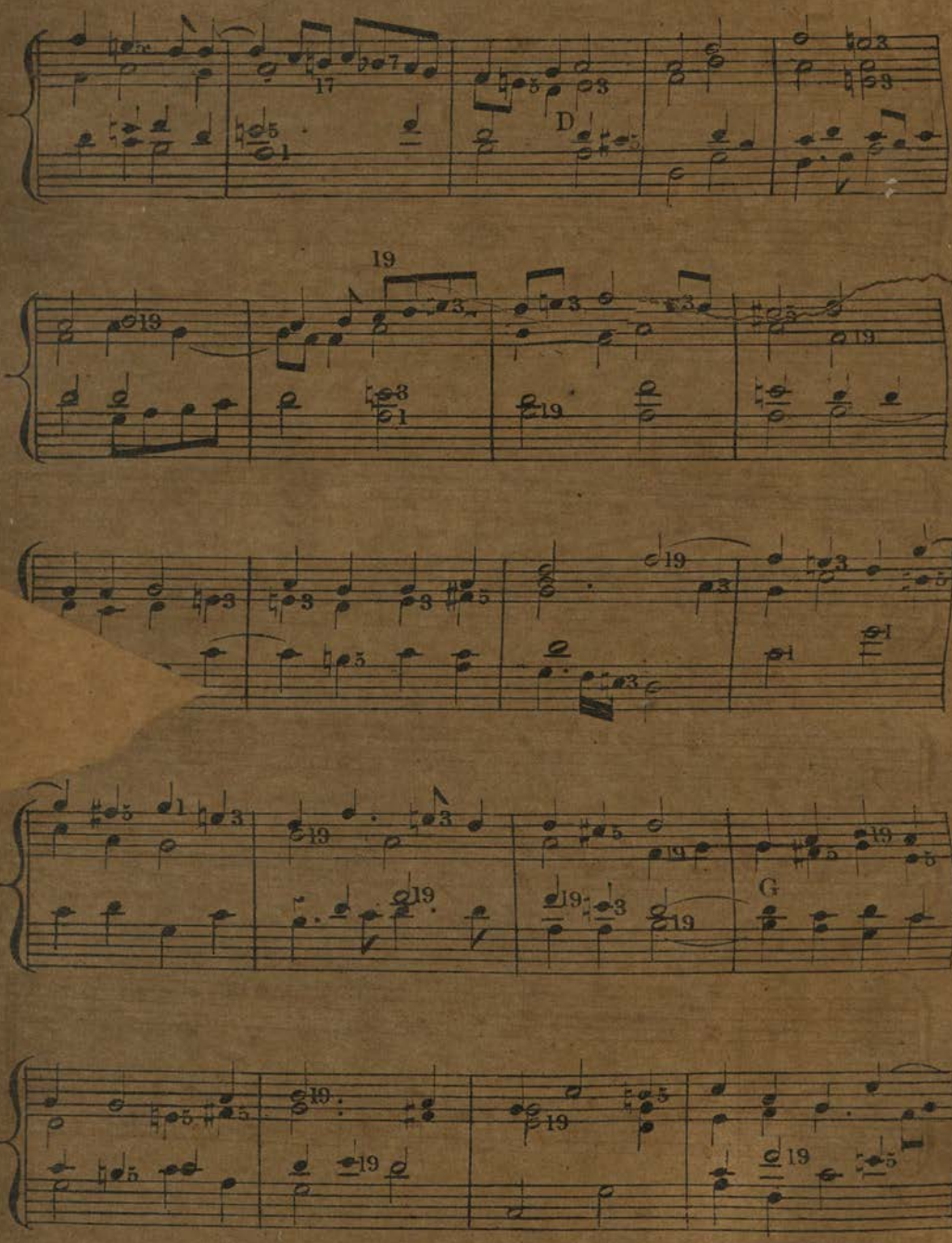

Ex 320.
Var: 4.
Bar 14.
§ 568.
C
C

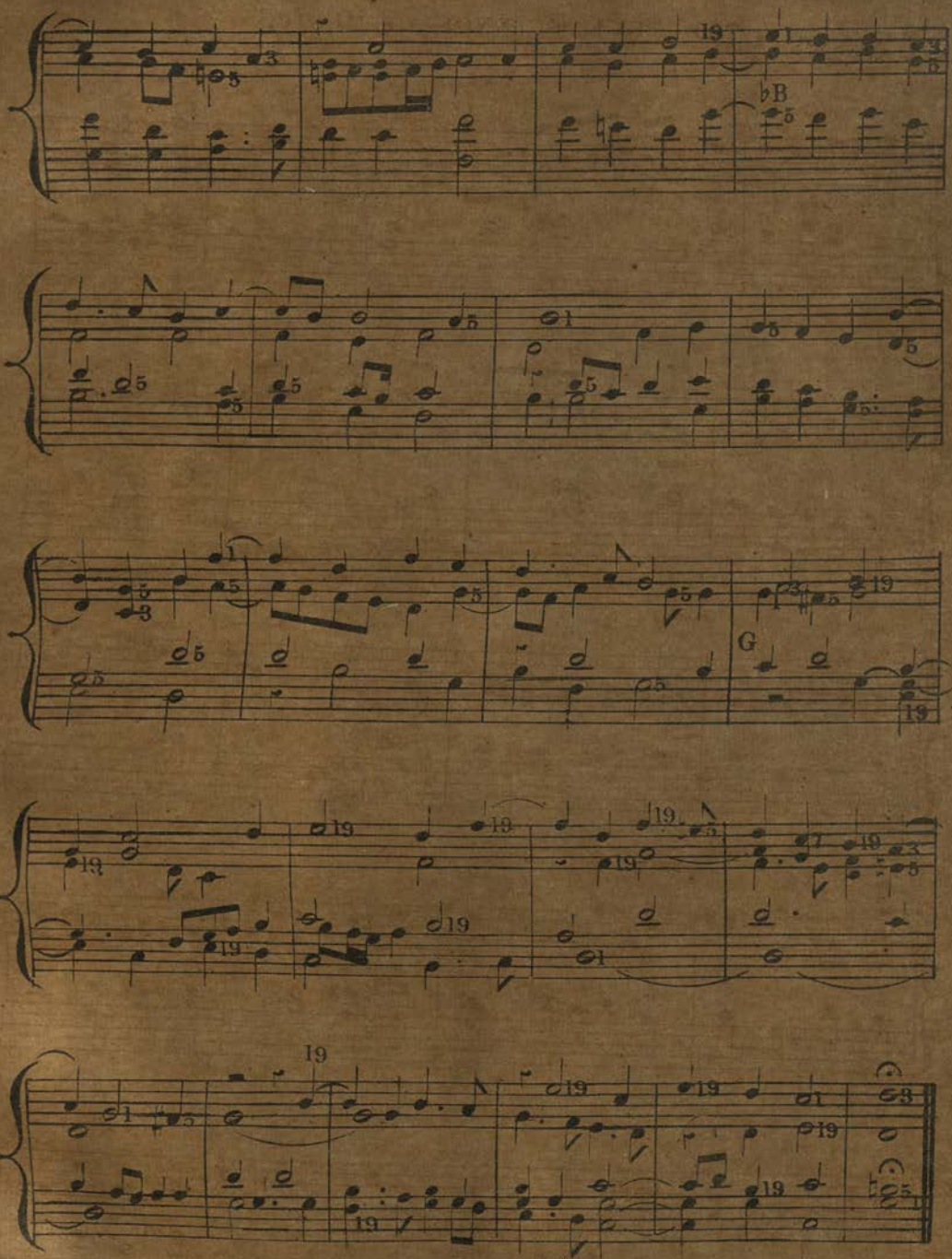
Nº 1. MOTTETTI a 5. DI G. PIER LUIGI DA PALESTRINA

Ex: 321.
§ 569.




D
G
D
F

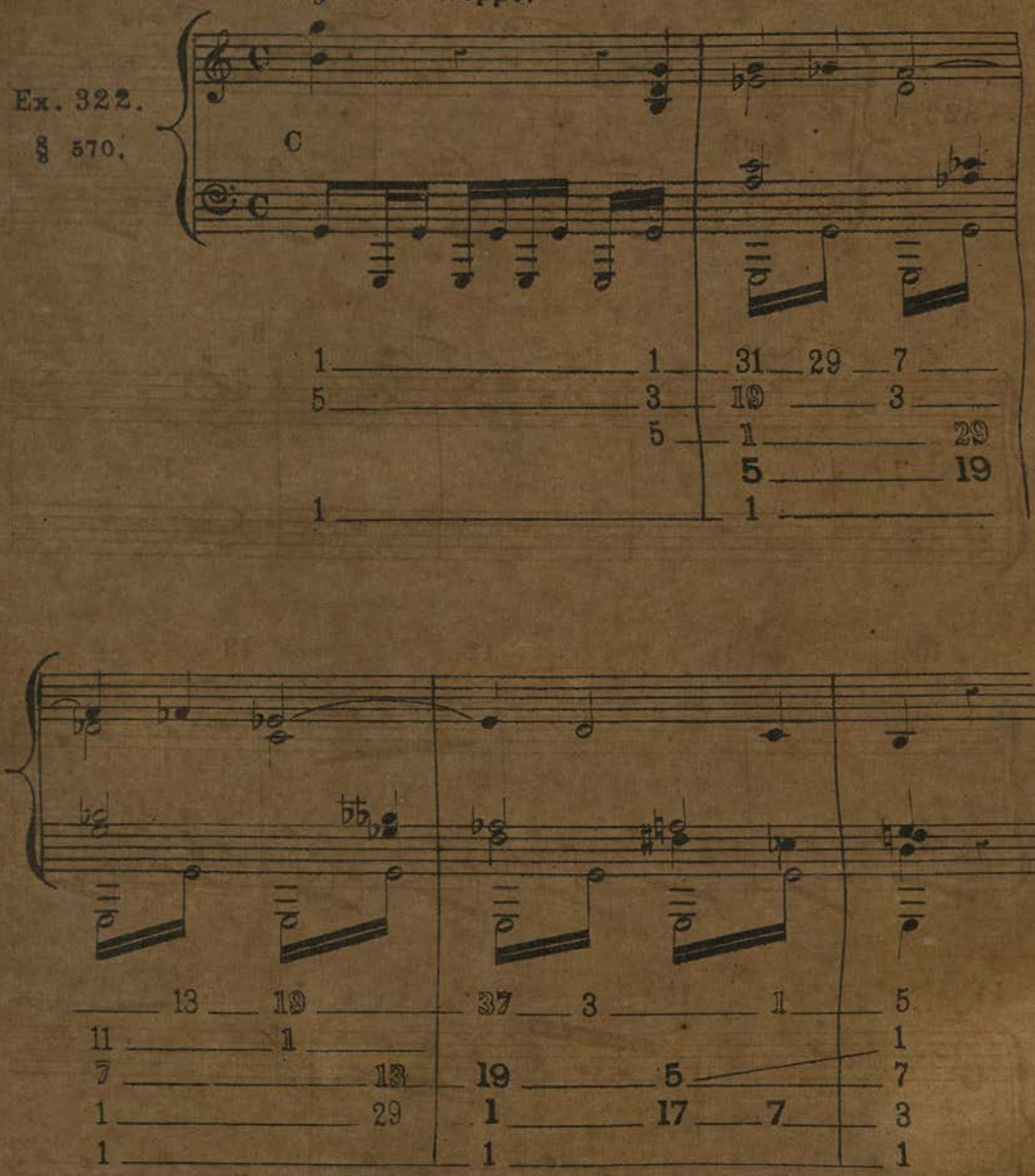

D
G


bB
G

THE TEMPEST, FROM GLI UOMINI DI PROMETEO

Allegro non troppo. BEETHOVEN.

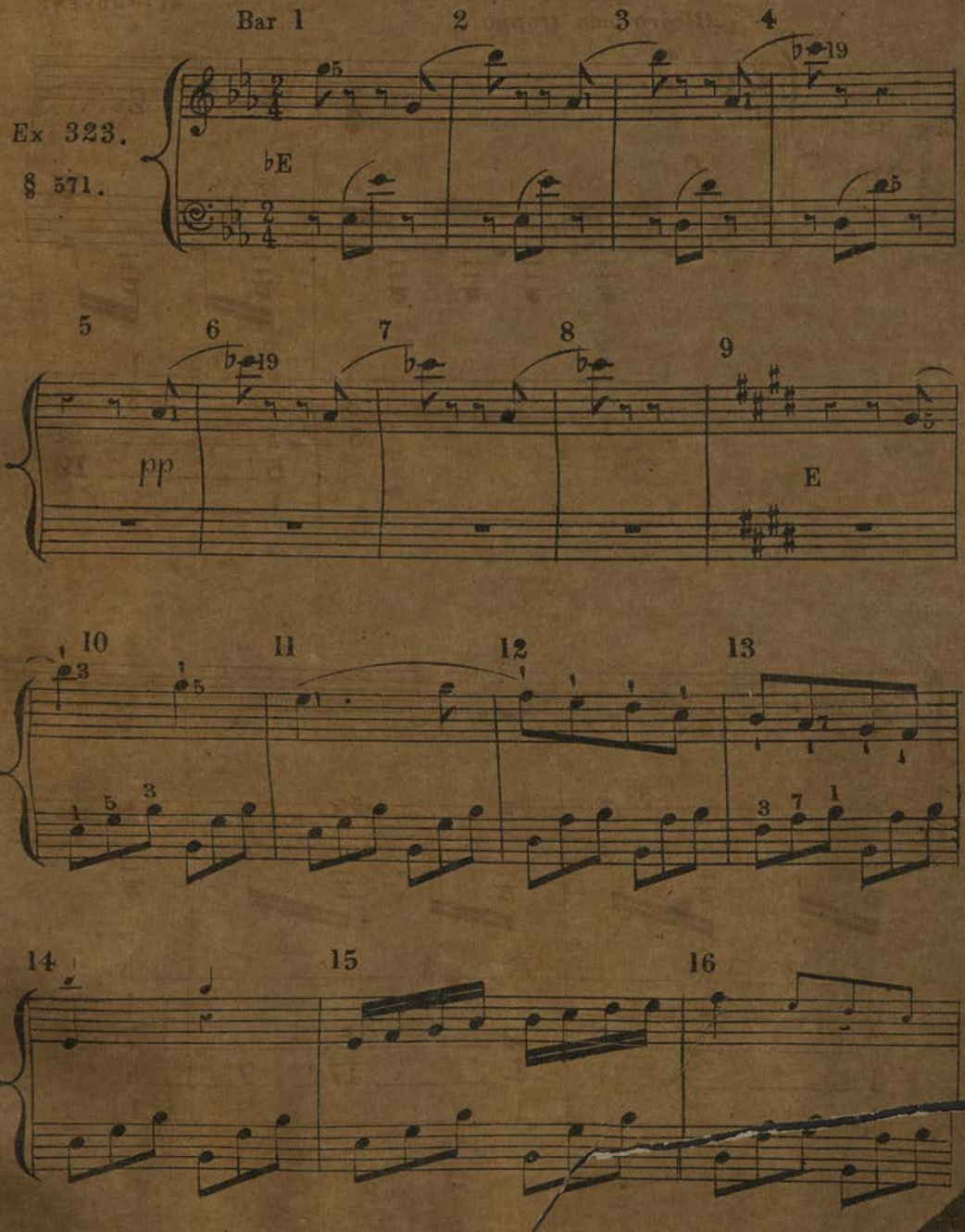
Ex. 322.
§ 570.

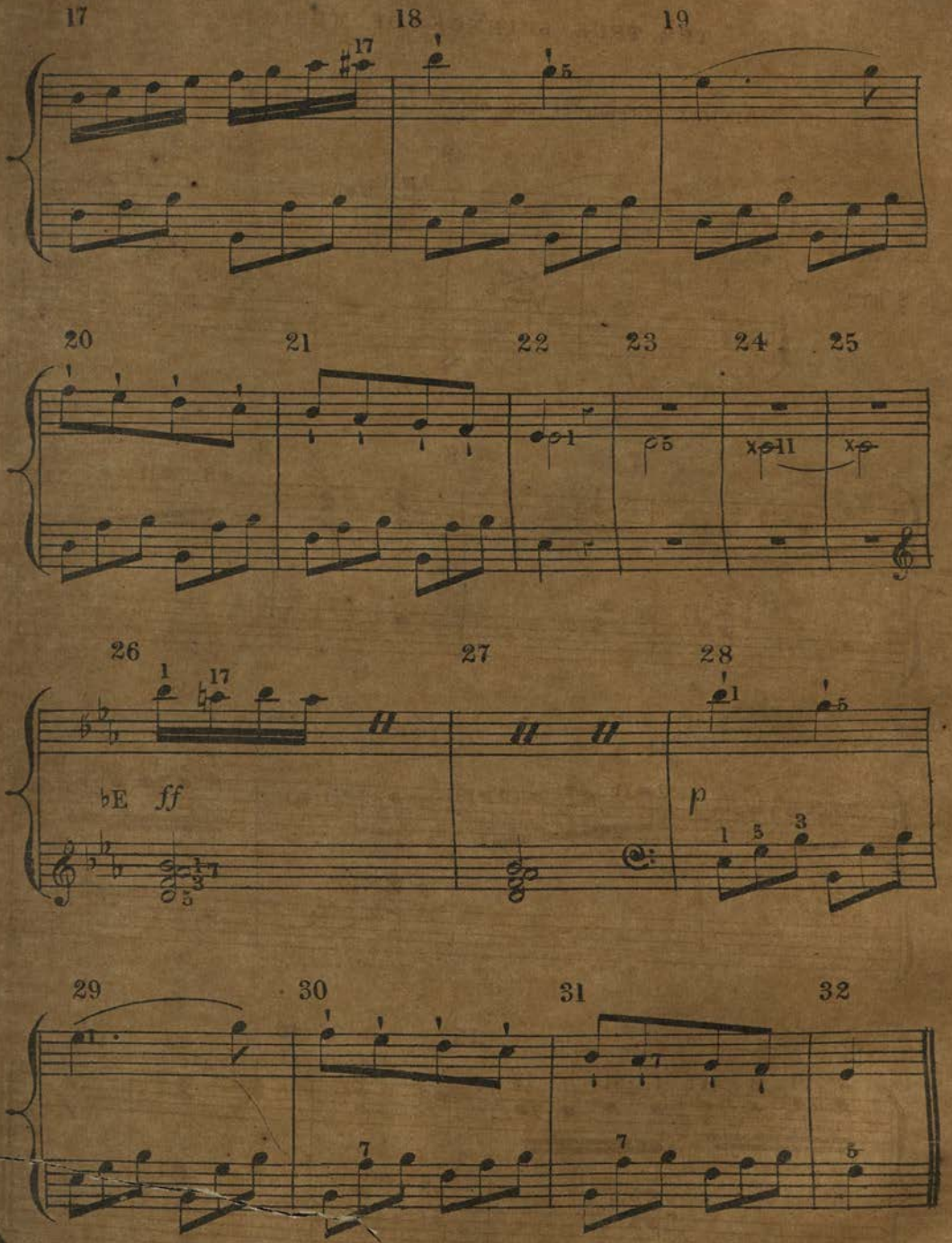


BEETHOVEN'S TRIO FOR PIANOFORTE, VIOLIN, AND VIOLONCELLO Op: 1 NO 1.

Ex 323.

§ 571.

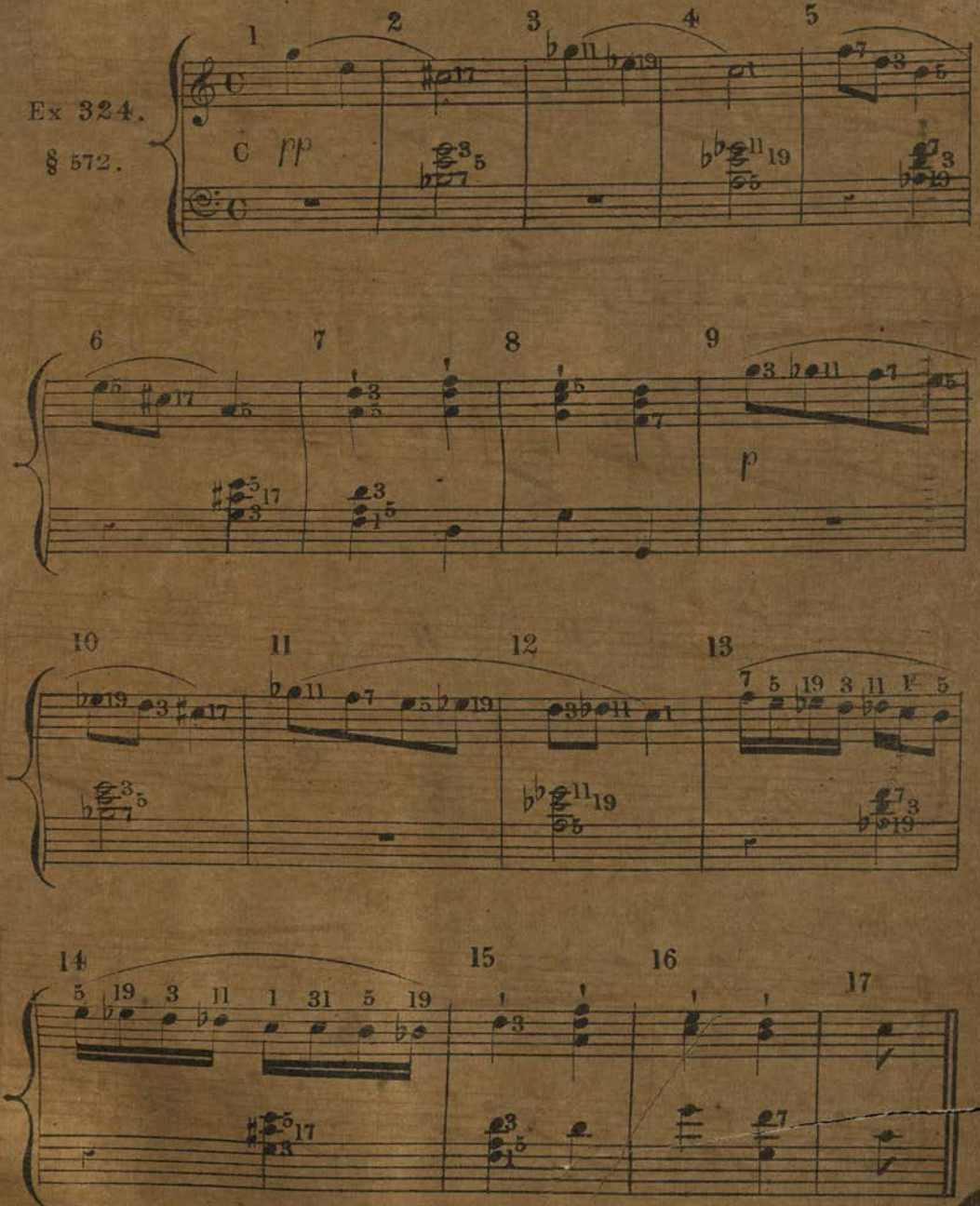



17
18
19
20
21
22
23
24
25
26
27
28
bE ff
p
29
30
31
32

Another portion of the above transposed from ♭B.

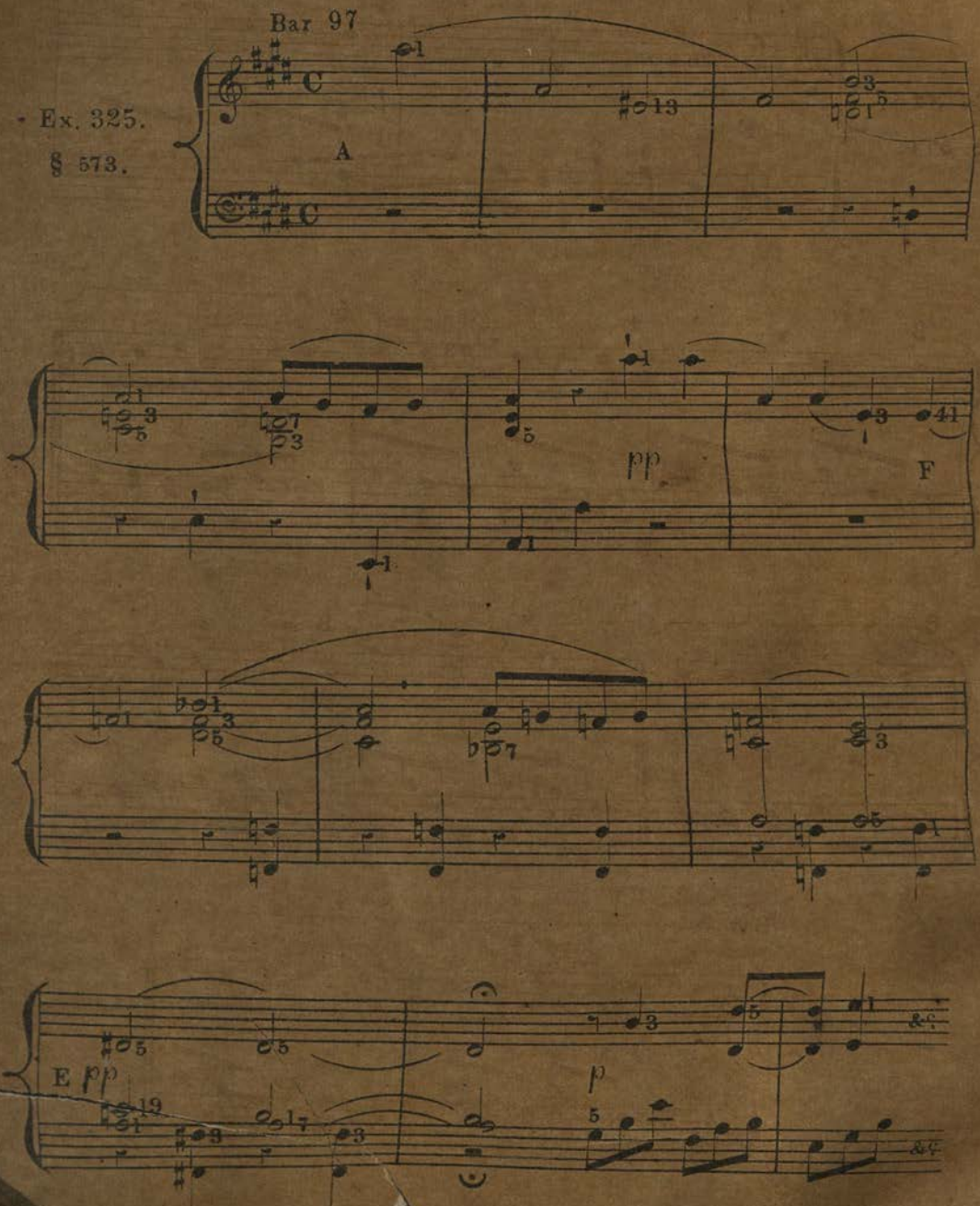
Ex 324.

§ 572.



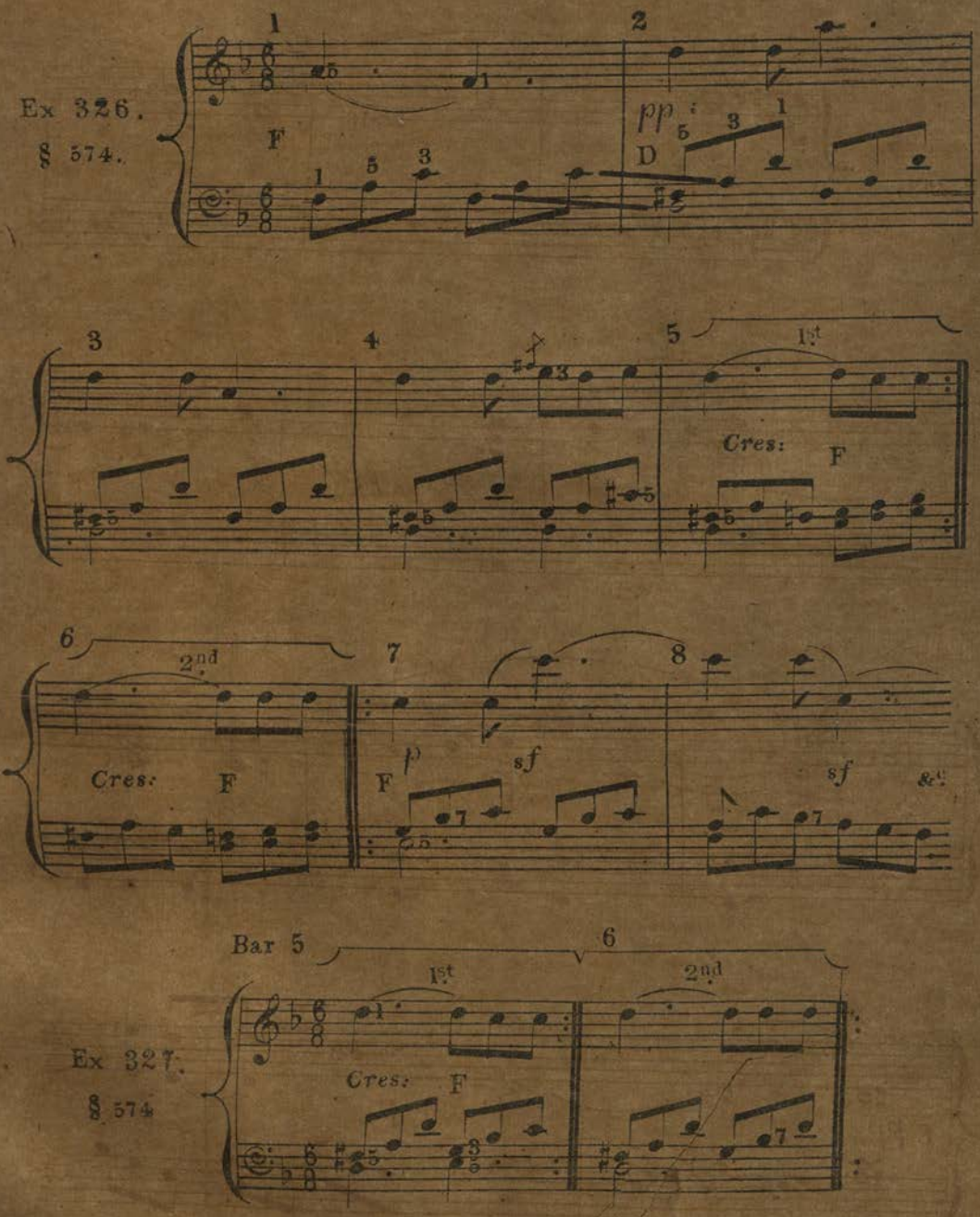
BEETHOVEN'S SONATA No. 1 Op: 4

Ex. 325.
§ 573.



BEETHOVEN'S AIRS ON BAGATELLES Nº 3.

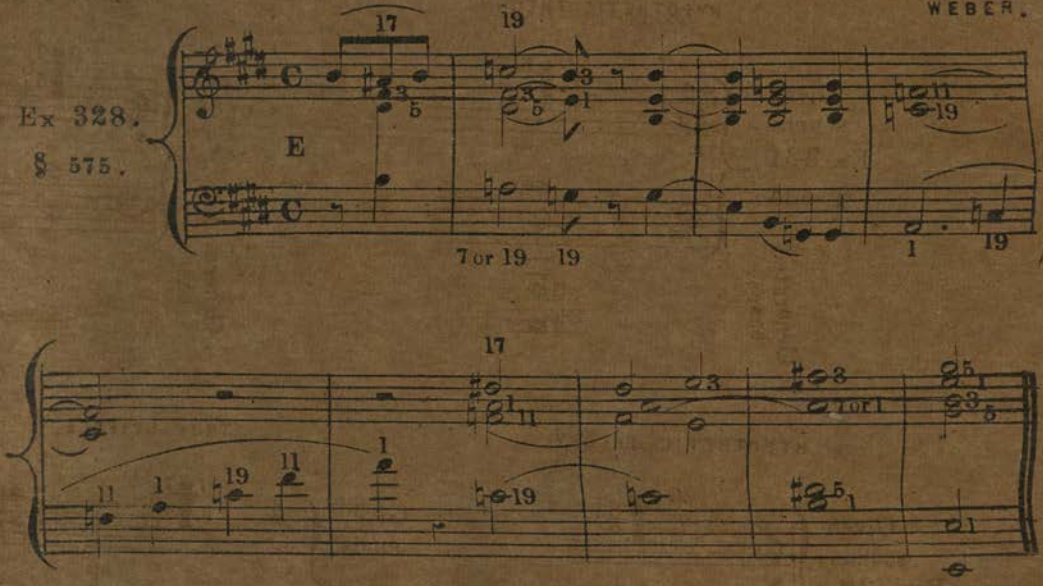
Ex 326.
§ 574.



Ex 327.
§ 574

SOFTLY SIGH'S THE VOICE OF EVENING. WEBER.

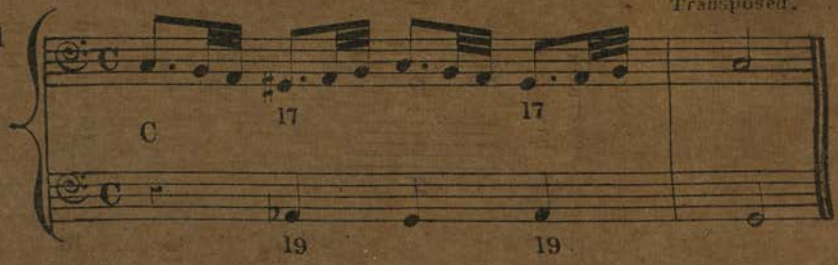
Ex 328.
§ 575.



WEBER'S DER FREISCHUTZ.

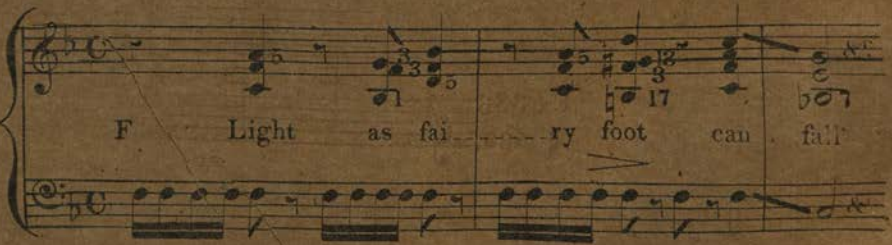
Violoncellos and Double bases.
Ex 329.
§ 576.
Trombones.

Transposed.

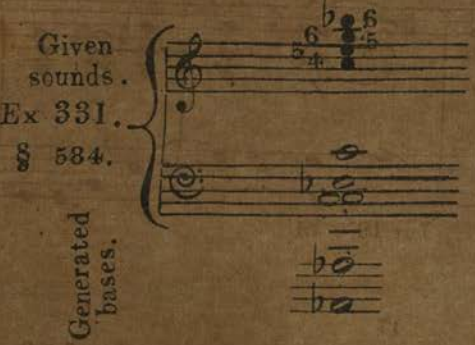


CHORUS IN WEBER'S OBERON.

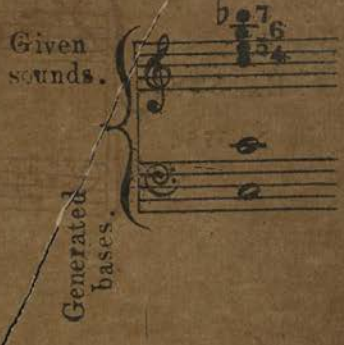
Ex 330.
§ 577.



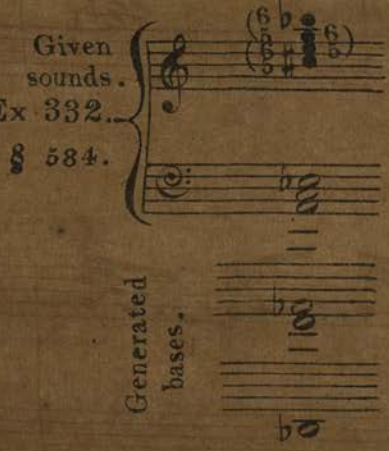
HYPOTHETIC THEORY.



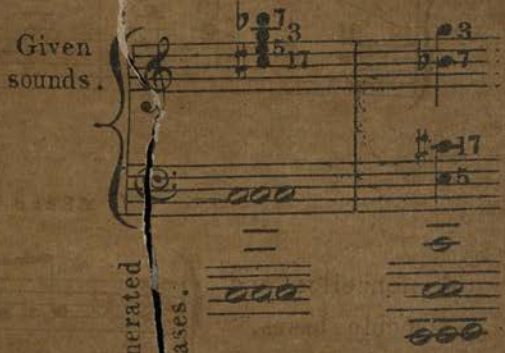
TRUE SCIENCE.



HYPOTHETIC THEORY.



TRUE SCIENCE.



BEETHOVEN'S

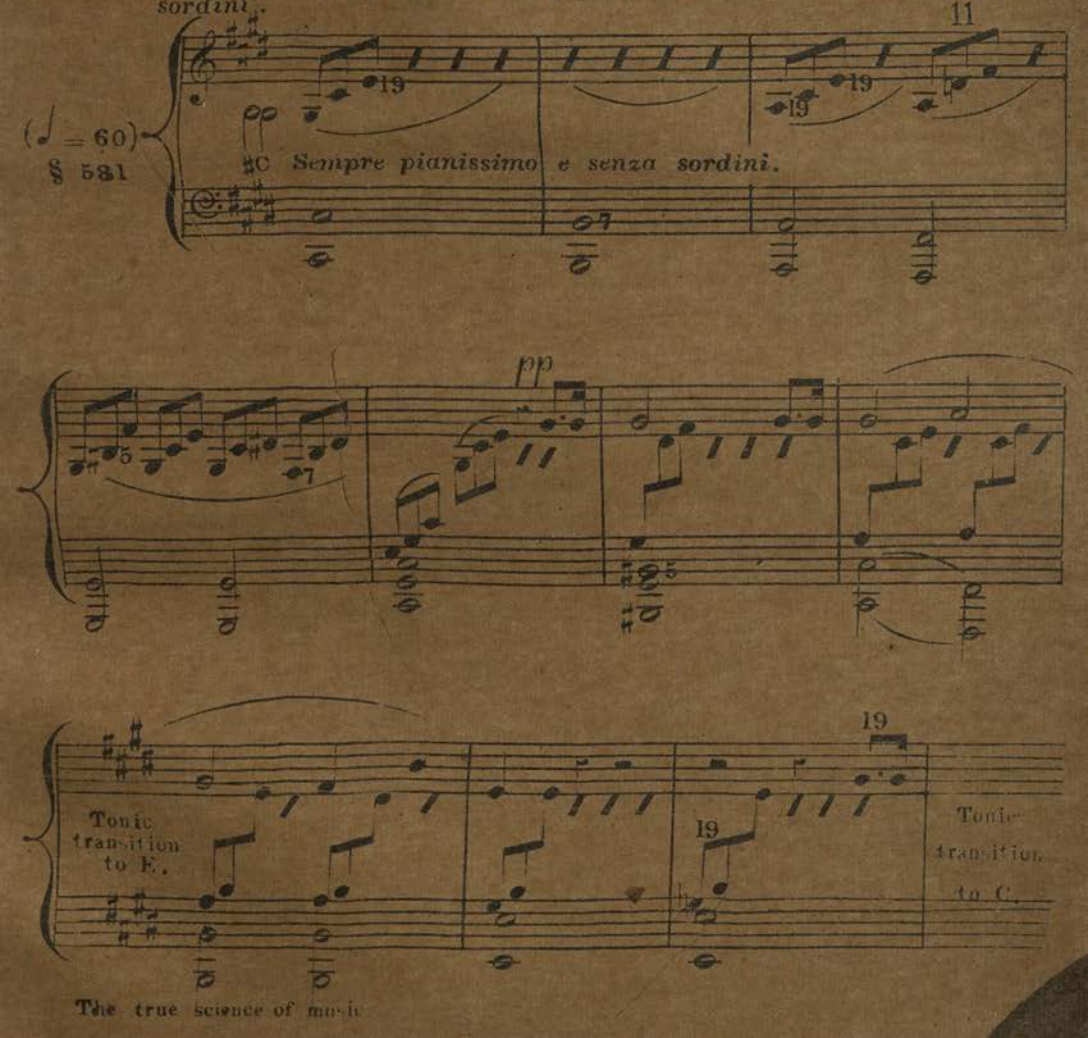
MOONLIGHT SONATA

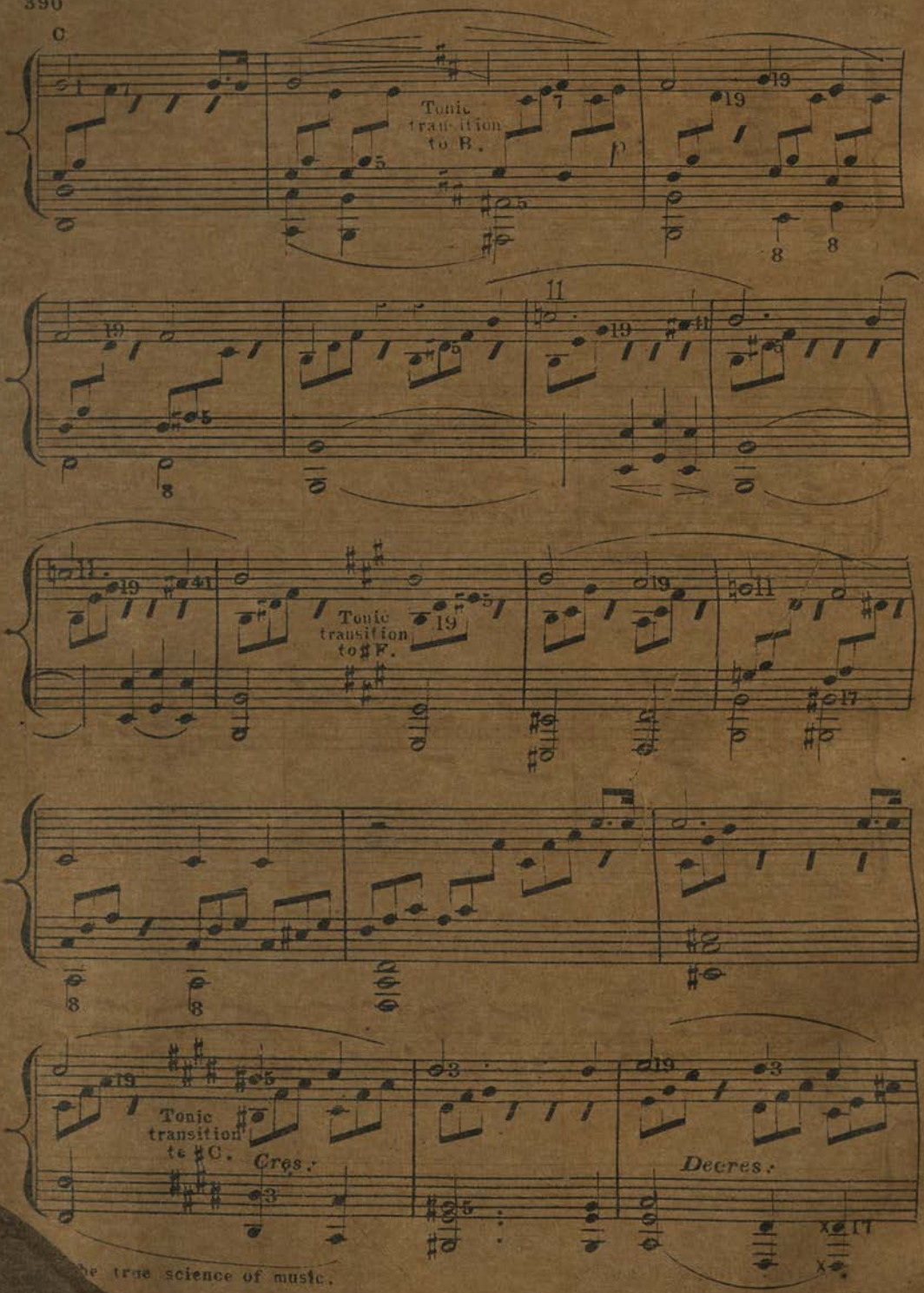
Op: 27

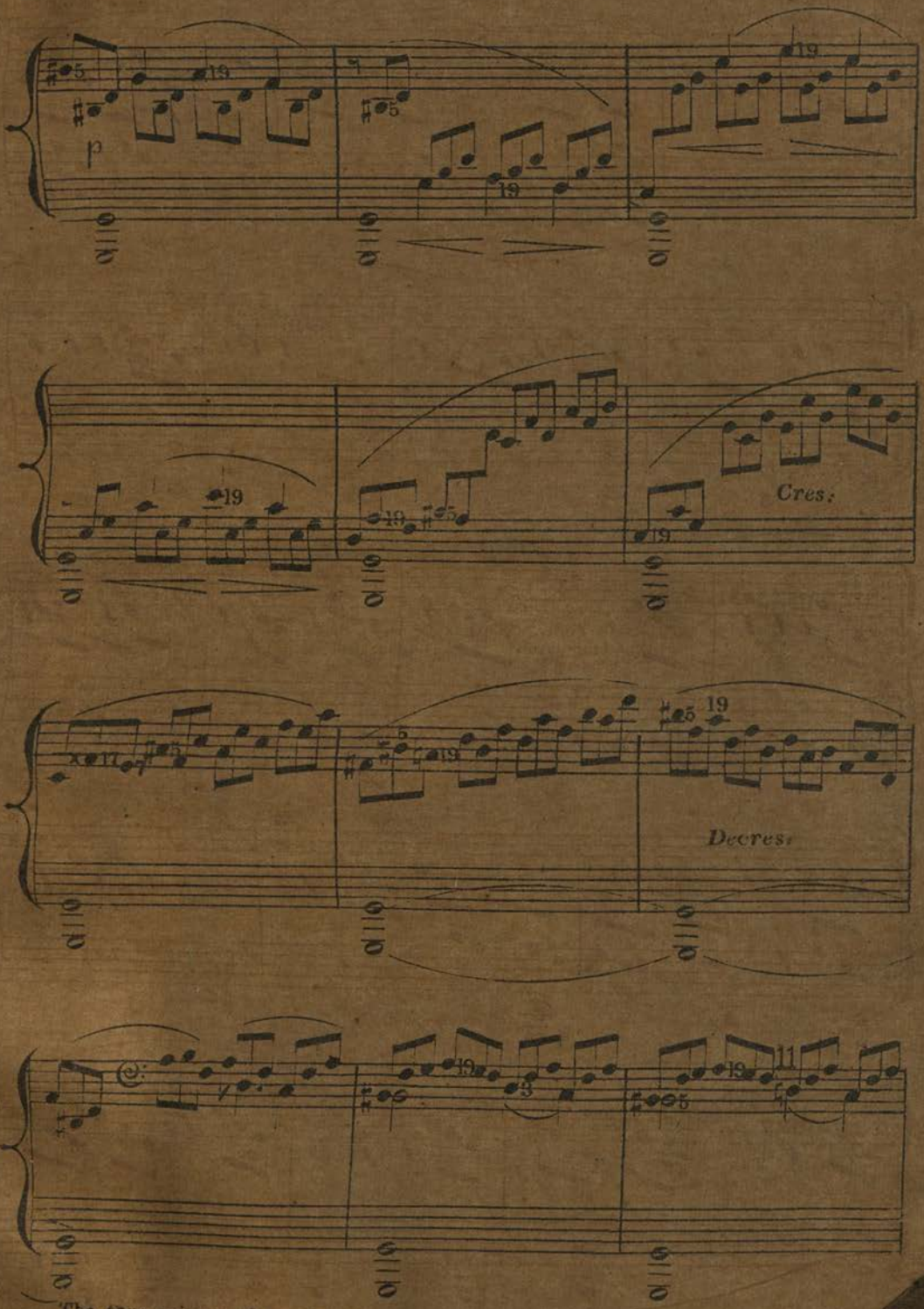
ANALYSED BY

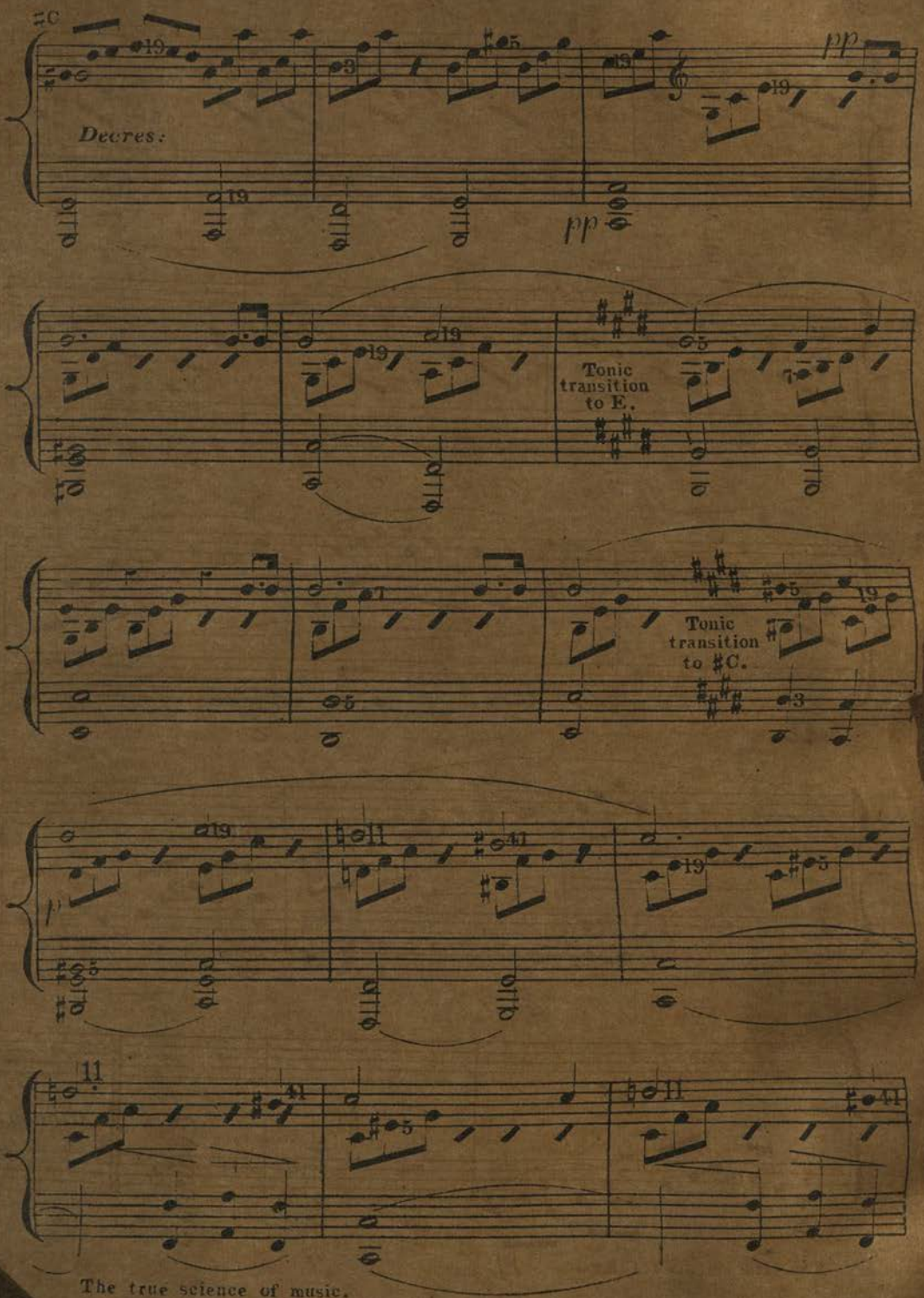
D. C. HEWITT.

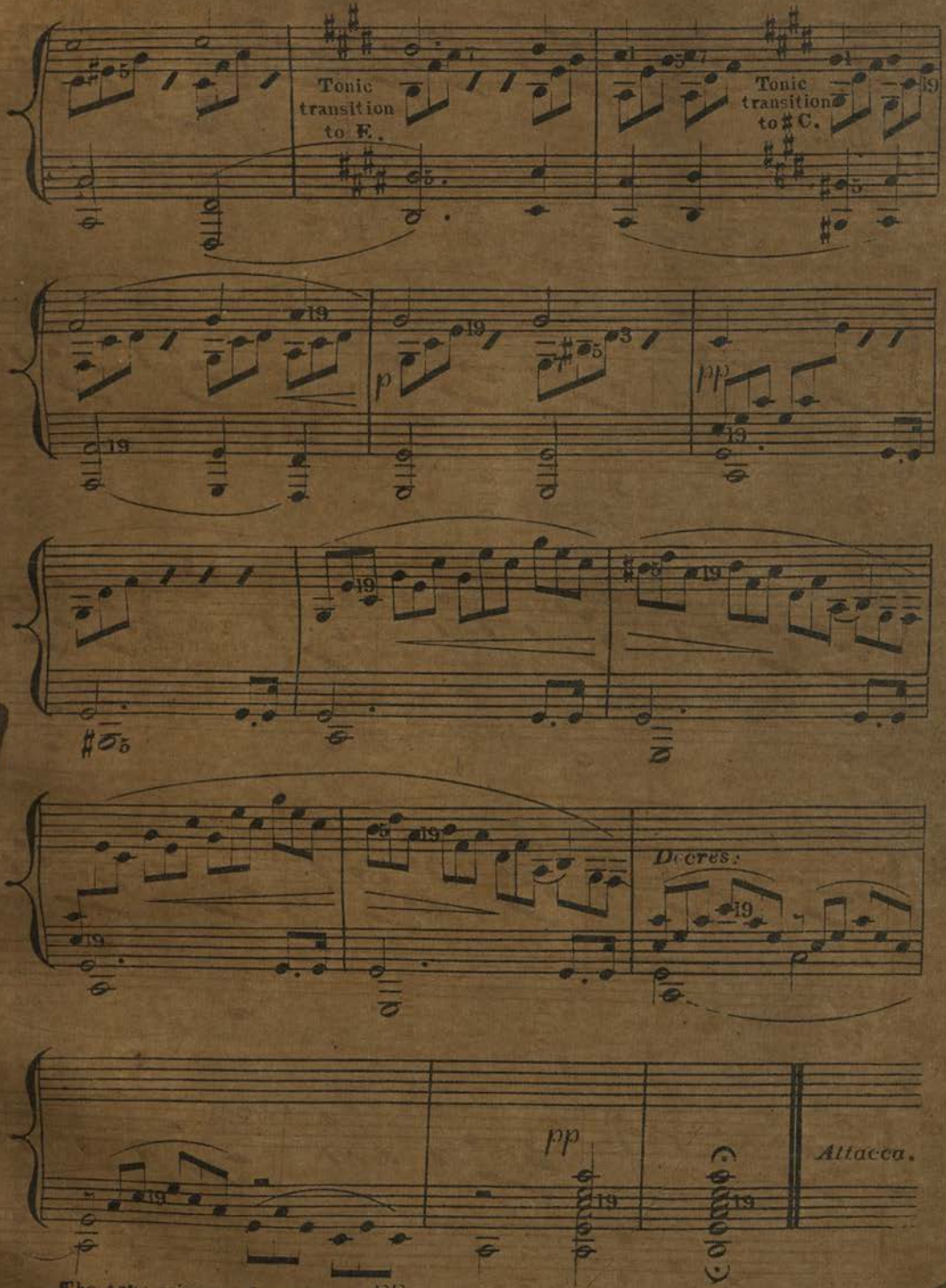
ADAGIO Si deve suonare tutto questo pezzo delicatissimamente e senza sordini.

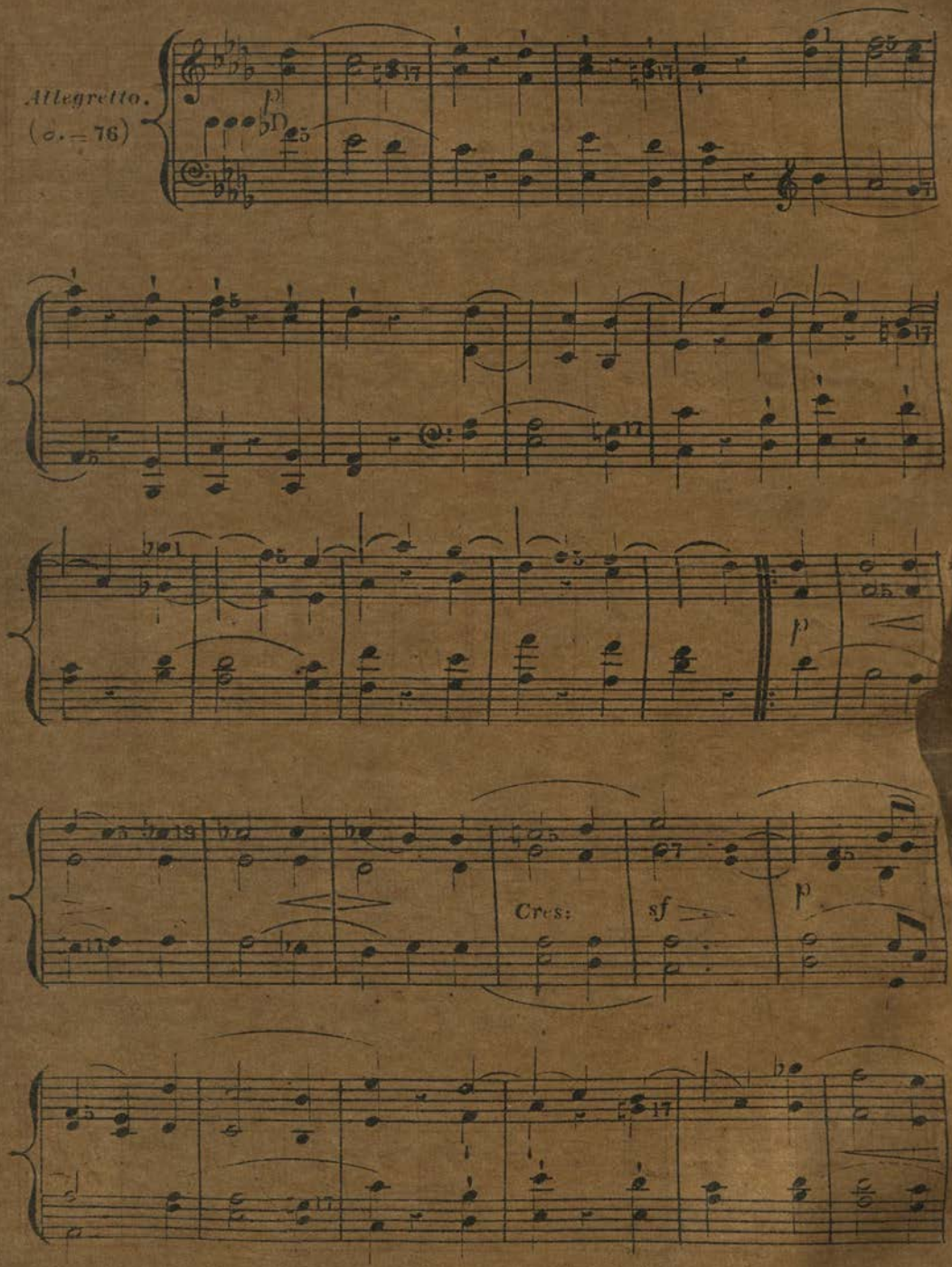


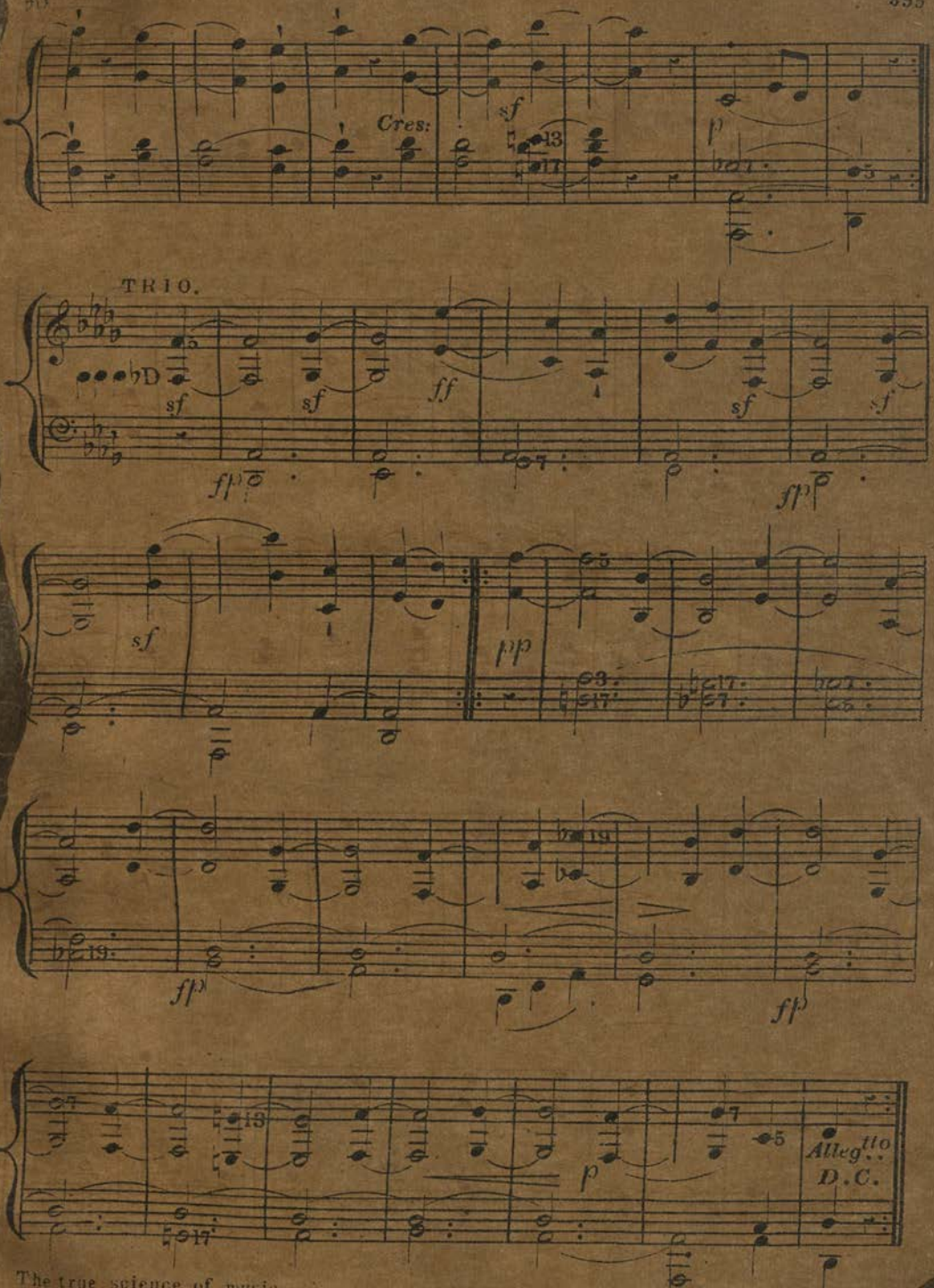

C
Tonic transition to B.
Tonic transition to ♯F.
Tonic transition to ♯C.
Cres.
Decres.

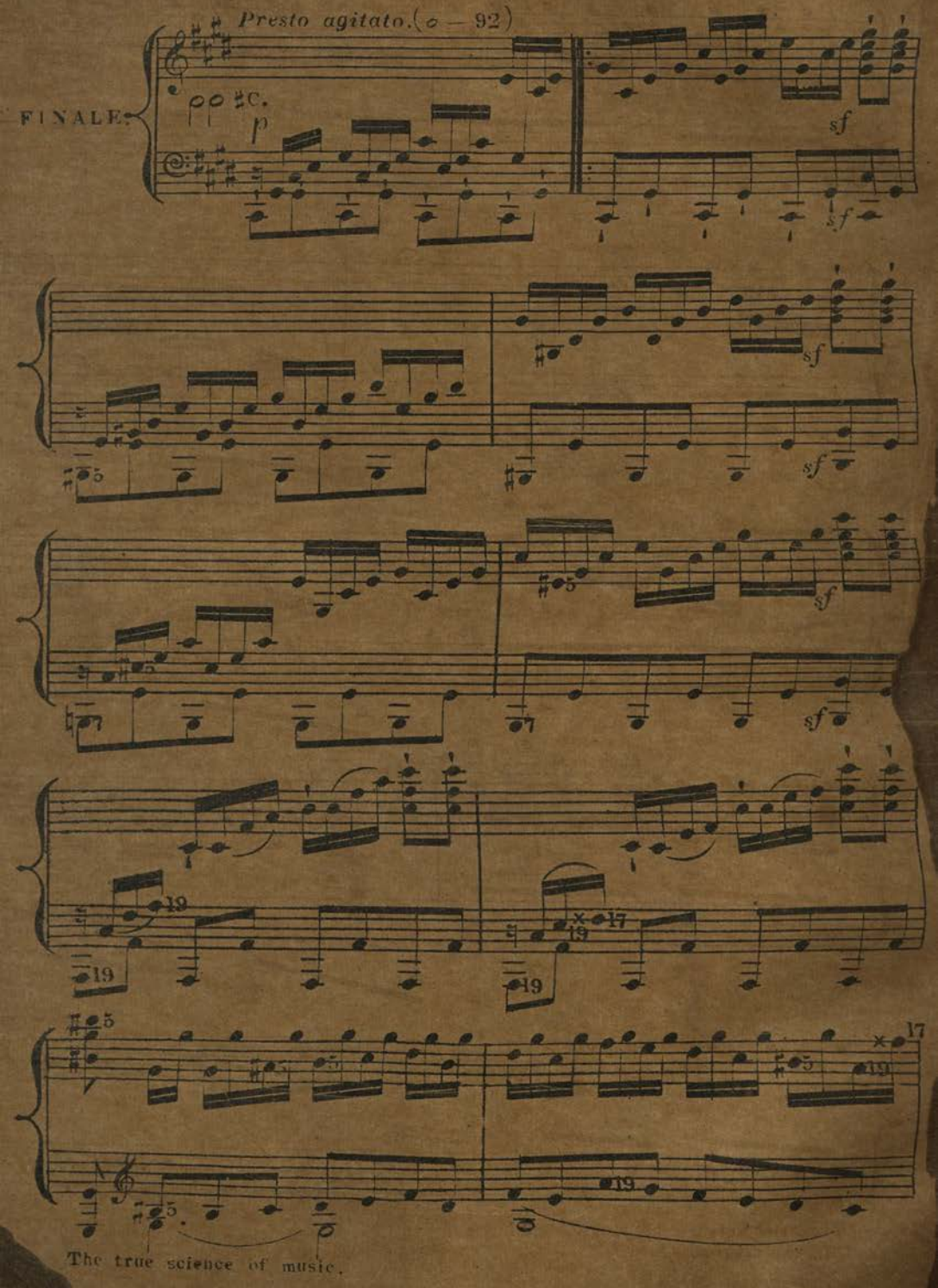

p
Cres:
Decres:

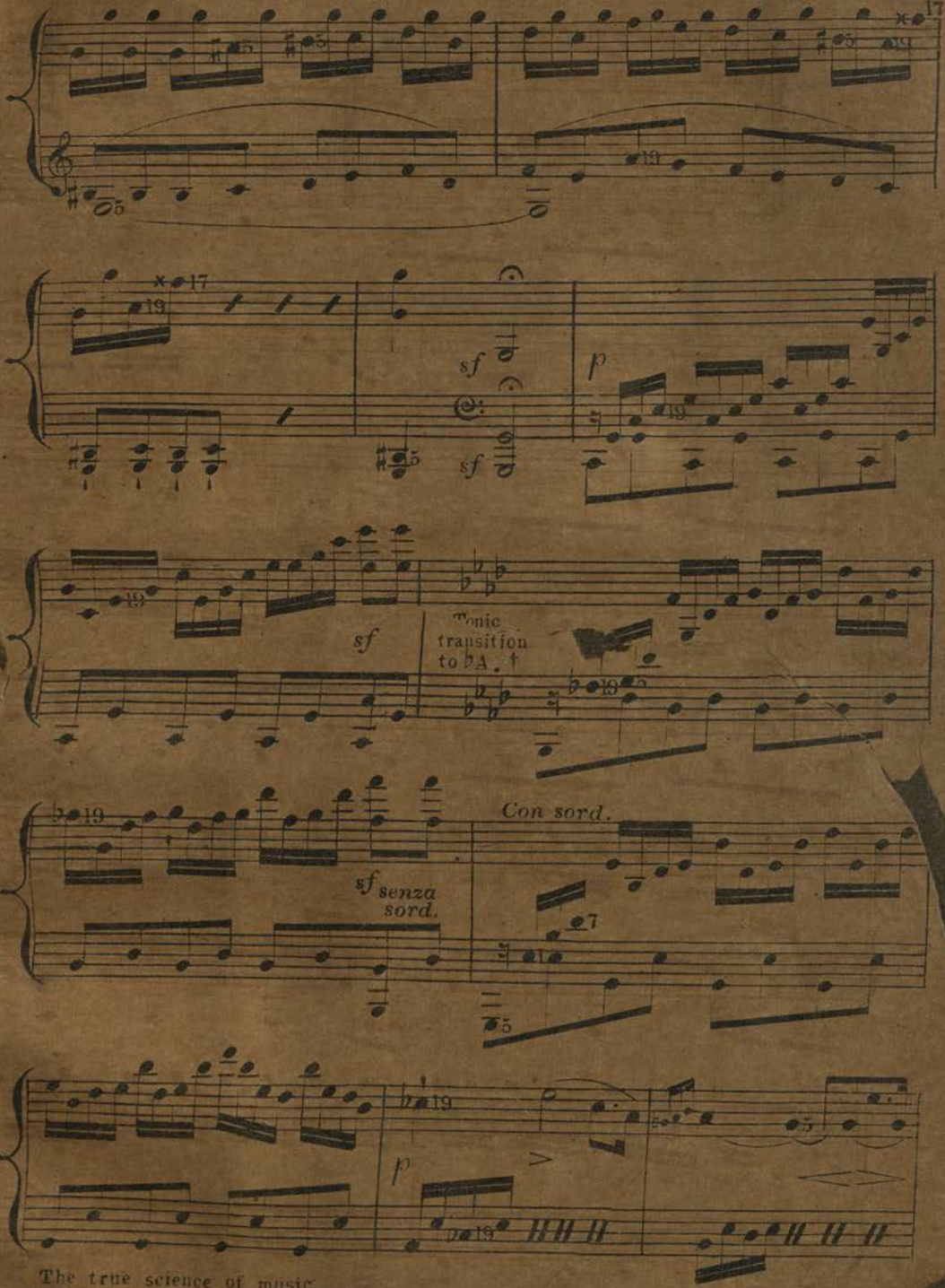

Decres.
pp
pp
Tonic transition to E.
Tonic transition to ♯C.
p


Tonic transition to E.
Tonic transition to ♯C.
p
pp
Decres:
pp
Attacca.

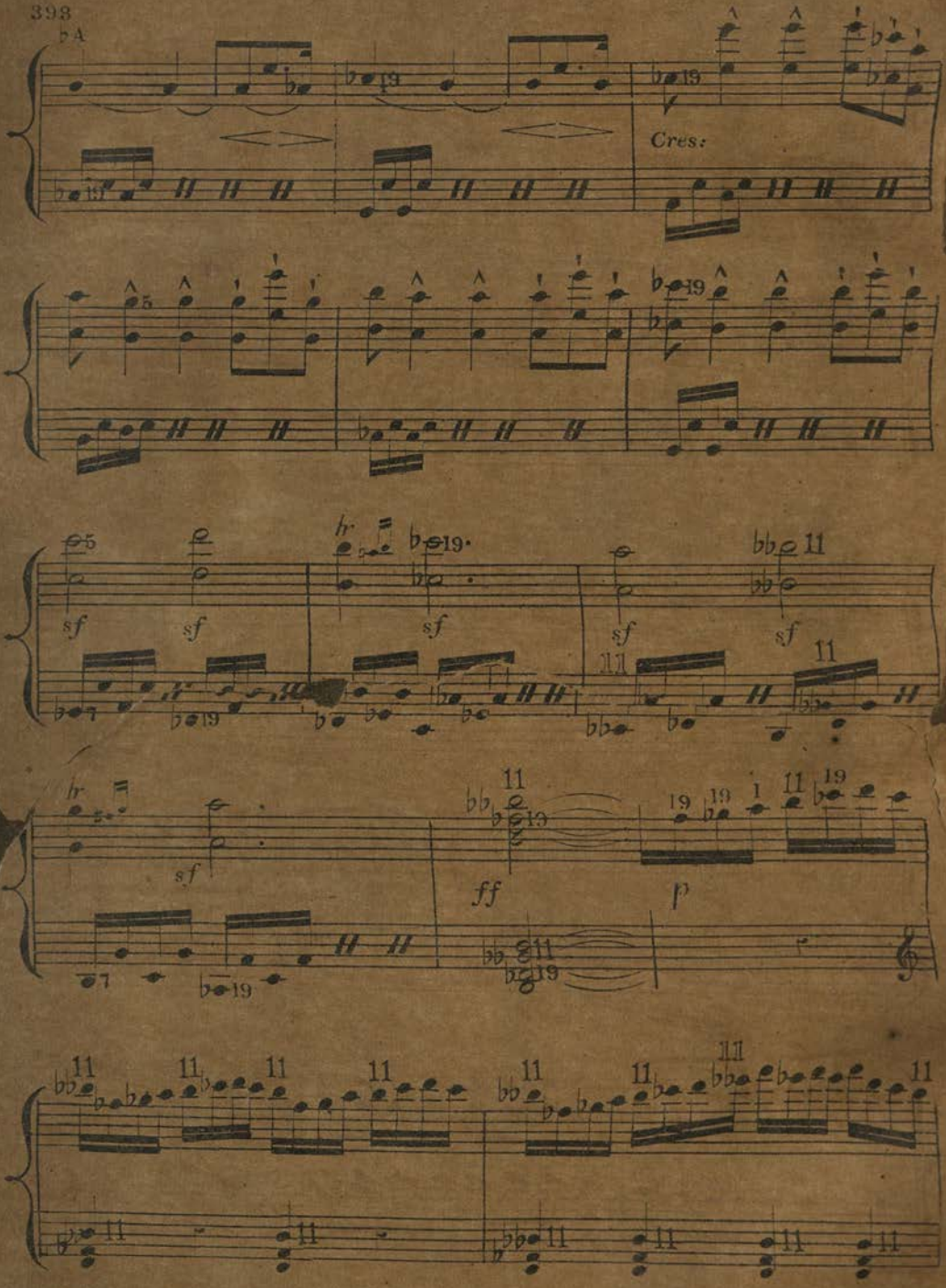

Allegretto.
(♩.=76)
p
Cres:
sf
p

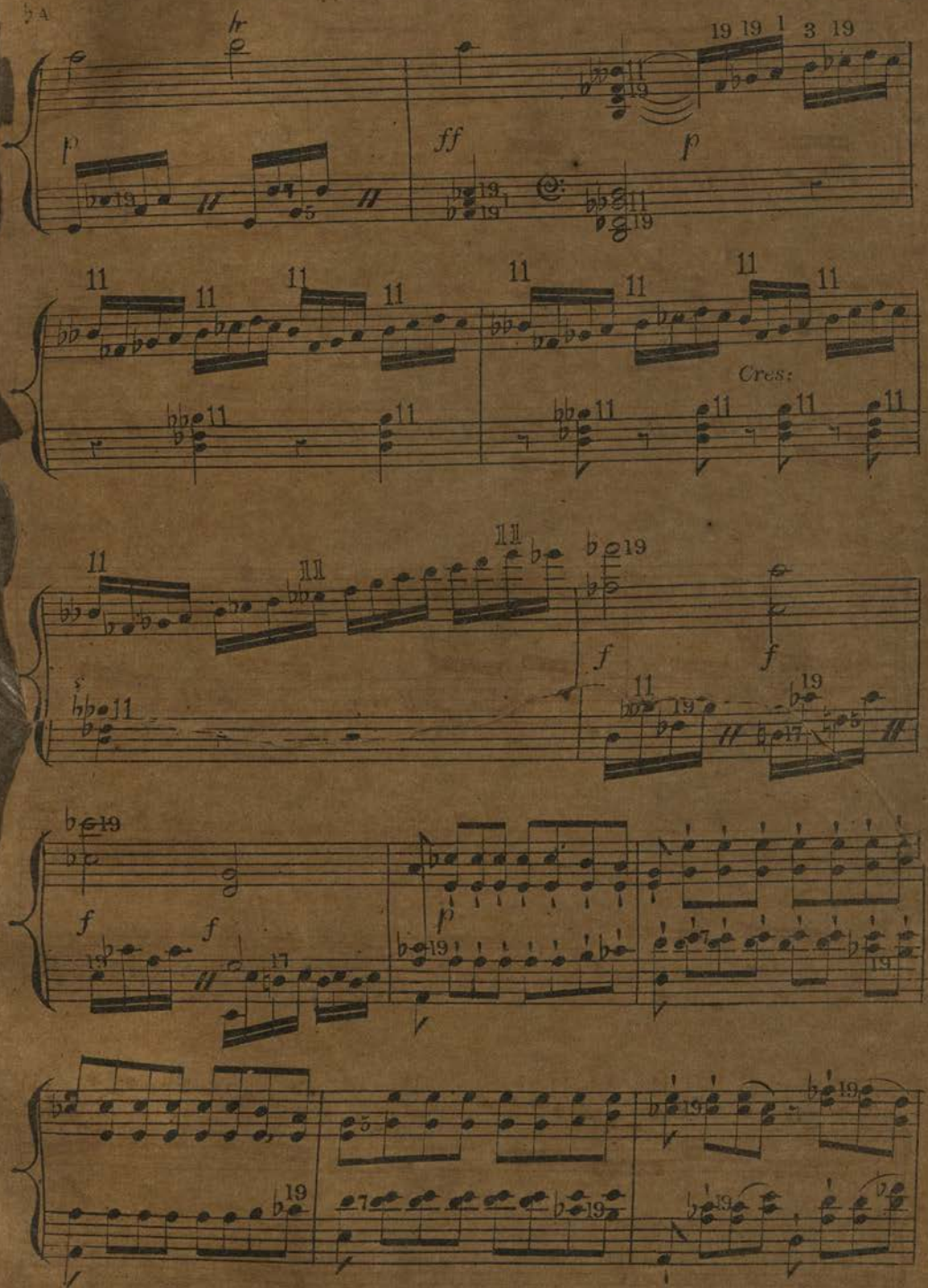

Cres:
sf
p
TRIO.
sf
sf
ff
sf
sf
fp
fp
sf
pp
fp
fp
Allegtto
D.C.

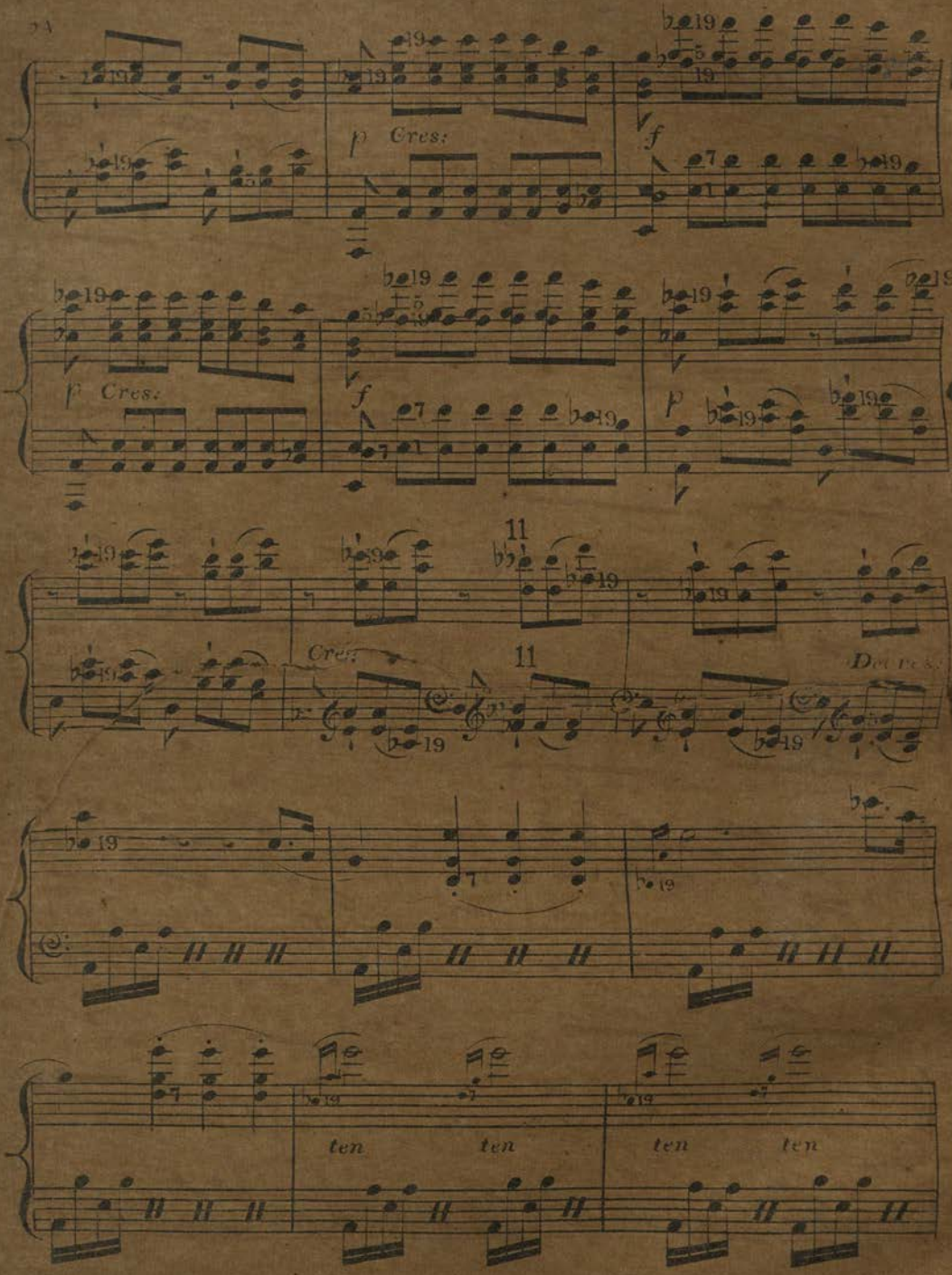

Presto agitato. (𝅗𝅥 = 92)
FINALE.
p
sf

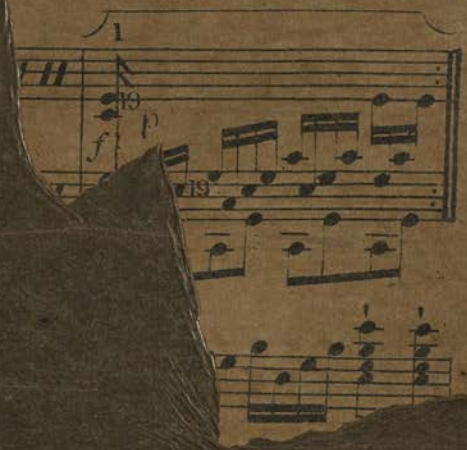


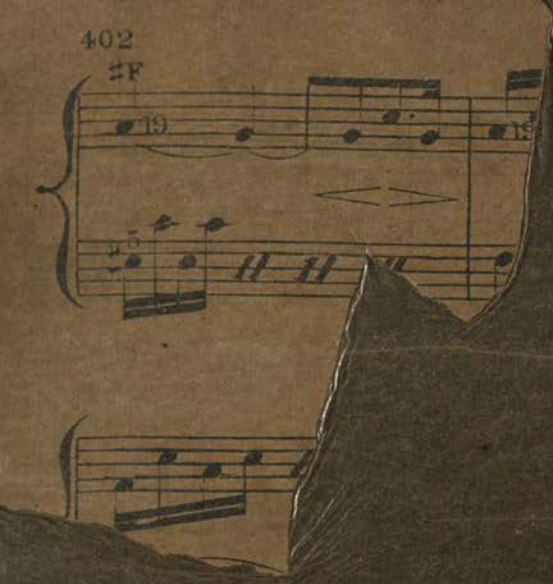
The true science of music.
† In the original the key is ♯G.

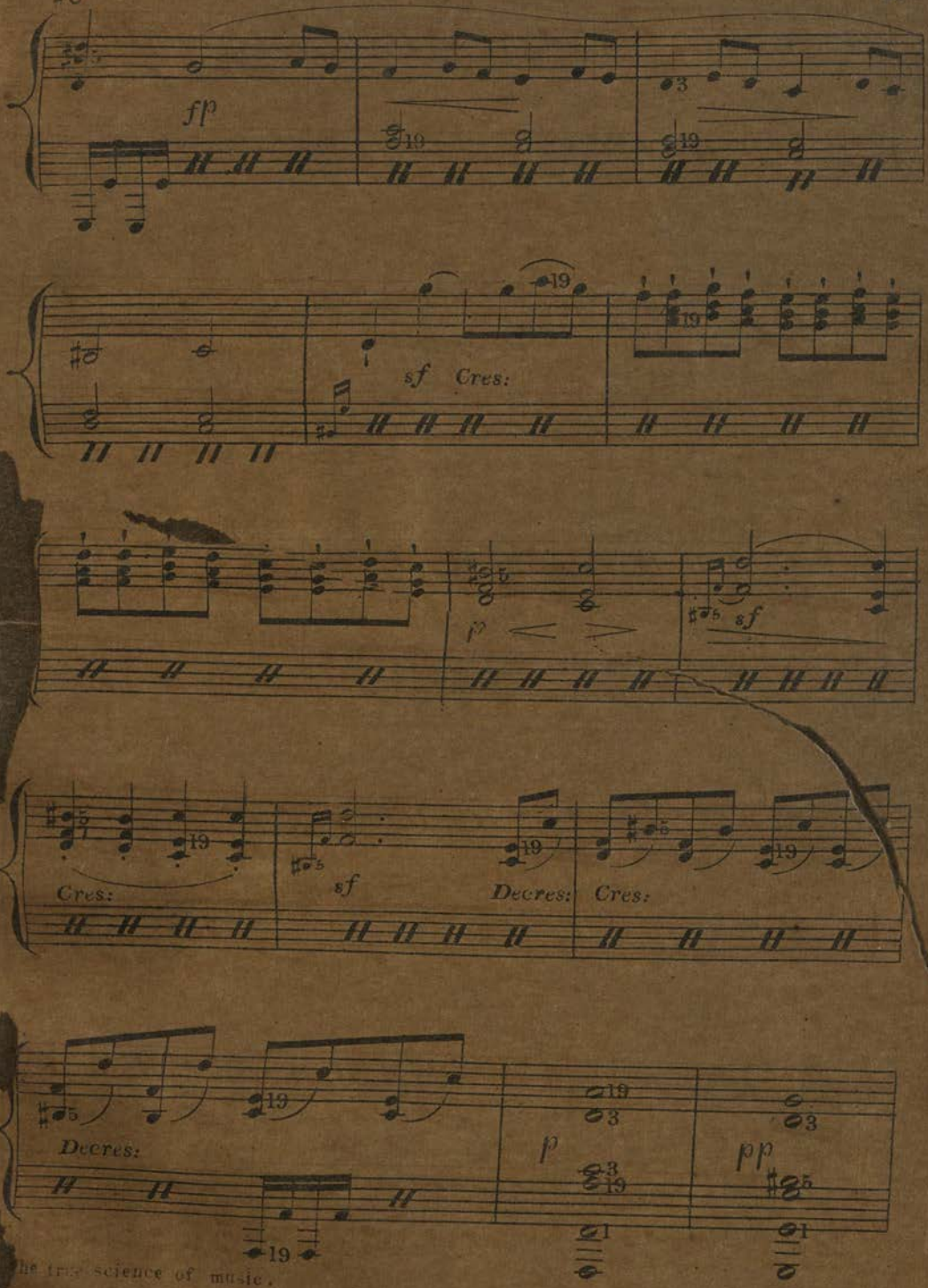

A
Cres:
sf
ff
p

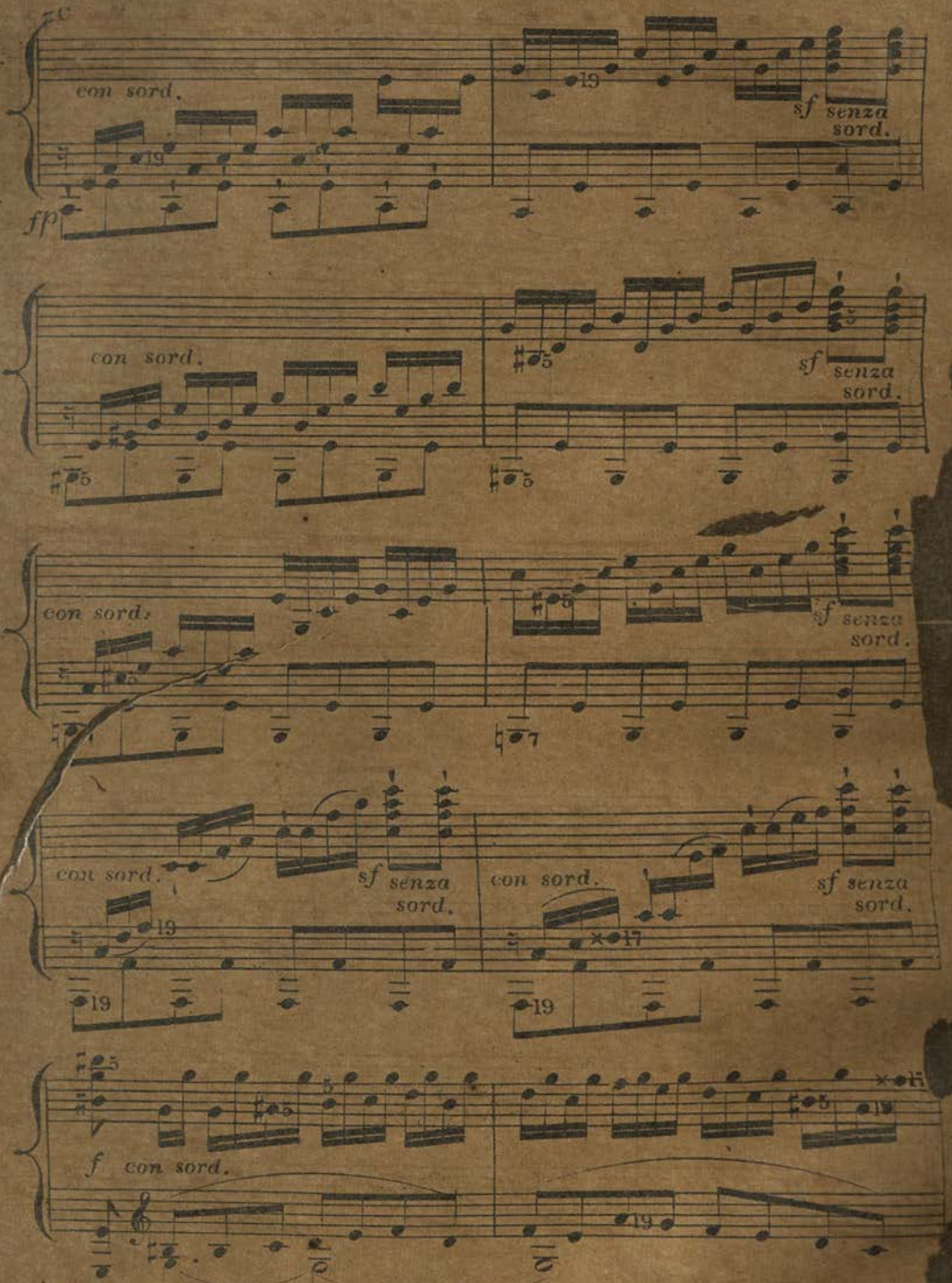

Cres:

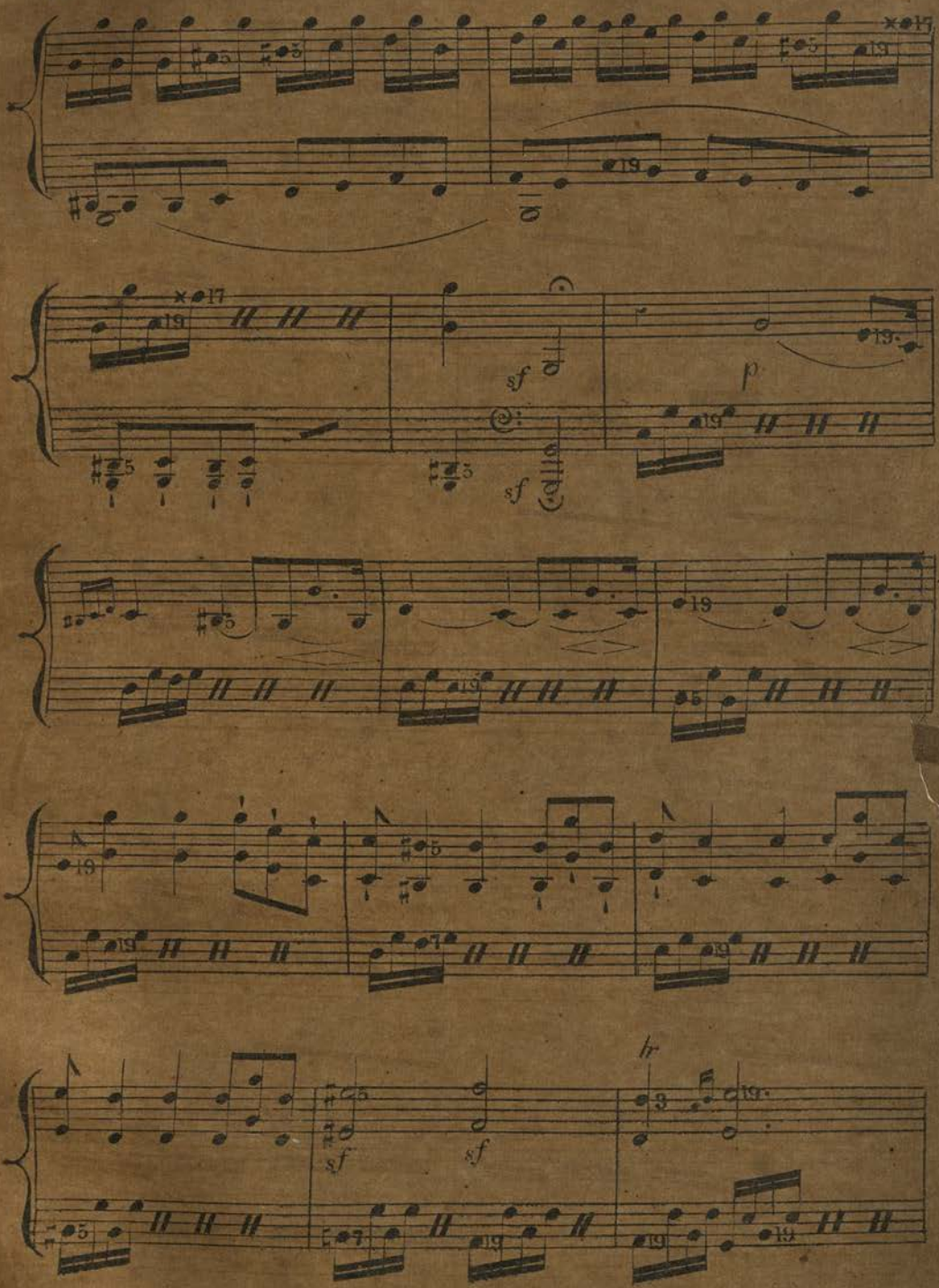

p Cres:
f
p Cres:
f
p
11
Cres:
11
ten
ten
ten
ten

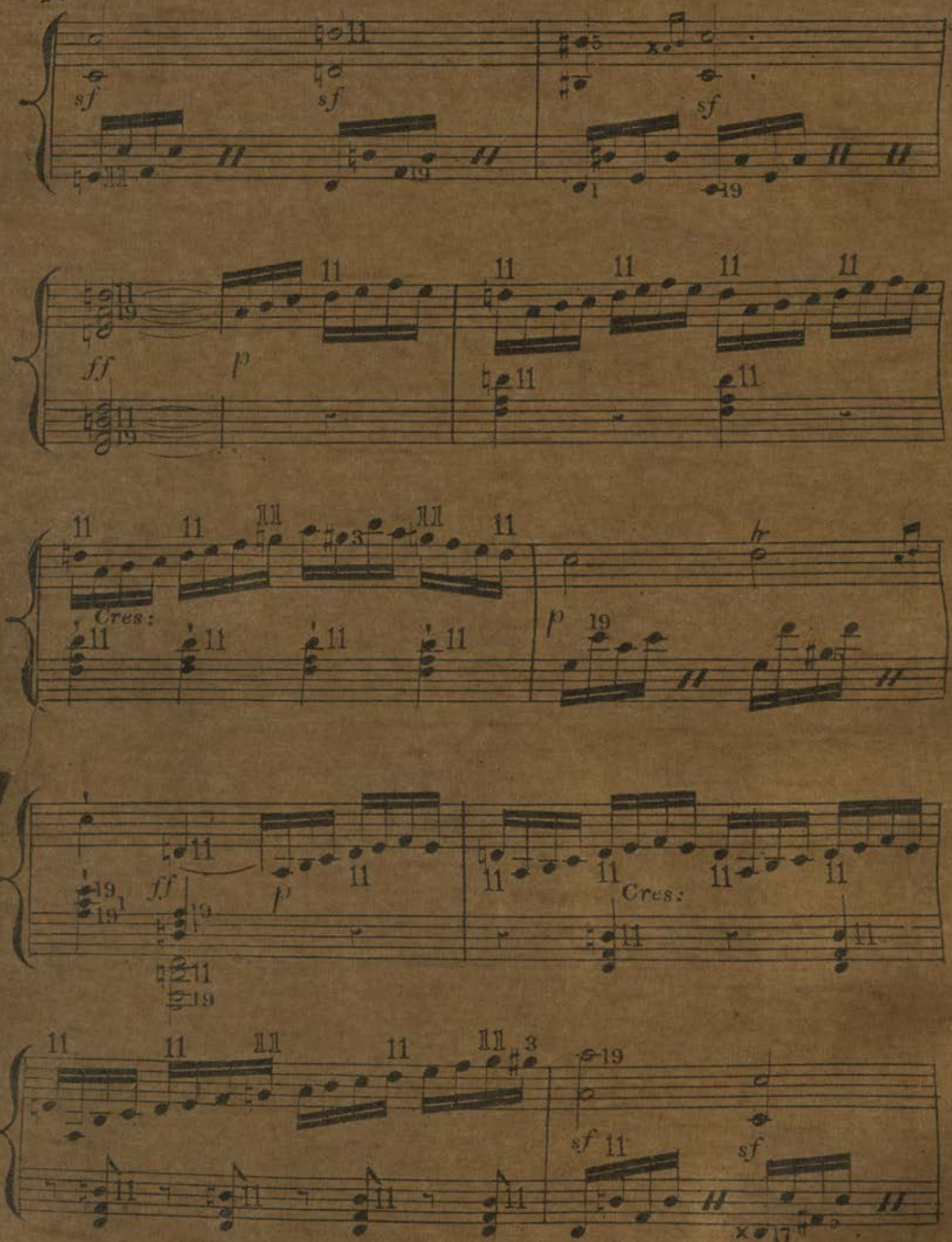

1
10
f
p
19

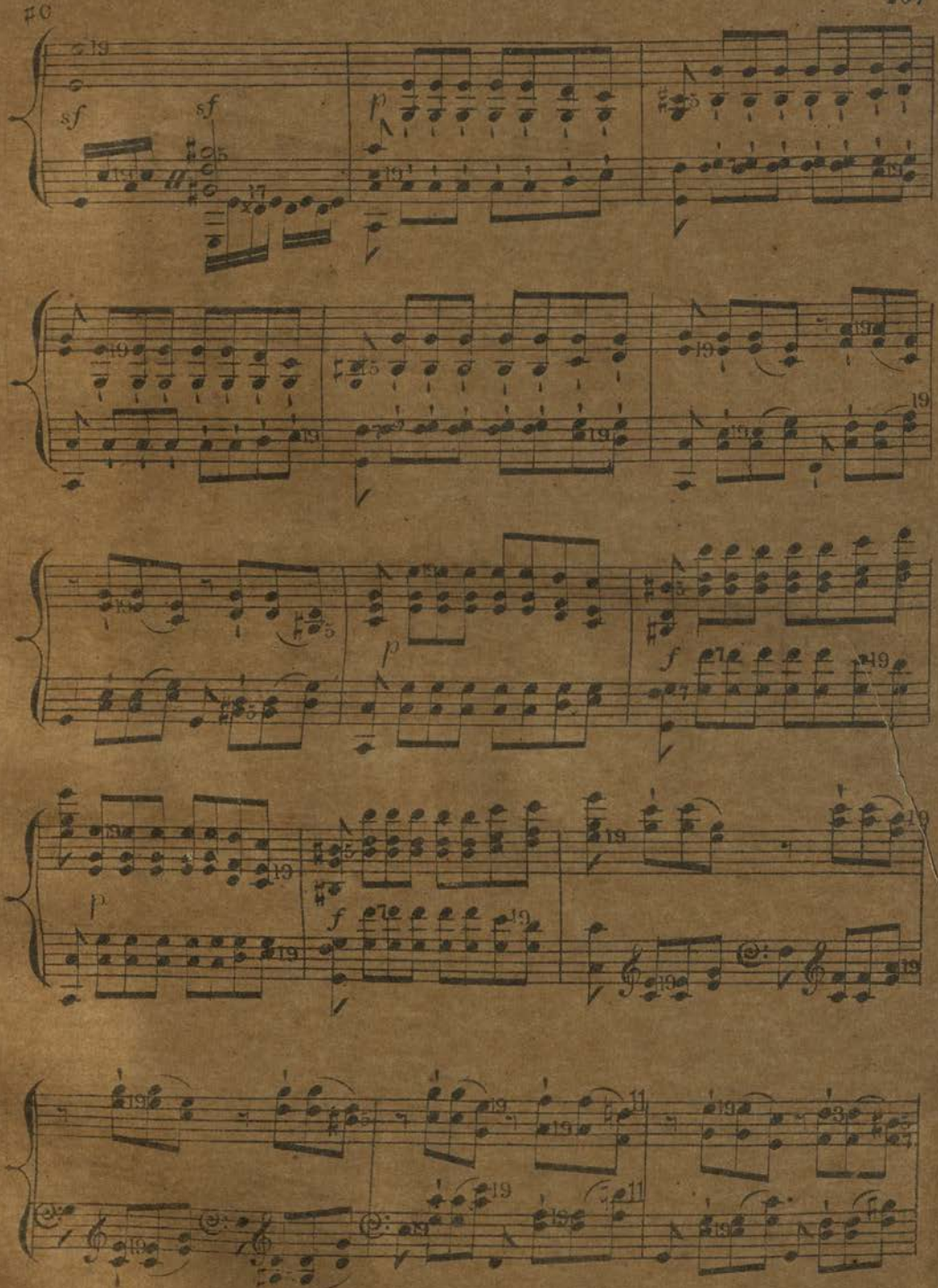

402
#F
19

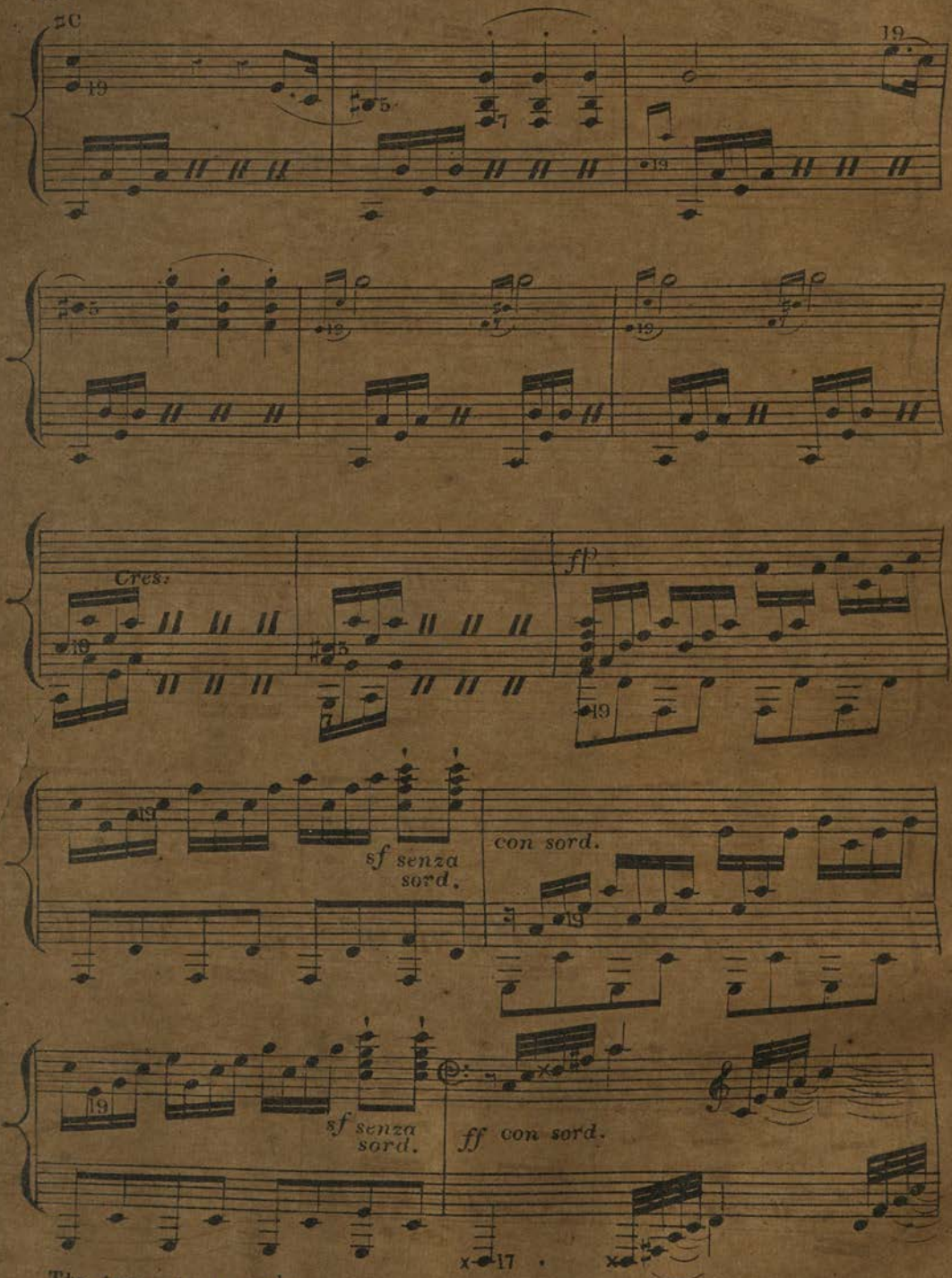

ff
sf Cres:
p
sf
Cres:
sf
Decres:
Cres:
Decres:
p
pp

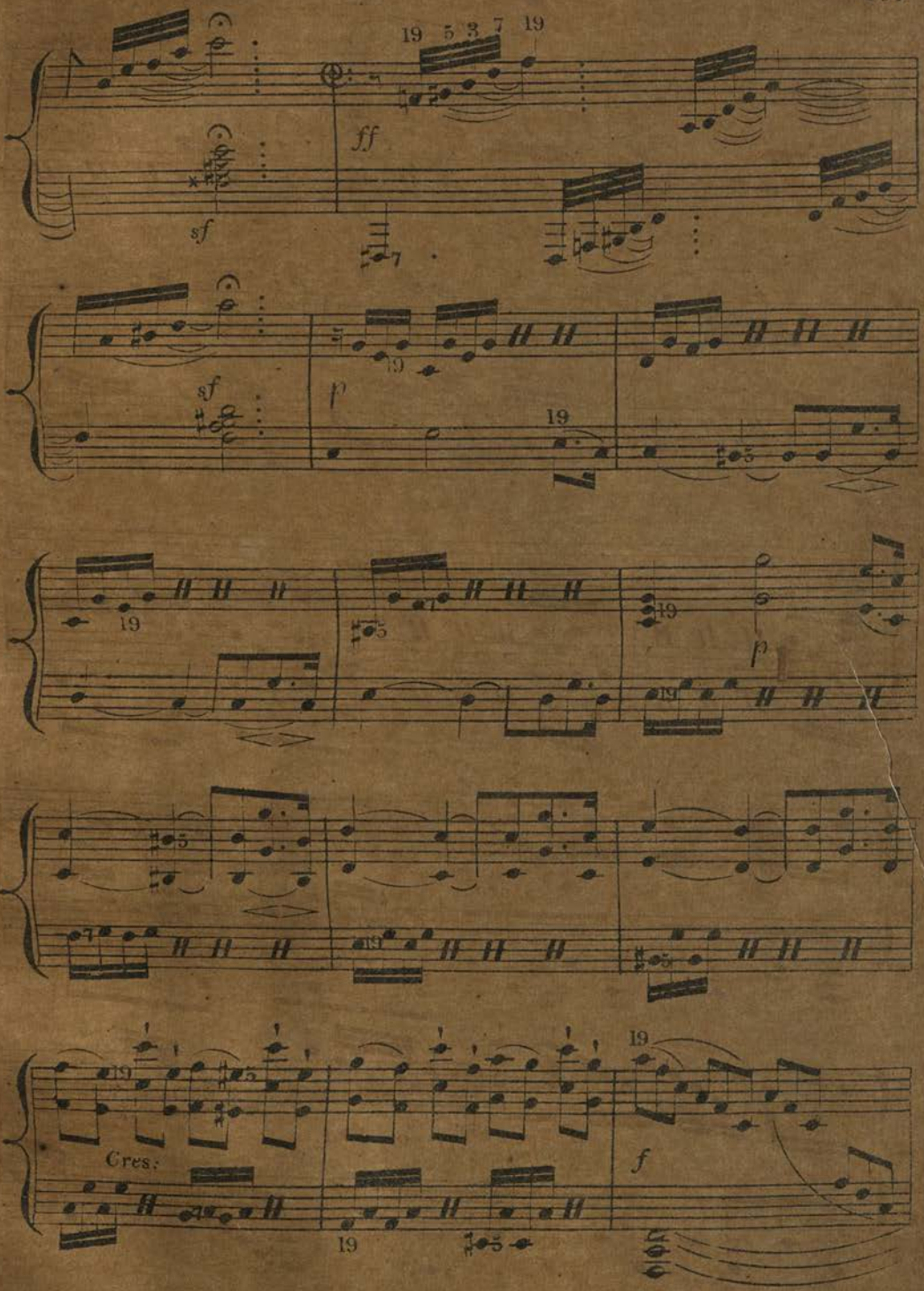

con sord.
sf senza sord.
fp
con sord.
sf senza sord.
con sord.
sf senza sord.
con sord.
sf senza sord.
con sord.
sf senza sord.
f con sord.

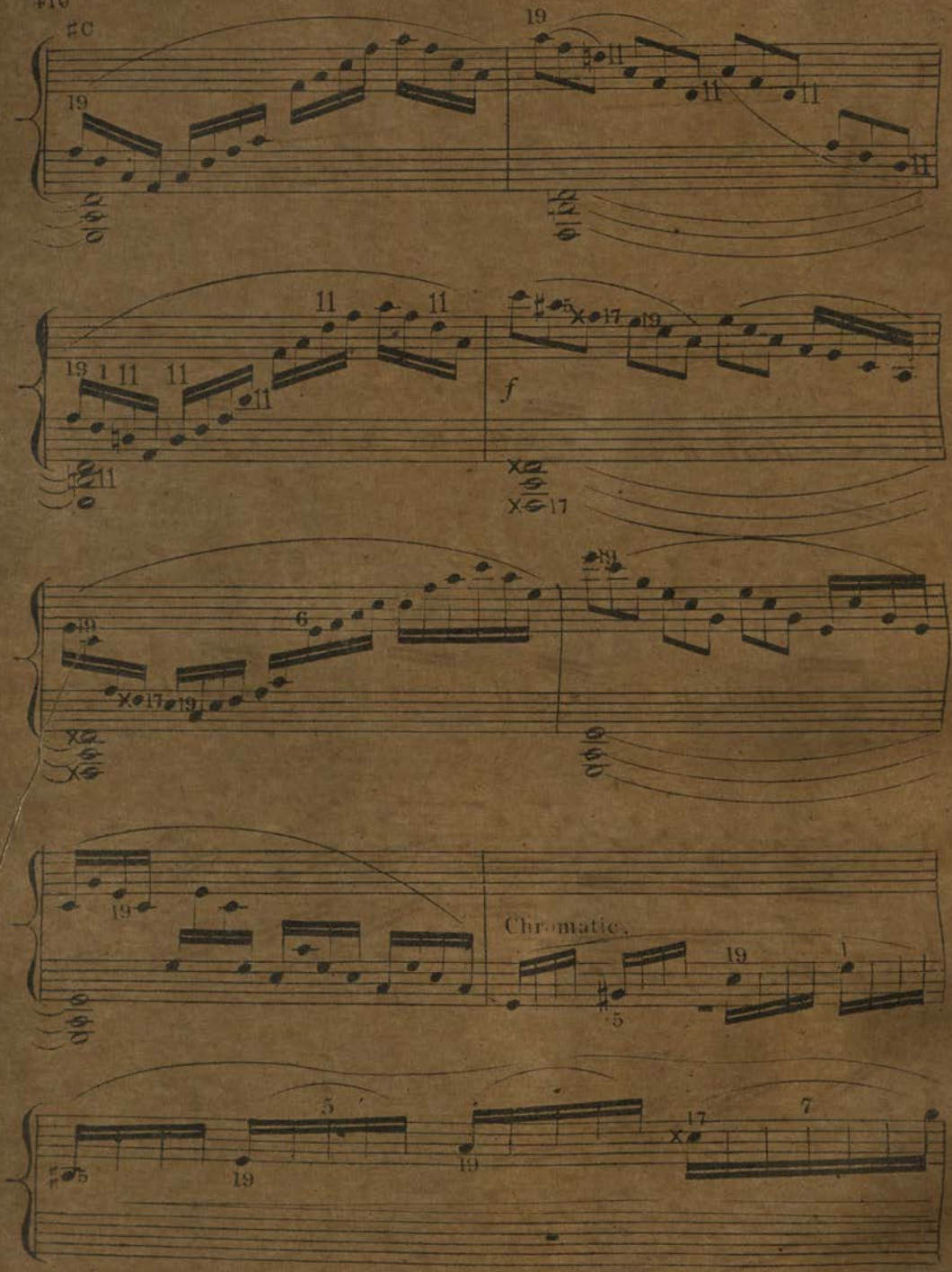

sf
p
sf
sf
sf
tr

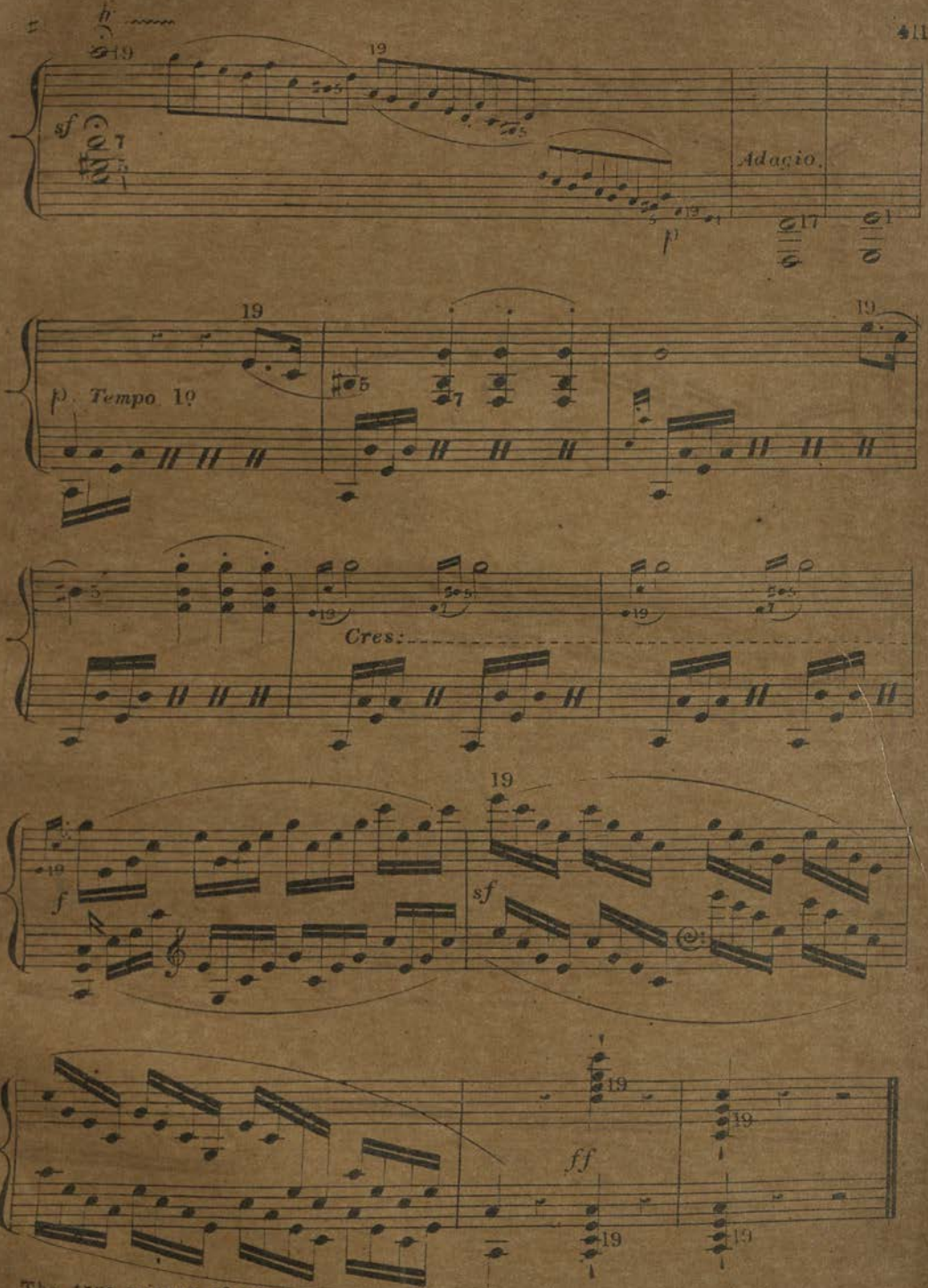

sf
sf
sf
ff
p
Cres:
p
ff
p
Cres:
sf
sf




Cres:
ff
sf senza sord.
con sord.
sf senza sord.
ff con sord.


19 5 3 7 19
ff
sf
sf
p
19
19
19
p
19
Cres:
f
19


Chromatic.


sf
Adagio.
p
p Tempo 1º
Cres:
f
sf
ff

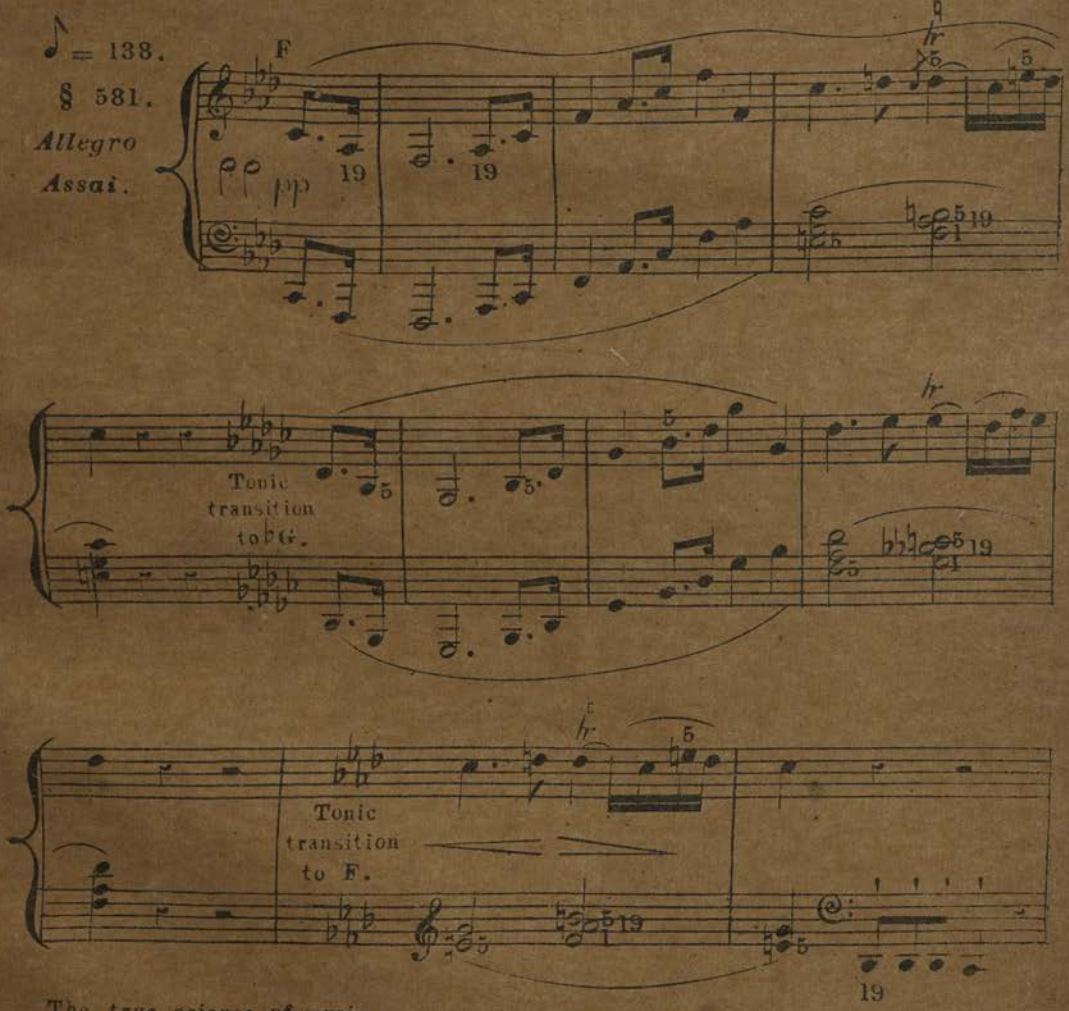
BEETHOVEN'S

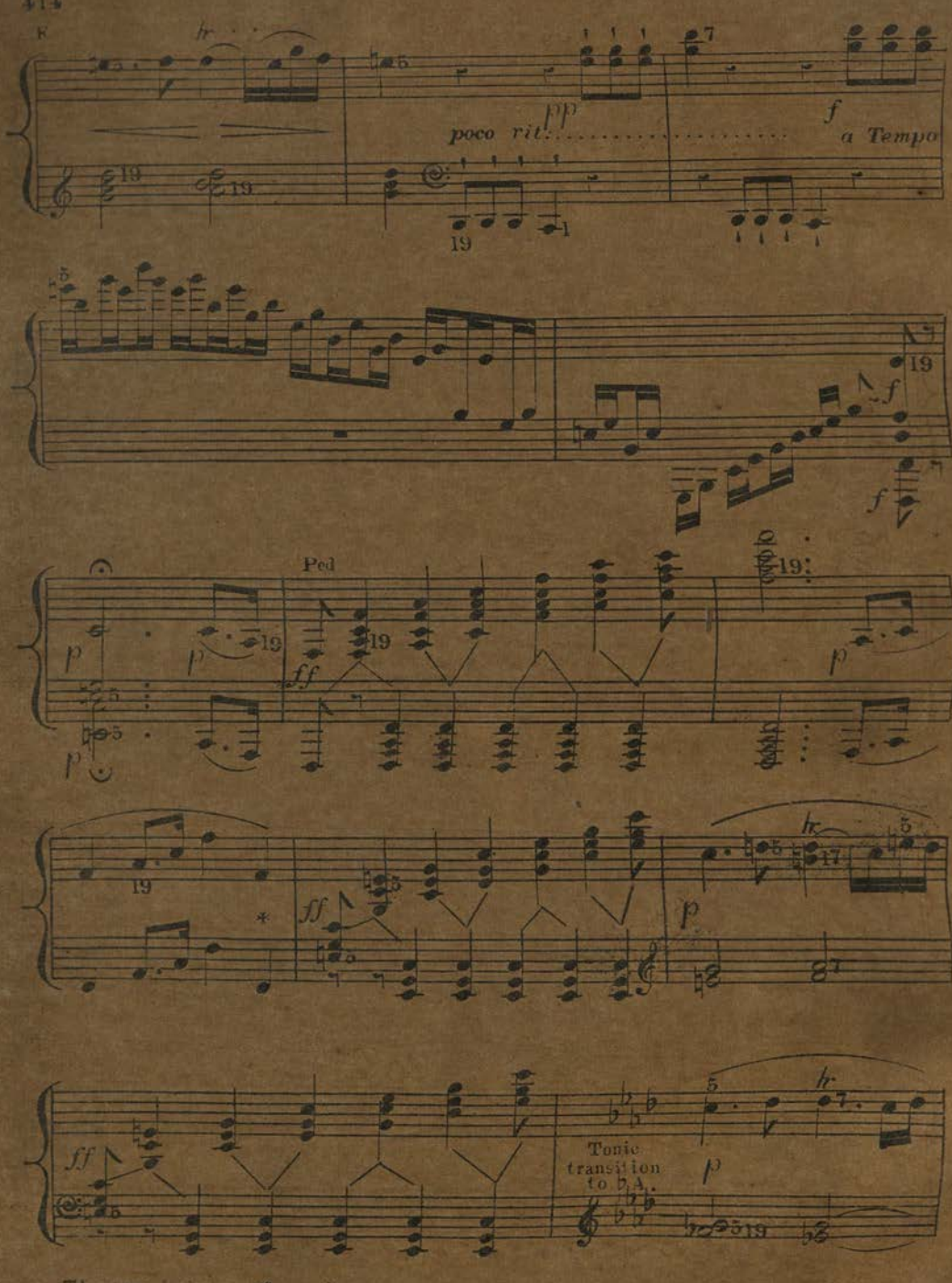
SONATA APPASSIONATA

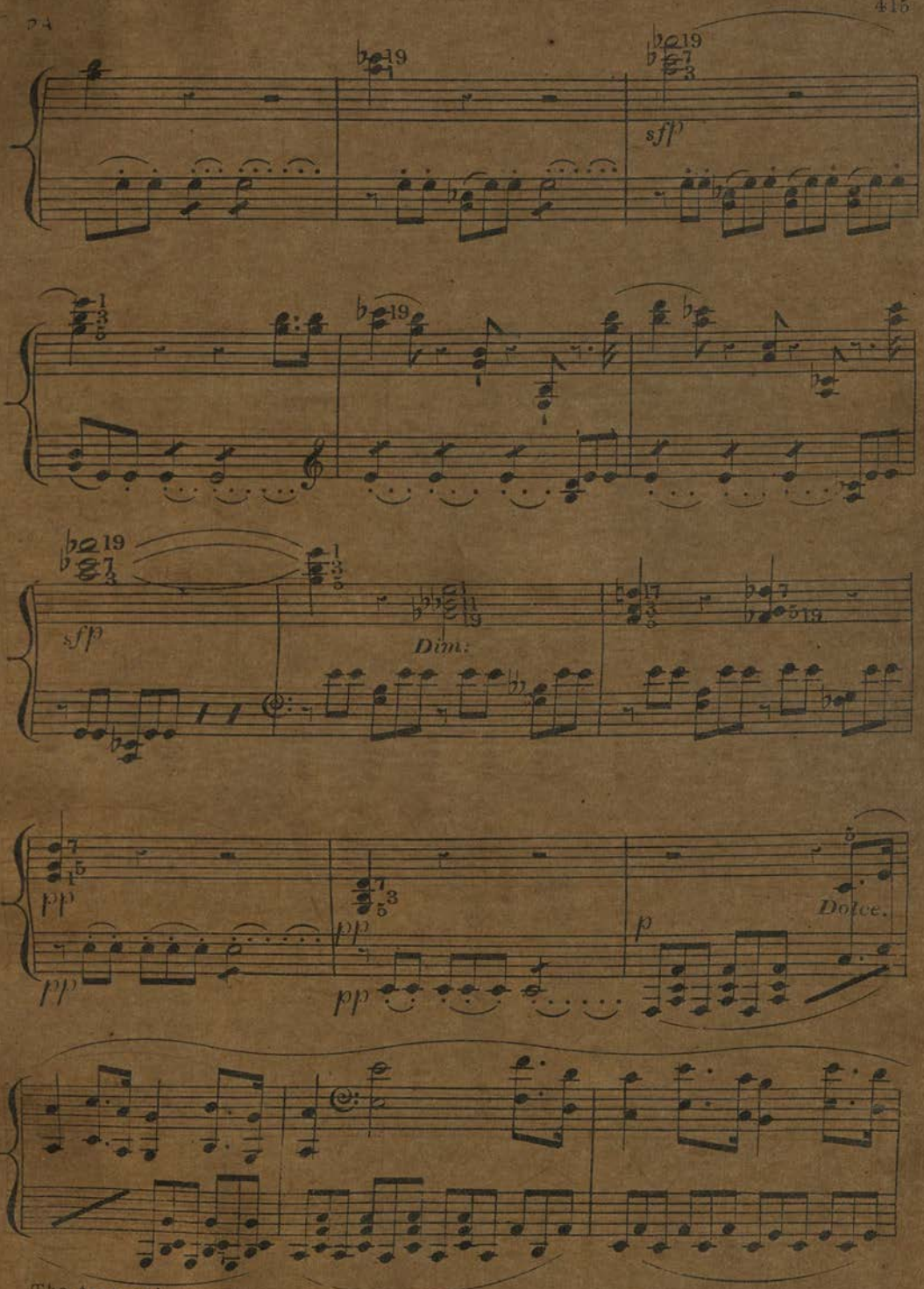
Op: 57.

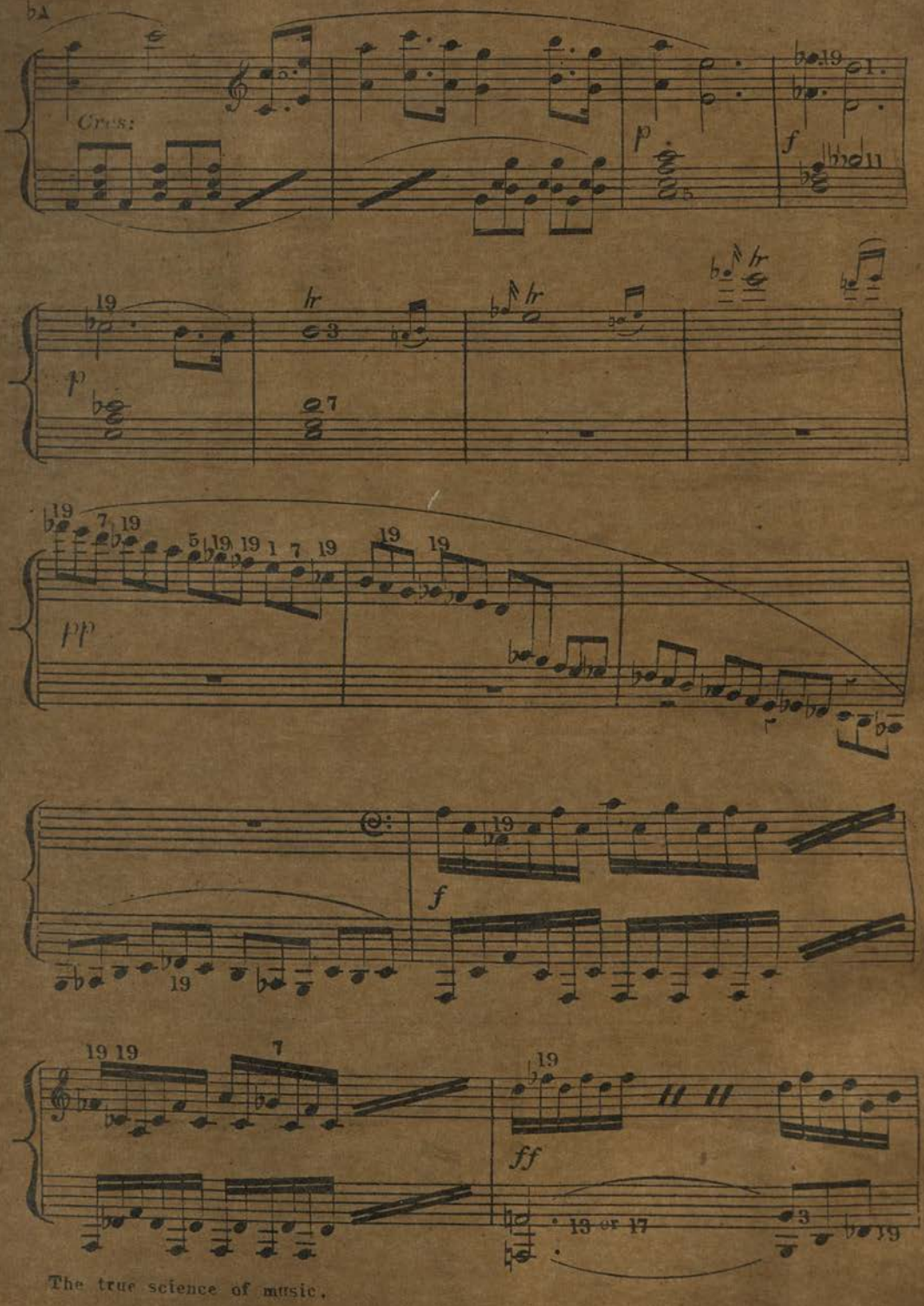
ANALYSED BY

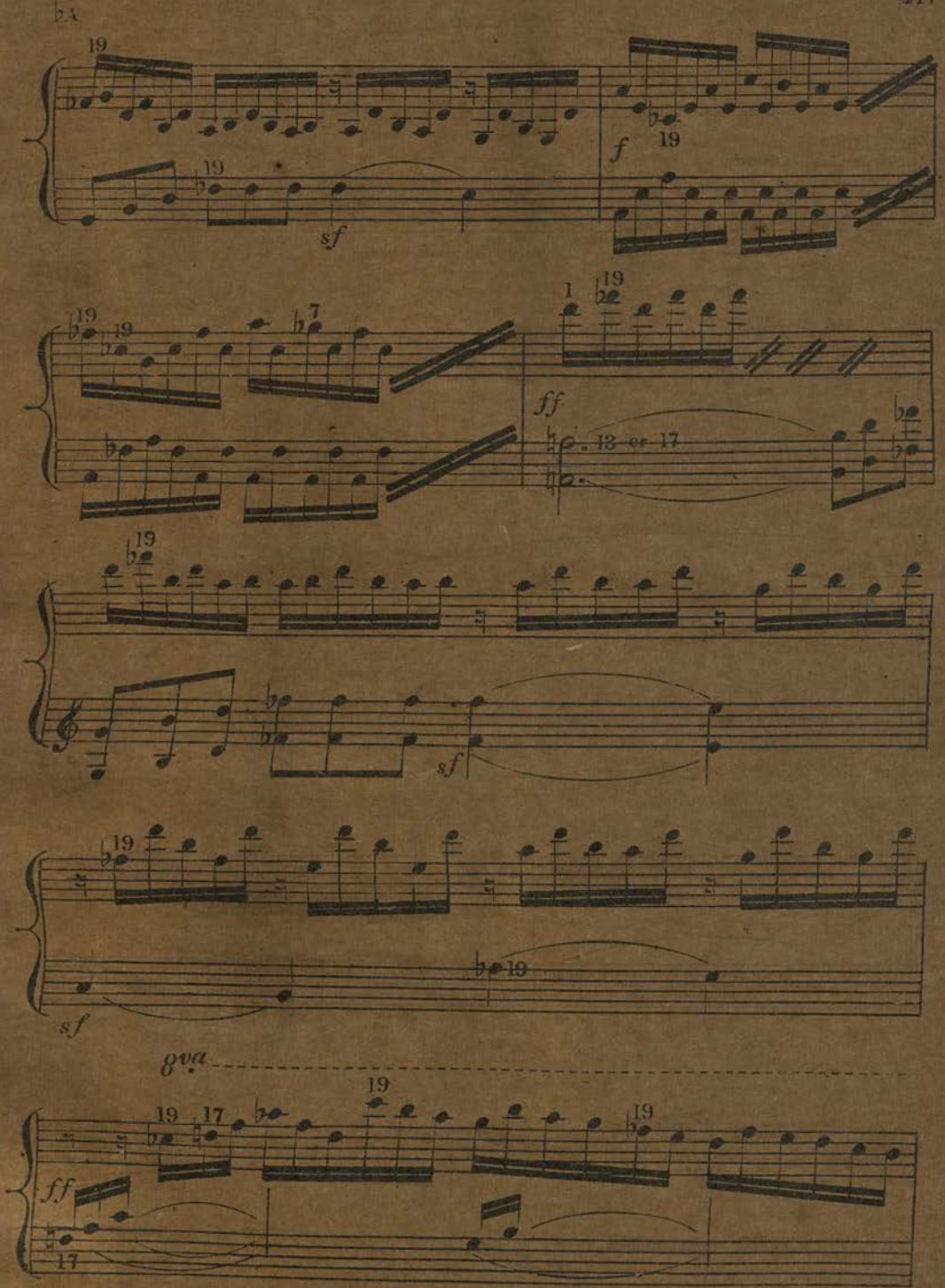
D. C. HEWITT.

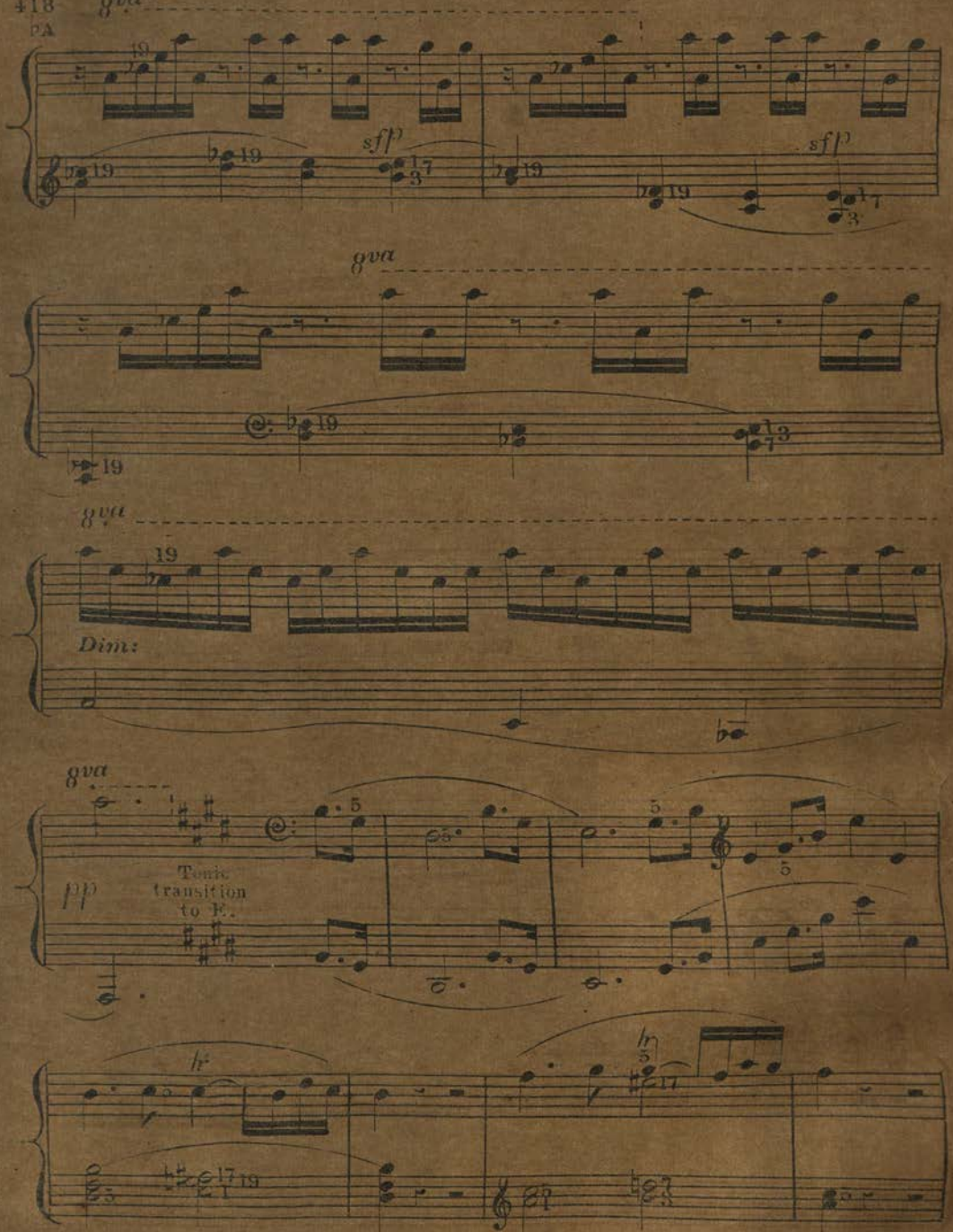



poco rit.
pp
f
a Tempo
Ped
ff
Tonic
transition
to b A.

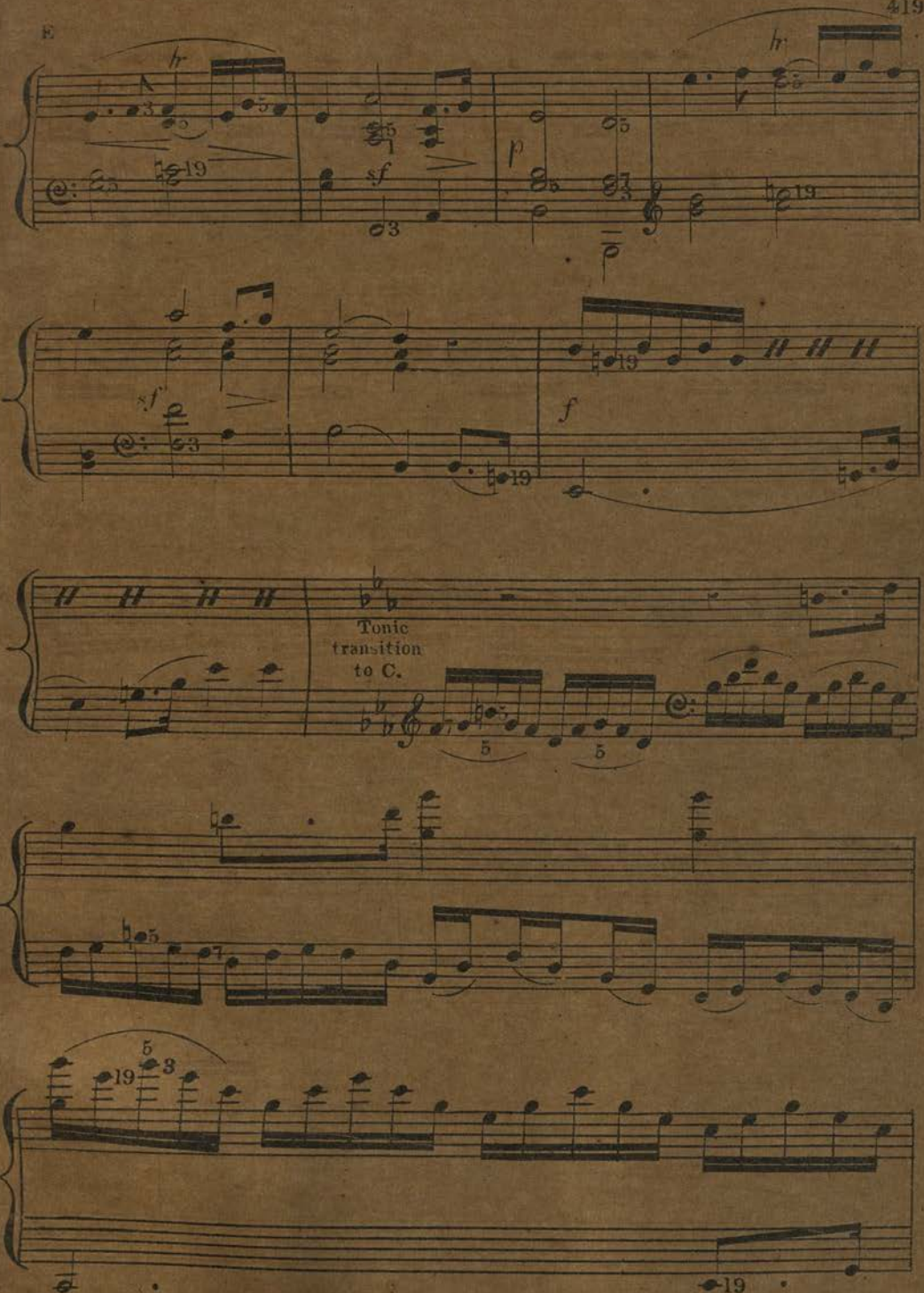

sff
sfp
Dim:
pp
pp
pp
p
Dolce.


Cres:
p
f
p
pp
f
ff
13 or 17

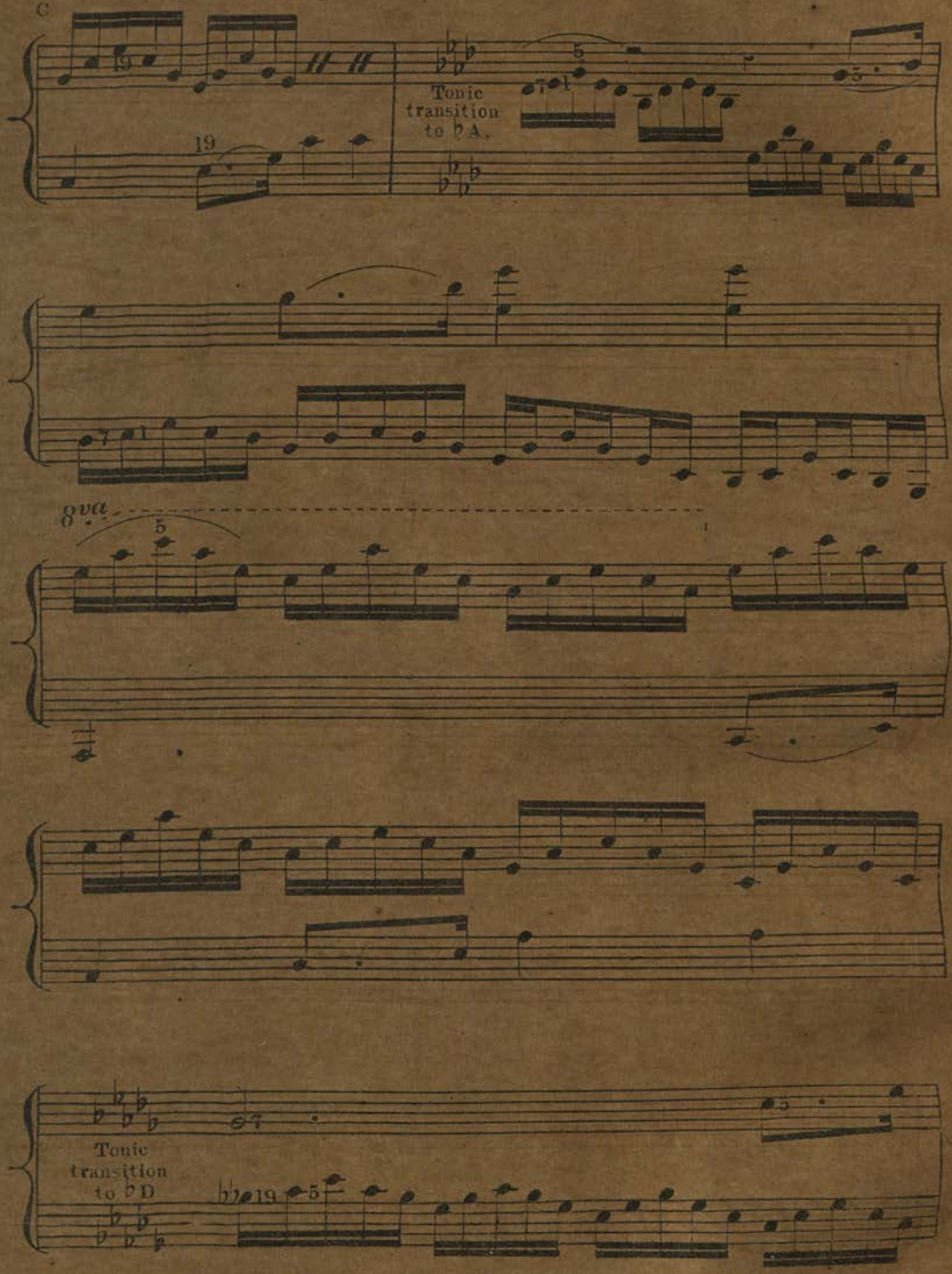

bA
19
19
sf
f
19
19
19
7
1
19
ff
18 or 17
19
sf
19
19
sf
8va
19
19
17
19
19
ff
17

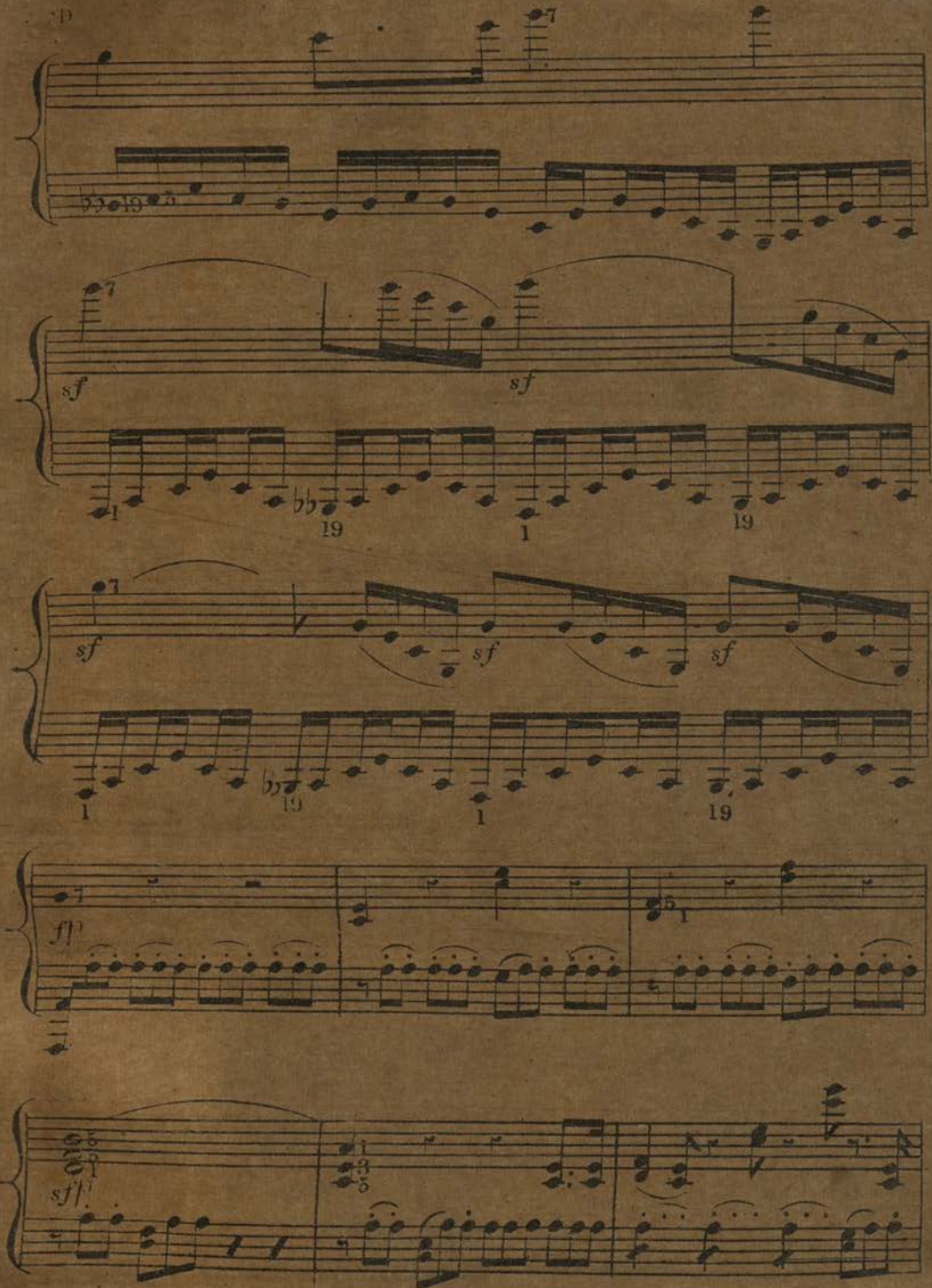

8va
sfp
sfp
8va
8va
Dim:
8va
pp
Tonic transition to E.
hr
hr

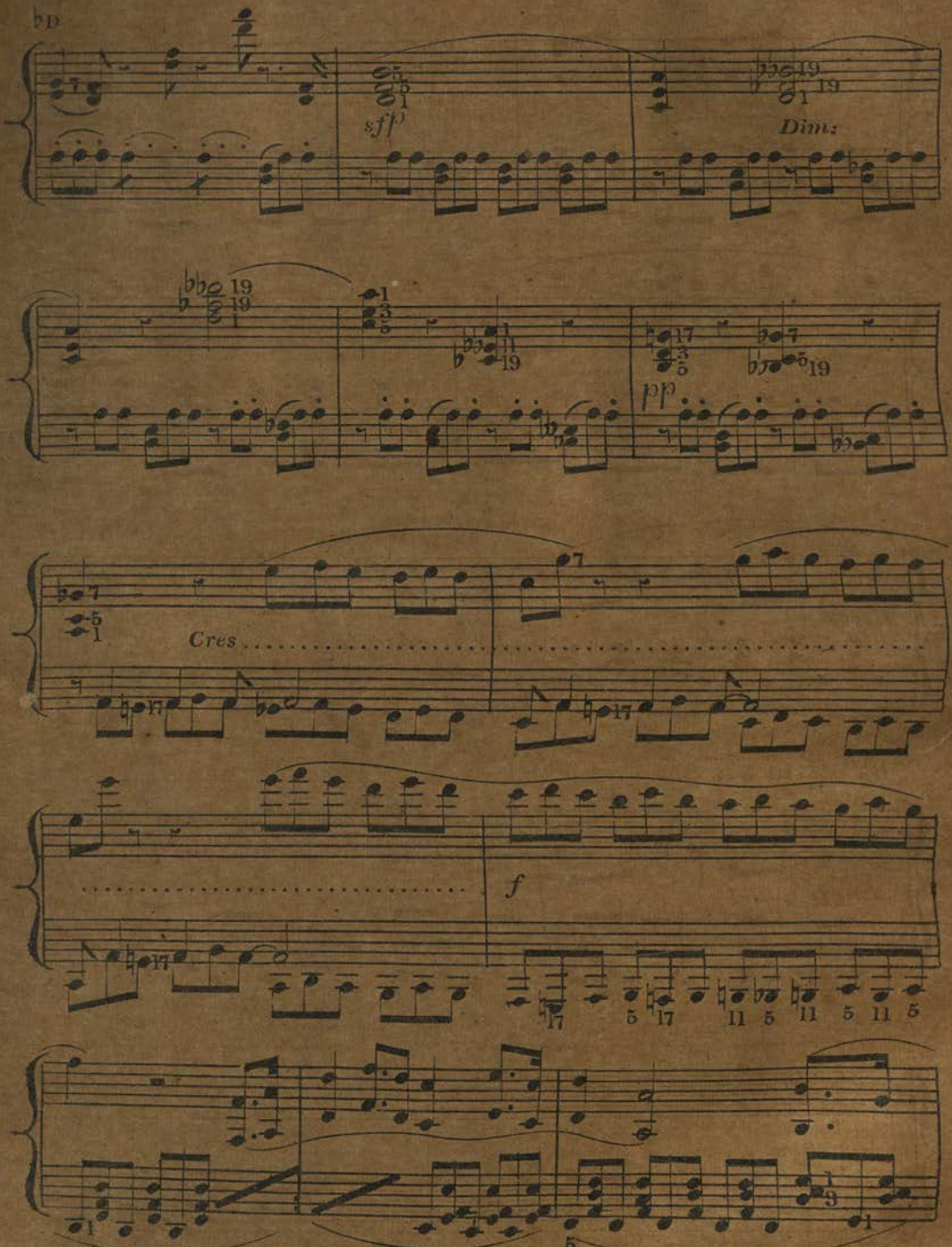
E

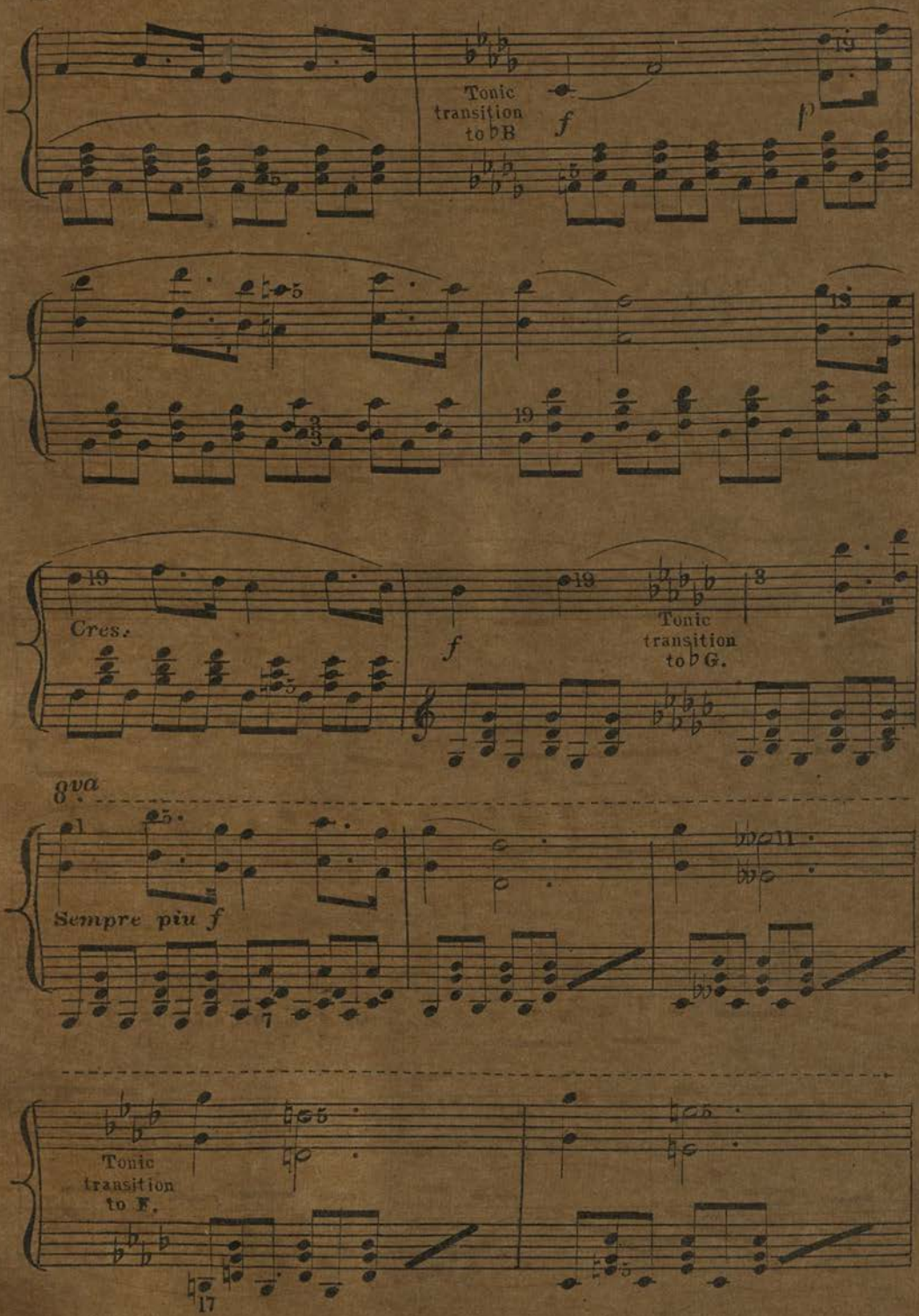


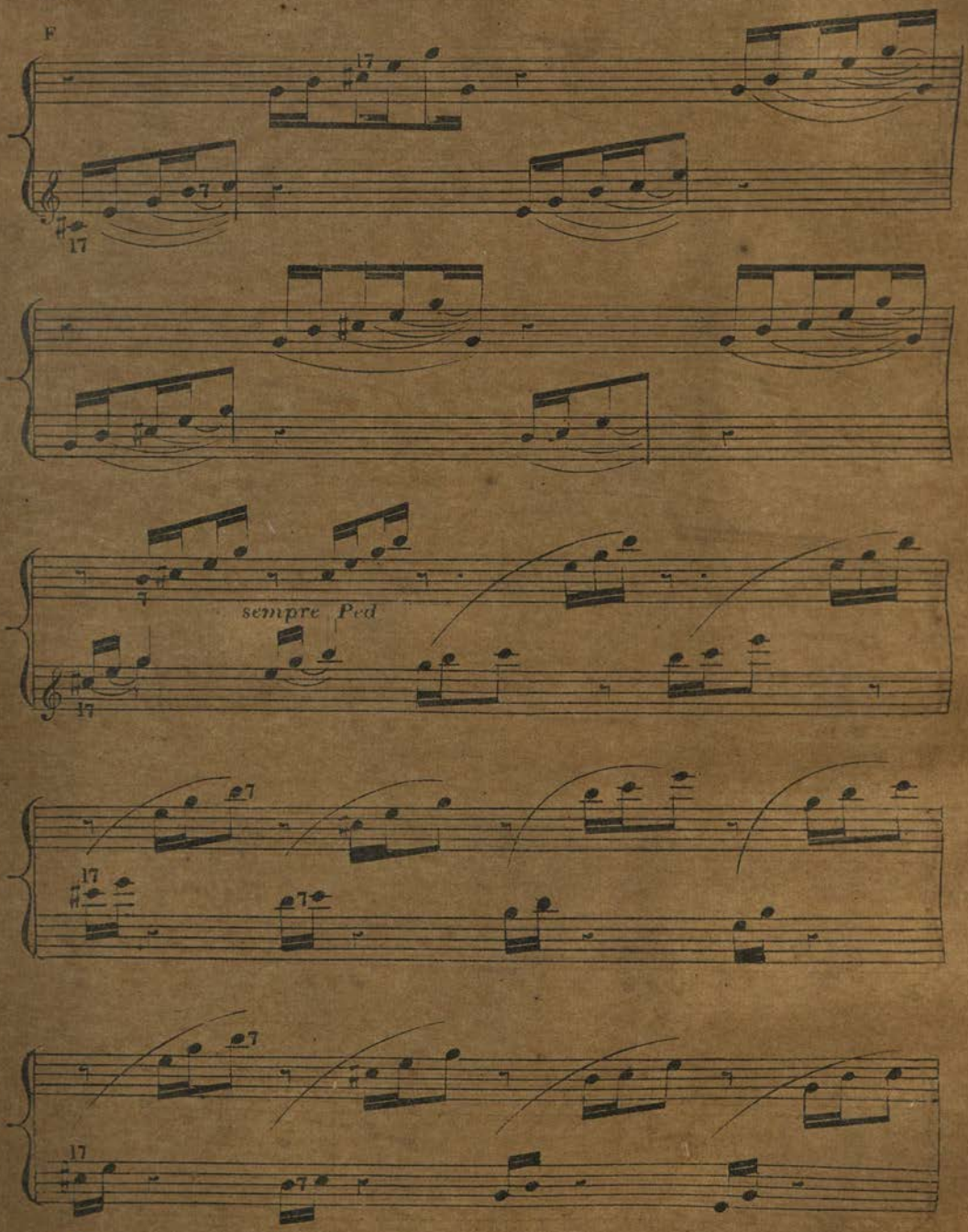
C

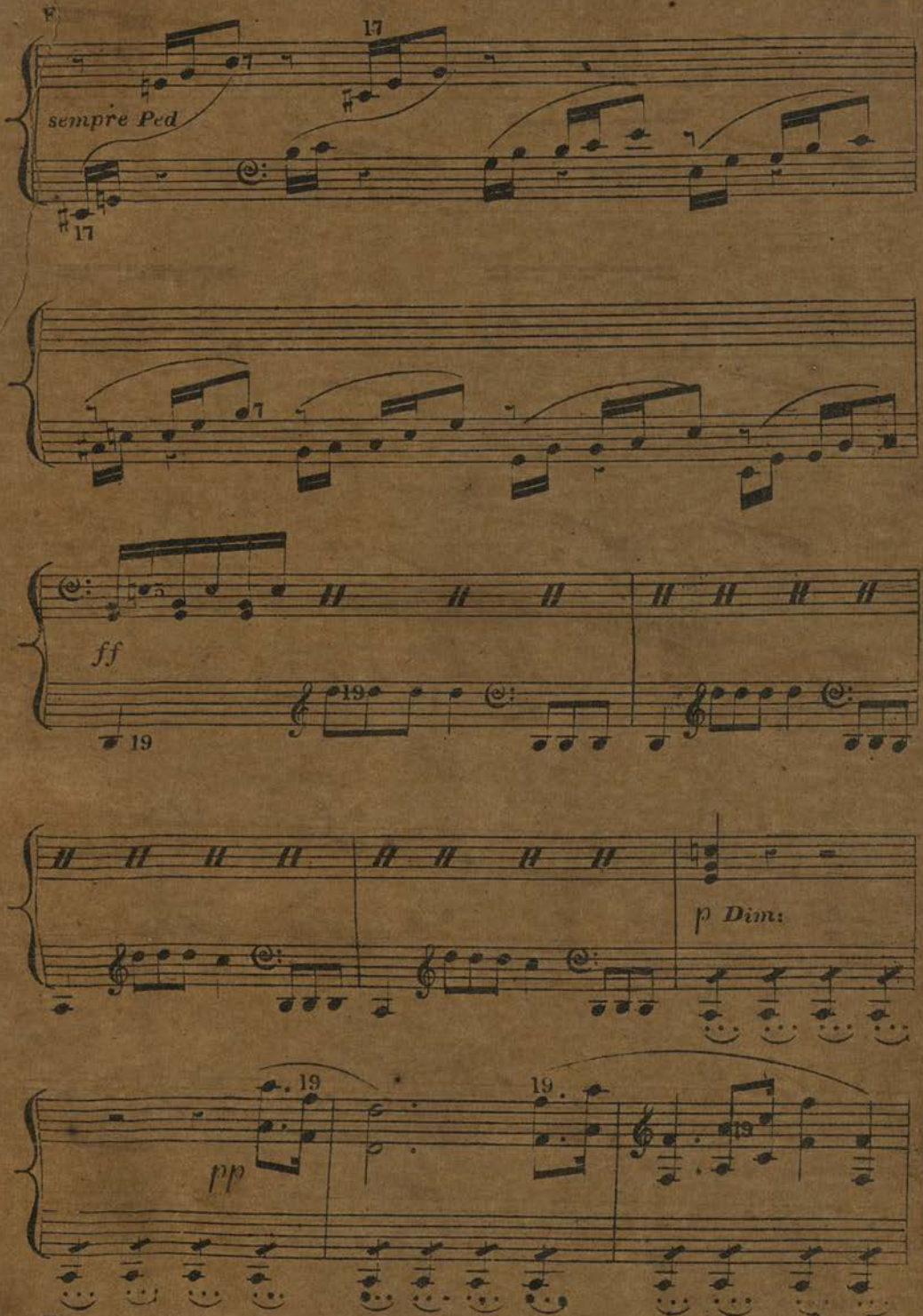


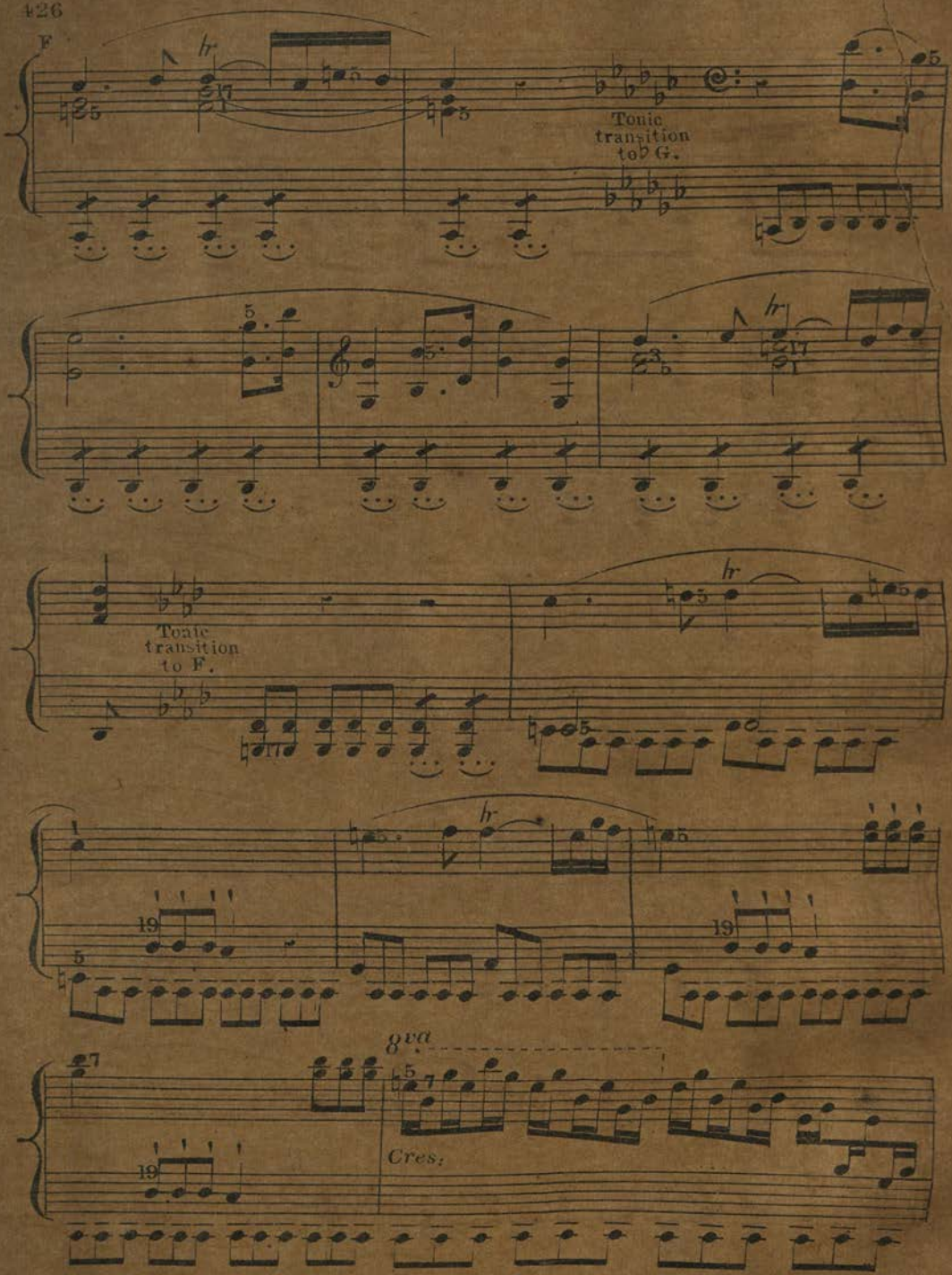

sf
sf
sf
sf
sf
ff
sff

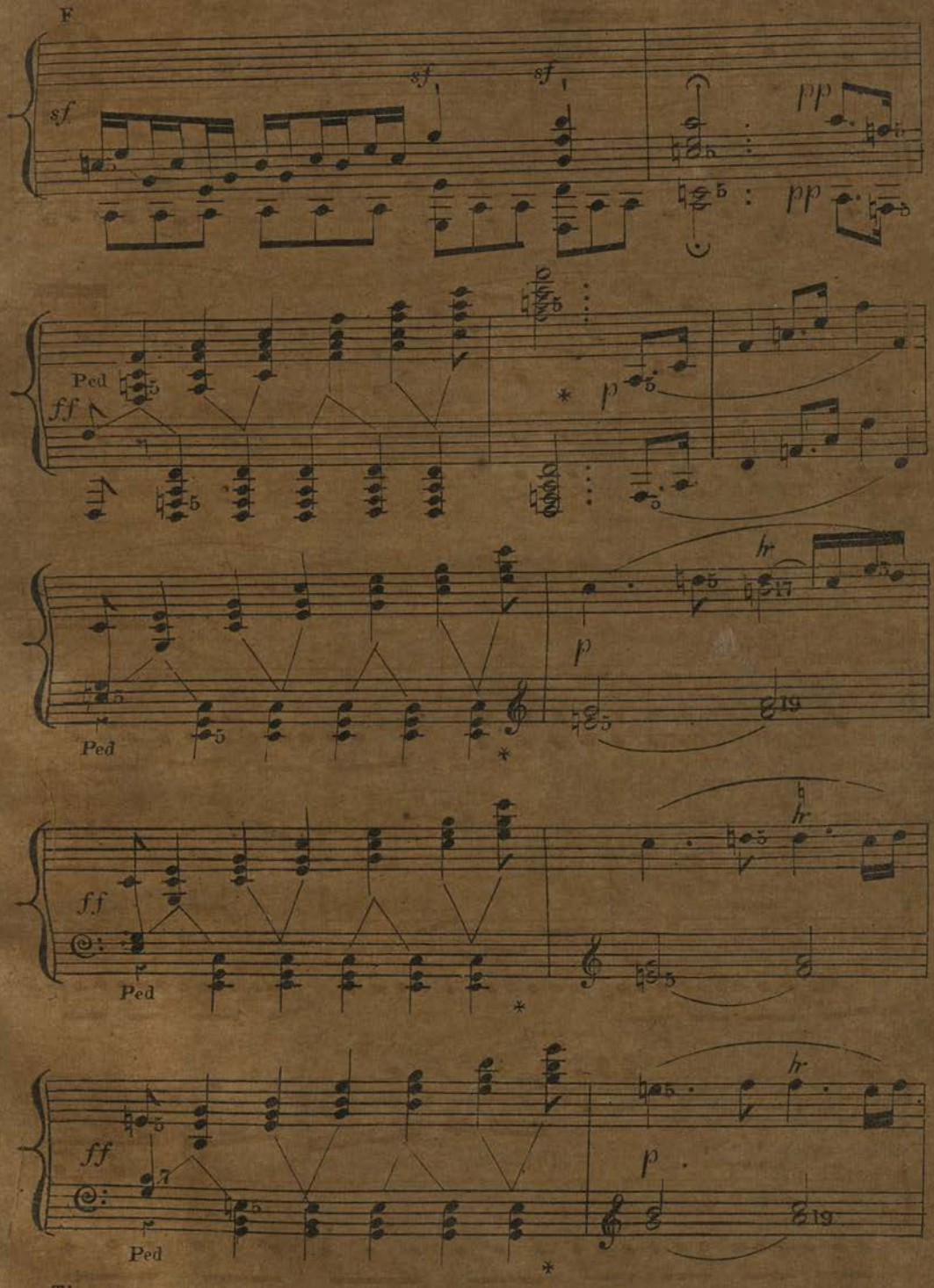

sff
Dim:
pp
Cres
f

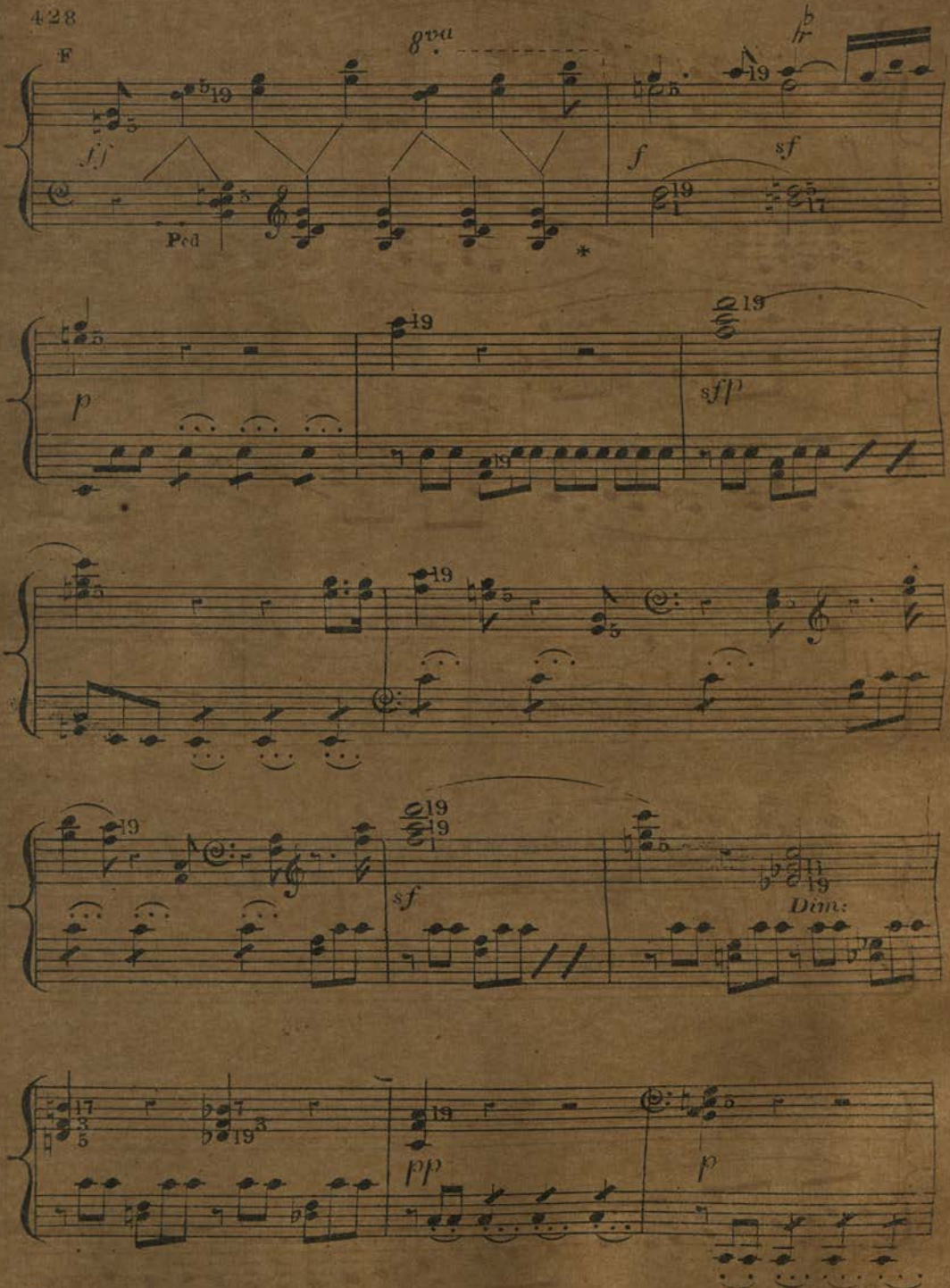

Tonic transition to ♭B
f
p
Cres.
f
Tonic transition to ♭G.
8va
Sempre più f
Tonic transition to F.

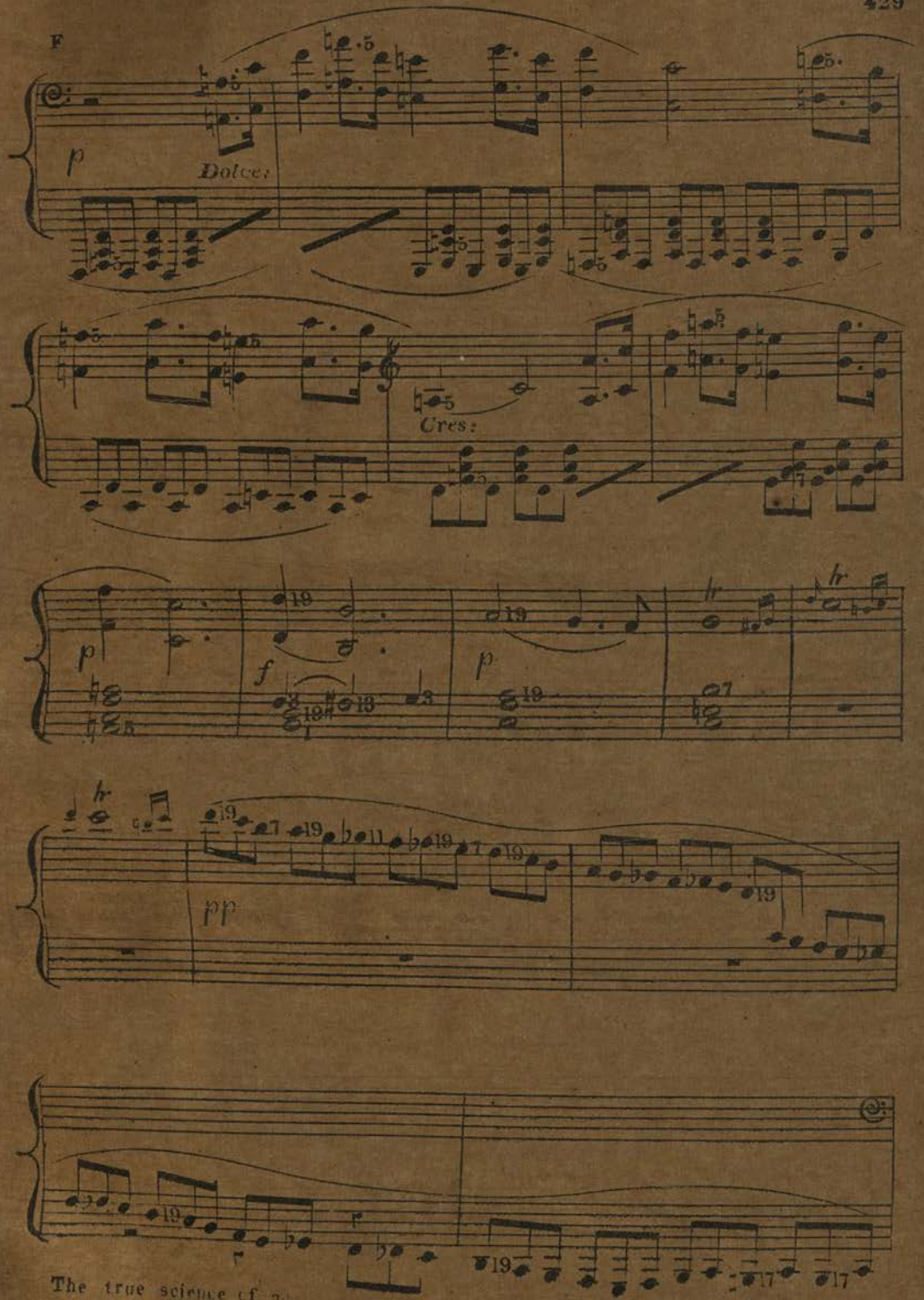

F
sempre Ped

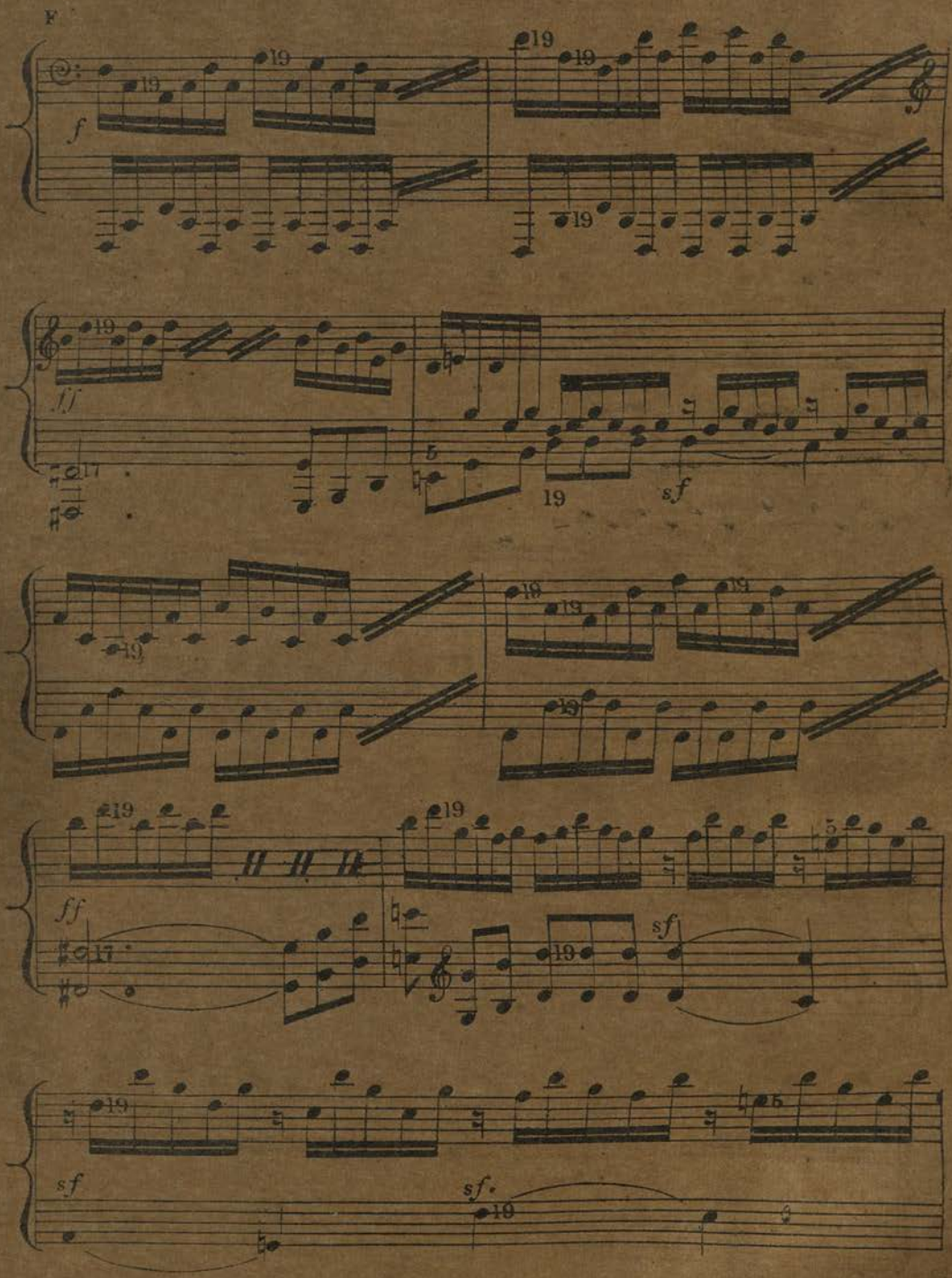

sempre Ped
ff
p Dim:
pp

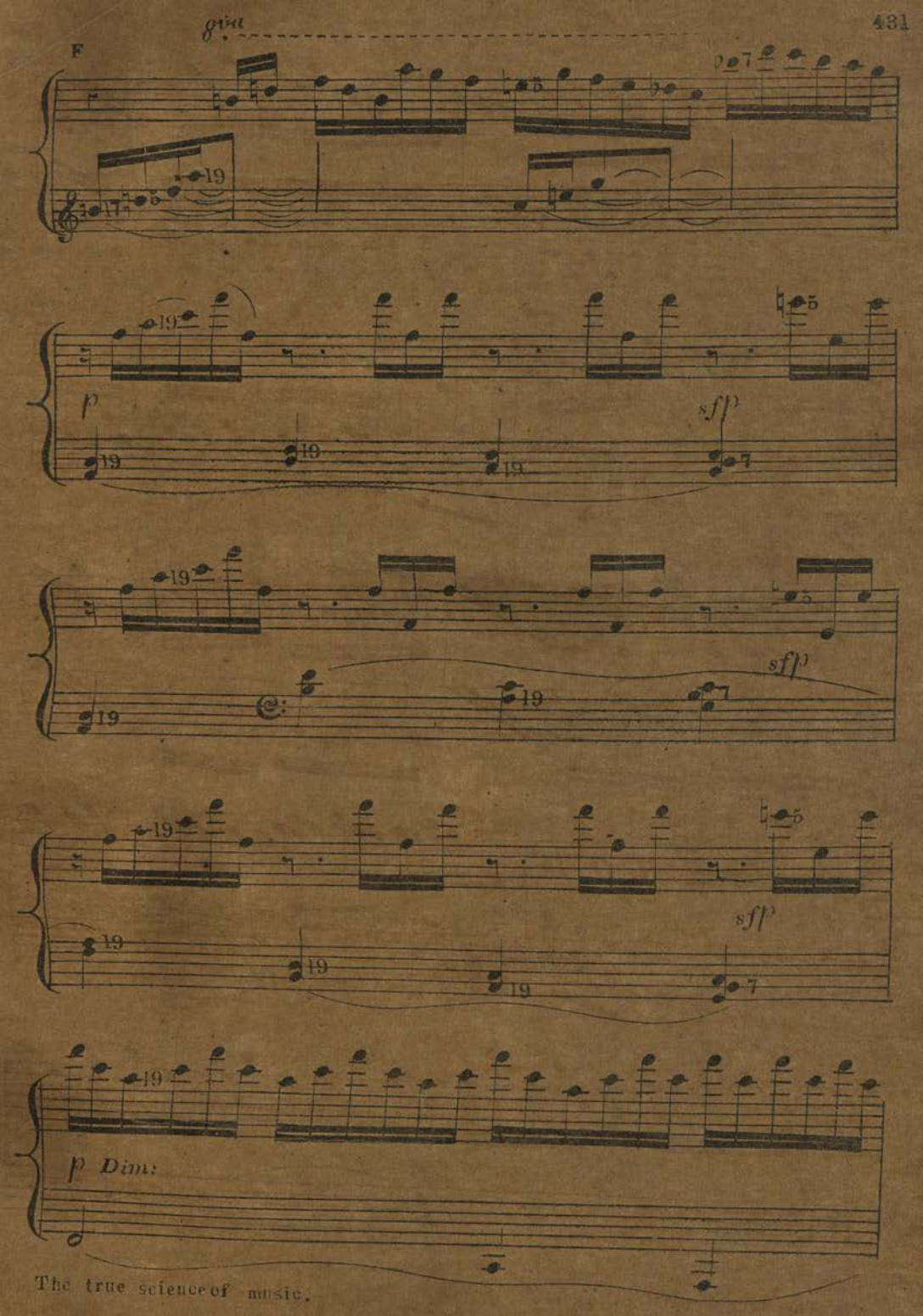

F
Tonic transition to G.
Tonic transition to F.
8va
Cres.

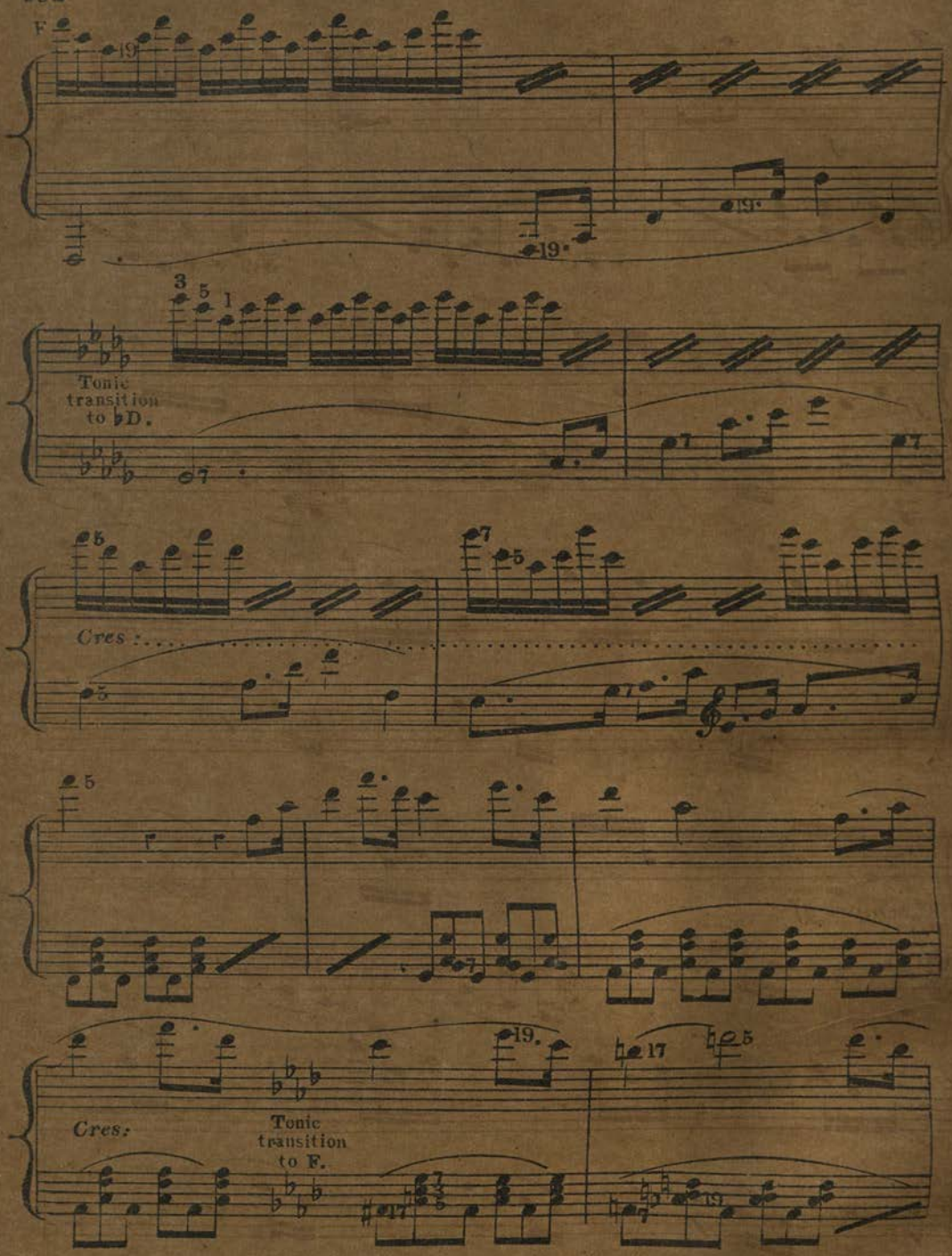

F
sf
sf
sf
pp
pp
Ped
ff
*
p
hr
p
Ped
*
hr
ff
Ped
*
hr
ff
p
Ped
*

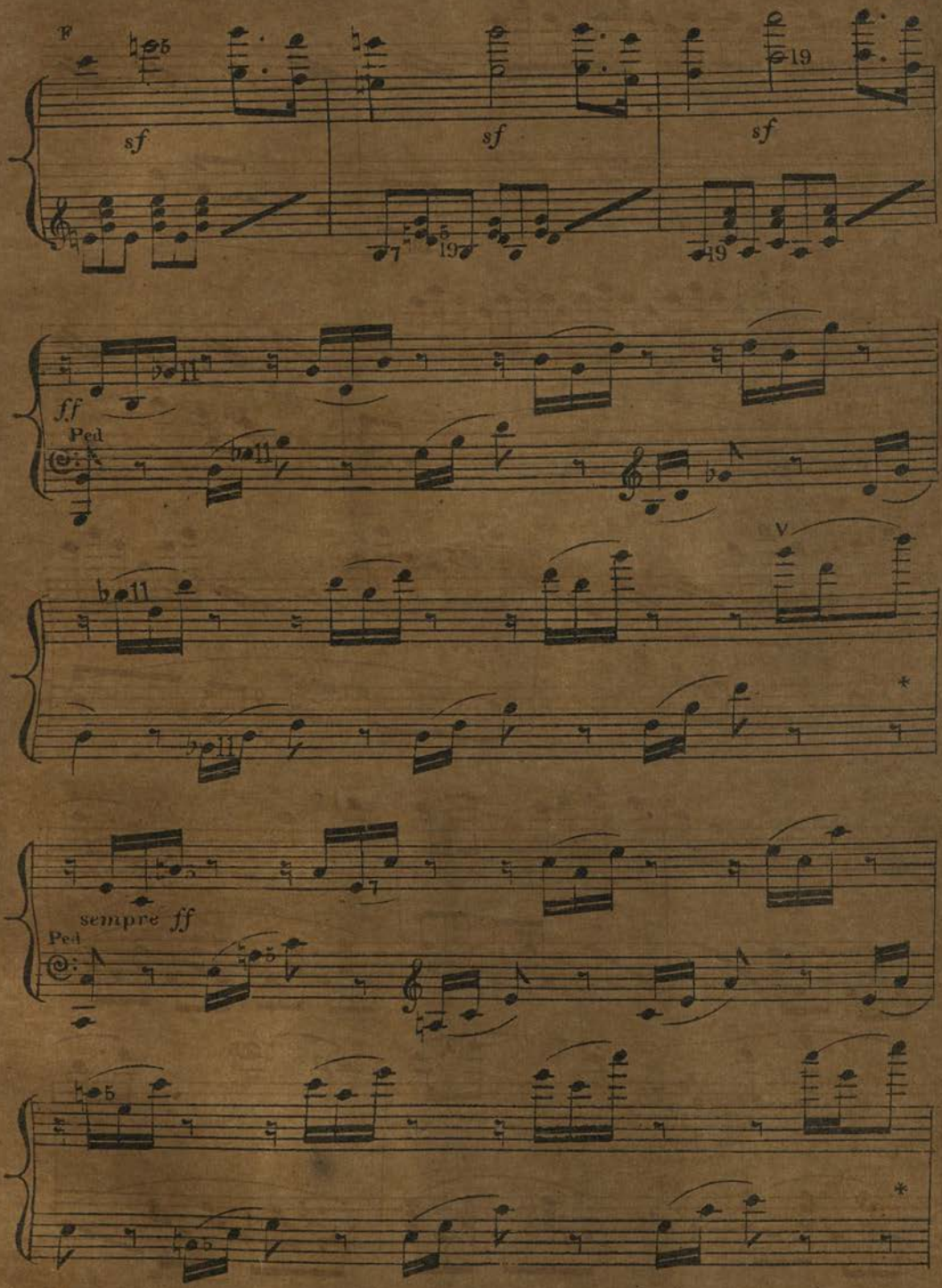

F
8va
ff
Ped
*
f
sf
p
sfp
sf
Dim:
pp
p

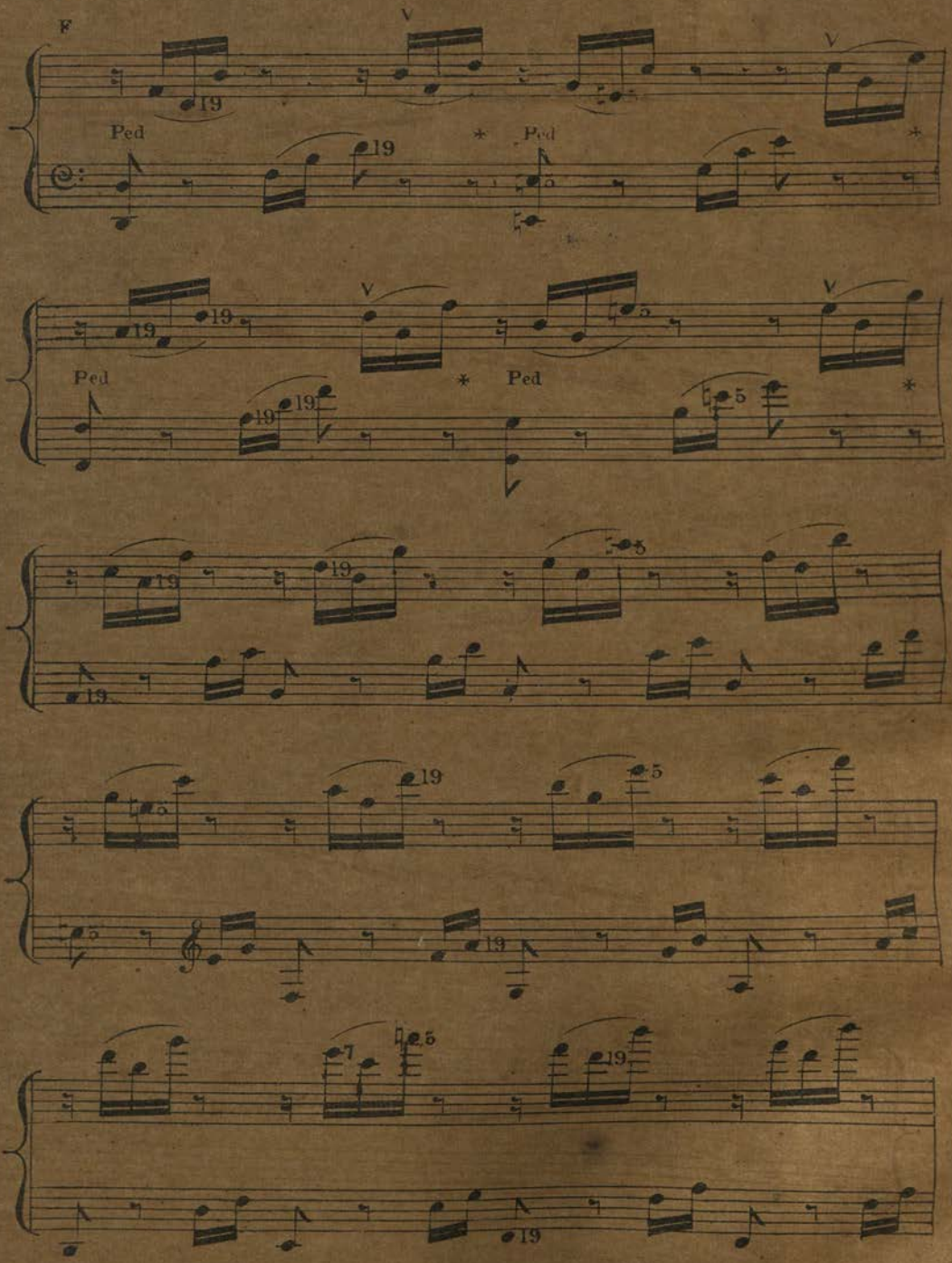

F
p
Dolce:
Cres:
p
f
p
pp

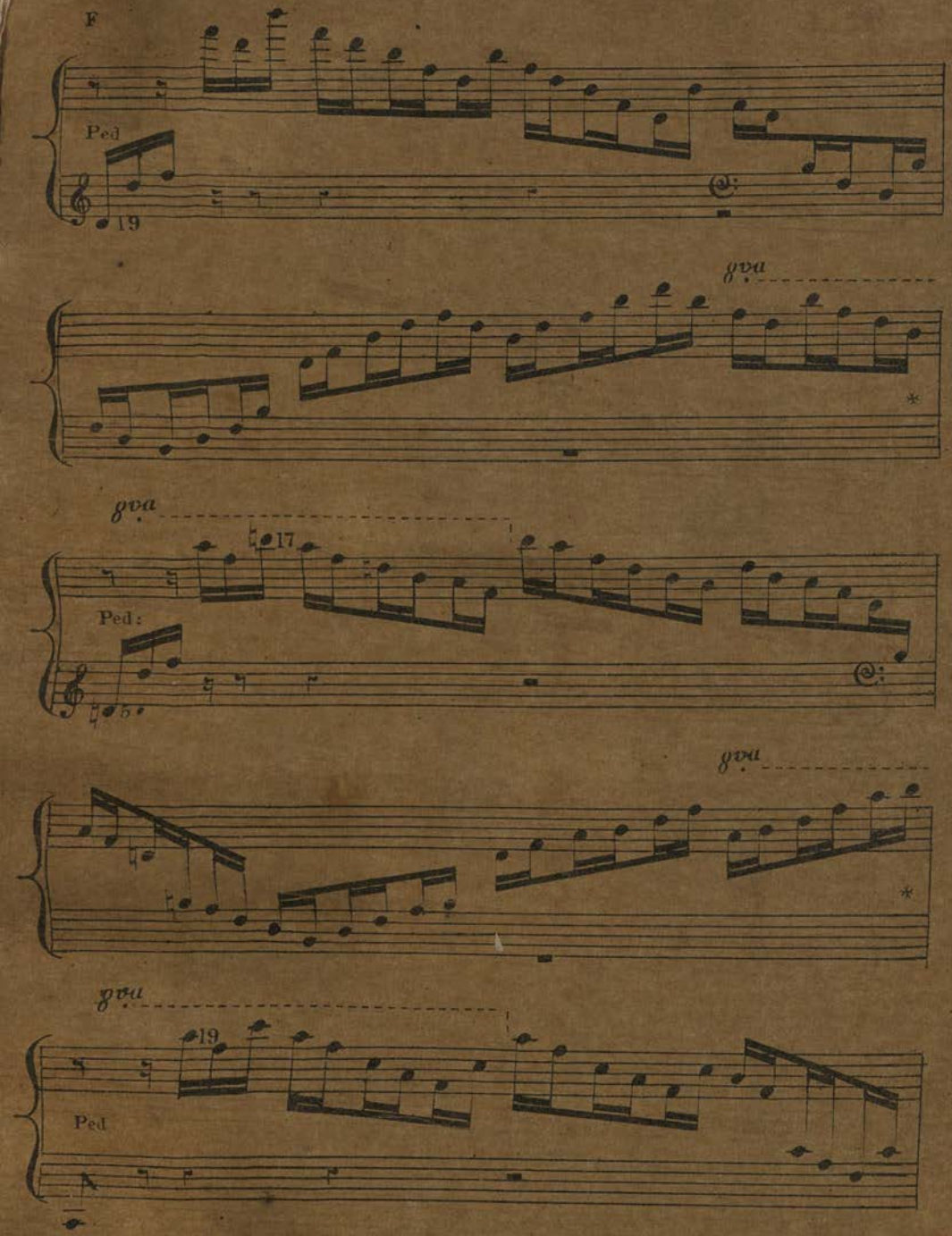

F
f
ff
sf
sf.

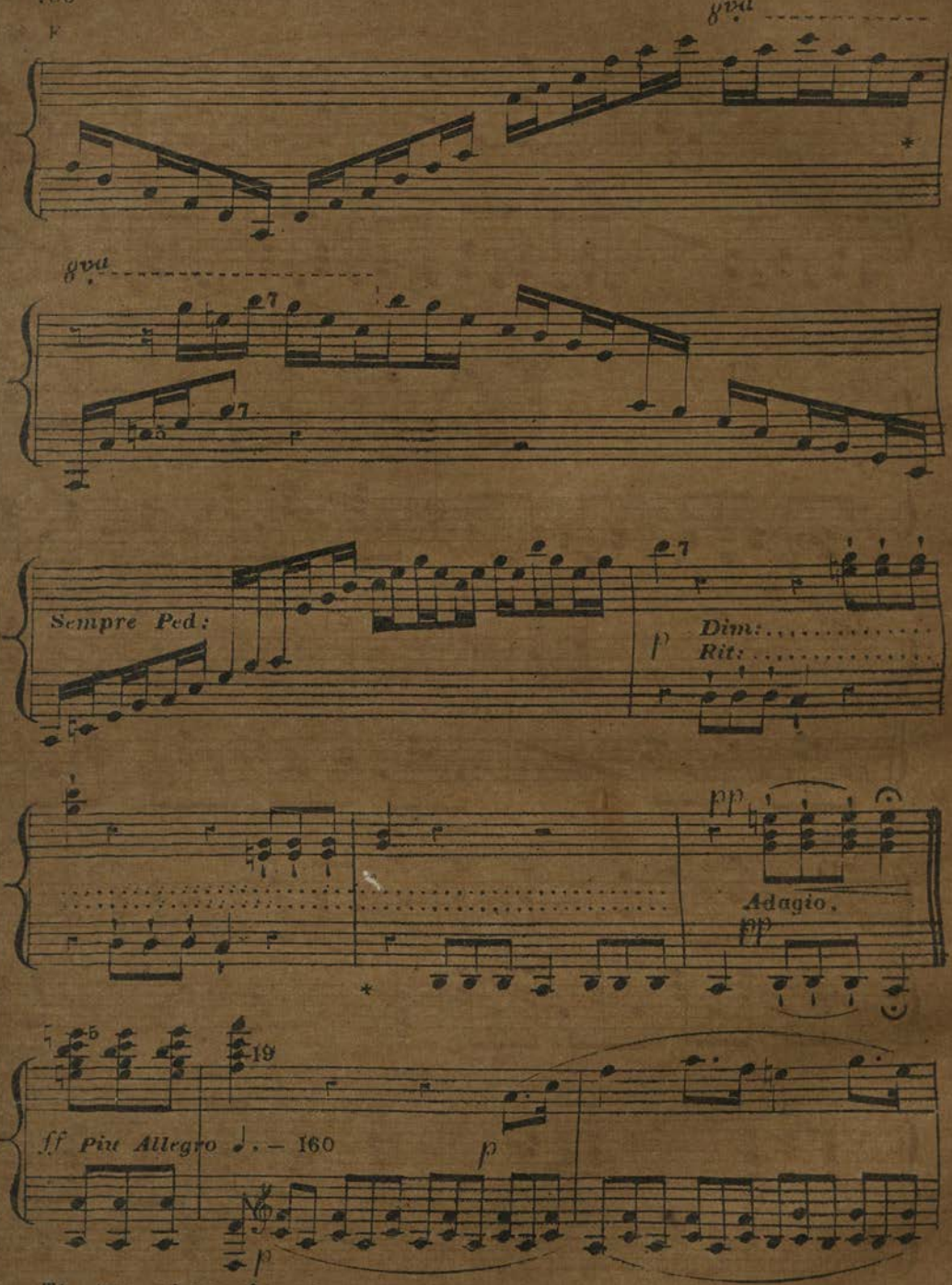

giu
F
p
sfp
sfp
sfp
p Dim:

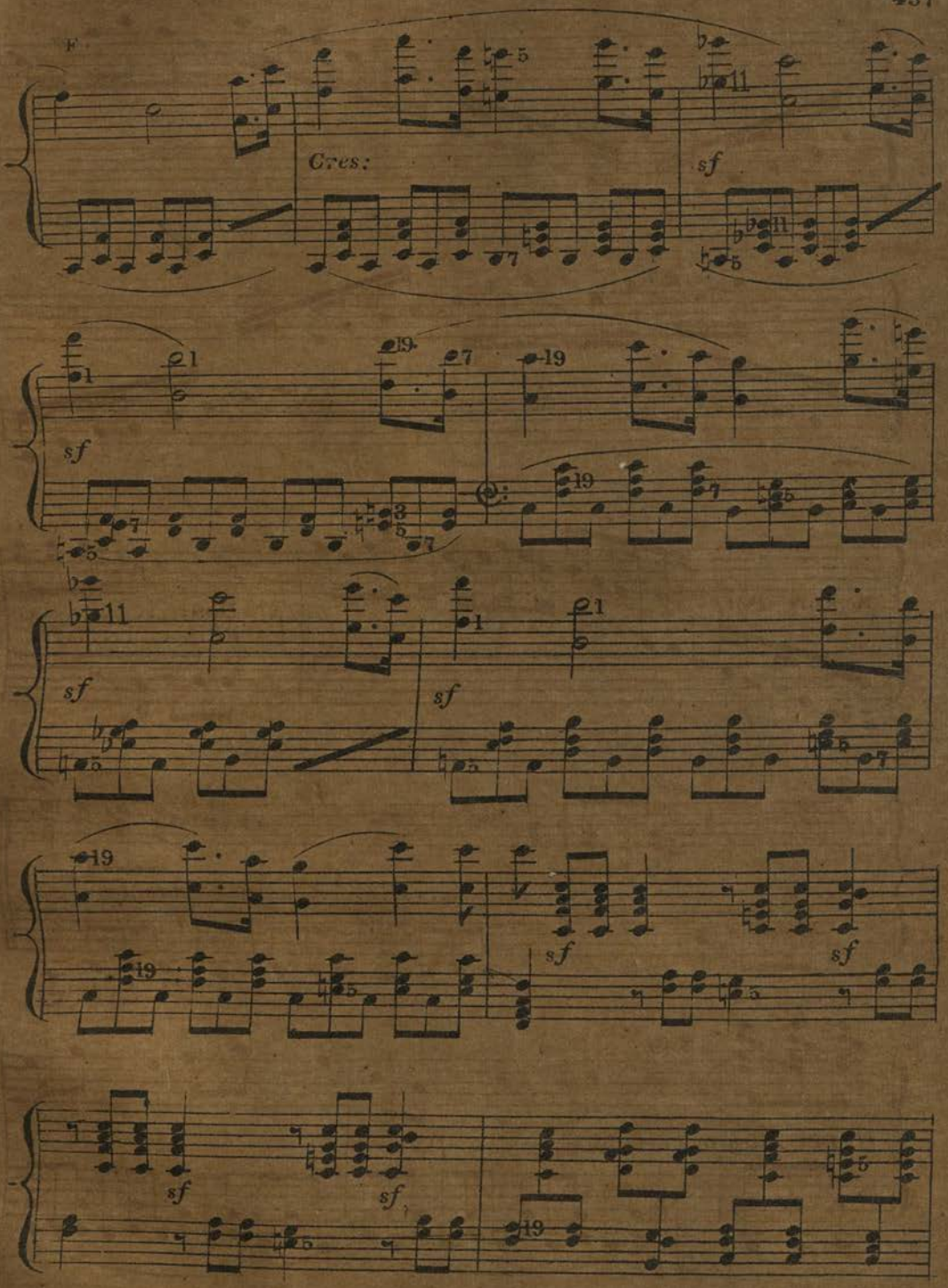

F.
Tonic transition to ♭D.
Cres:
Cres:
Tonic transition to F.


sf
sf
sf
ff
Ped
sempre ff
Ped

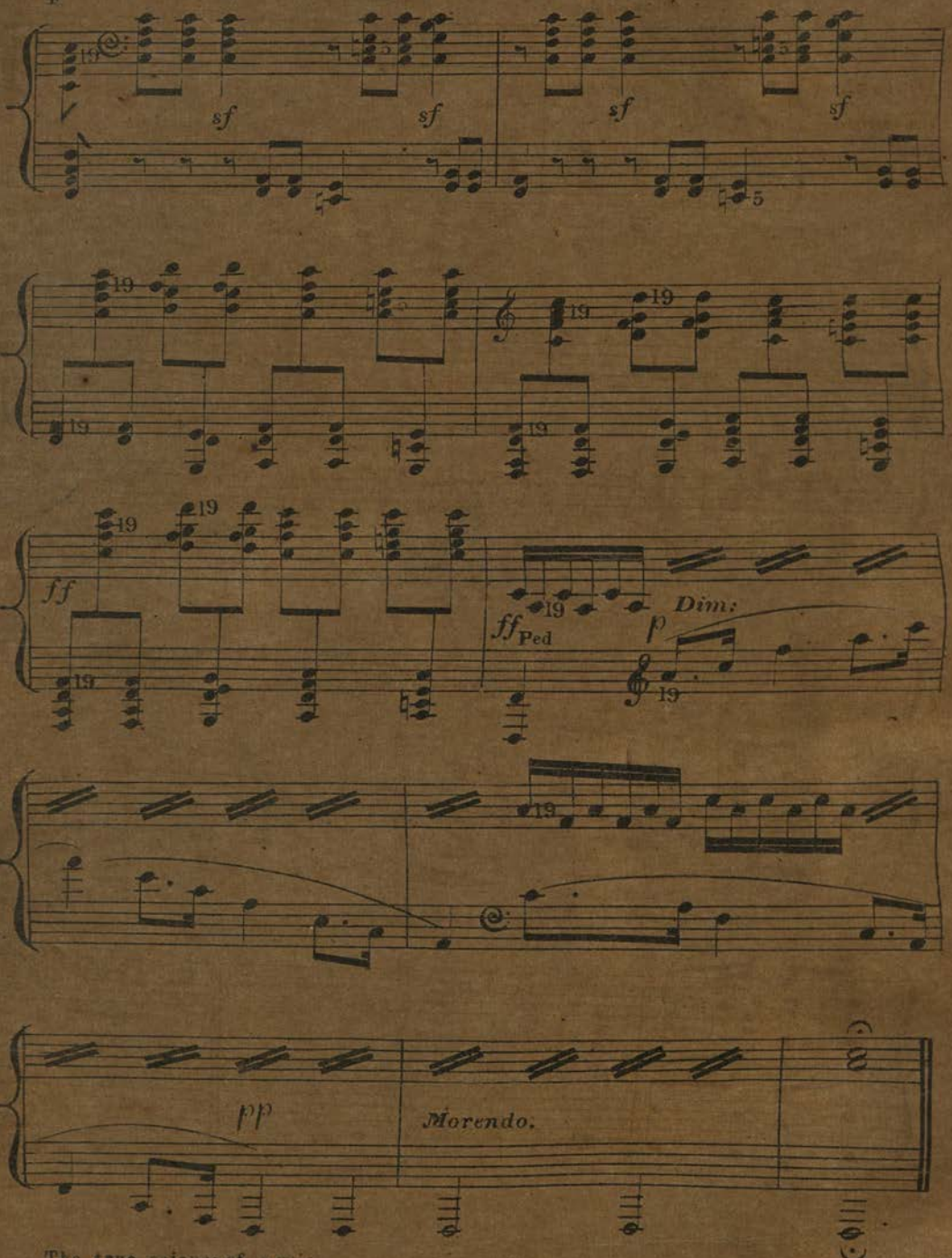

F
Ped
Ped
Ped
Ped

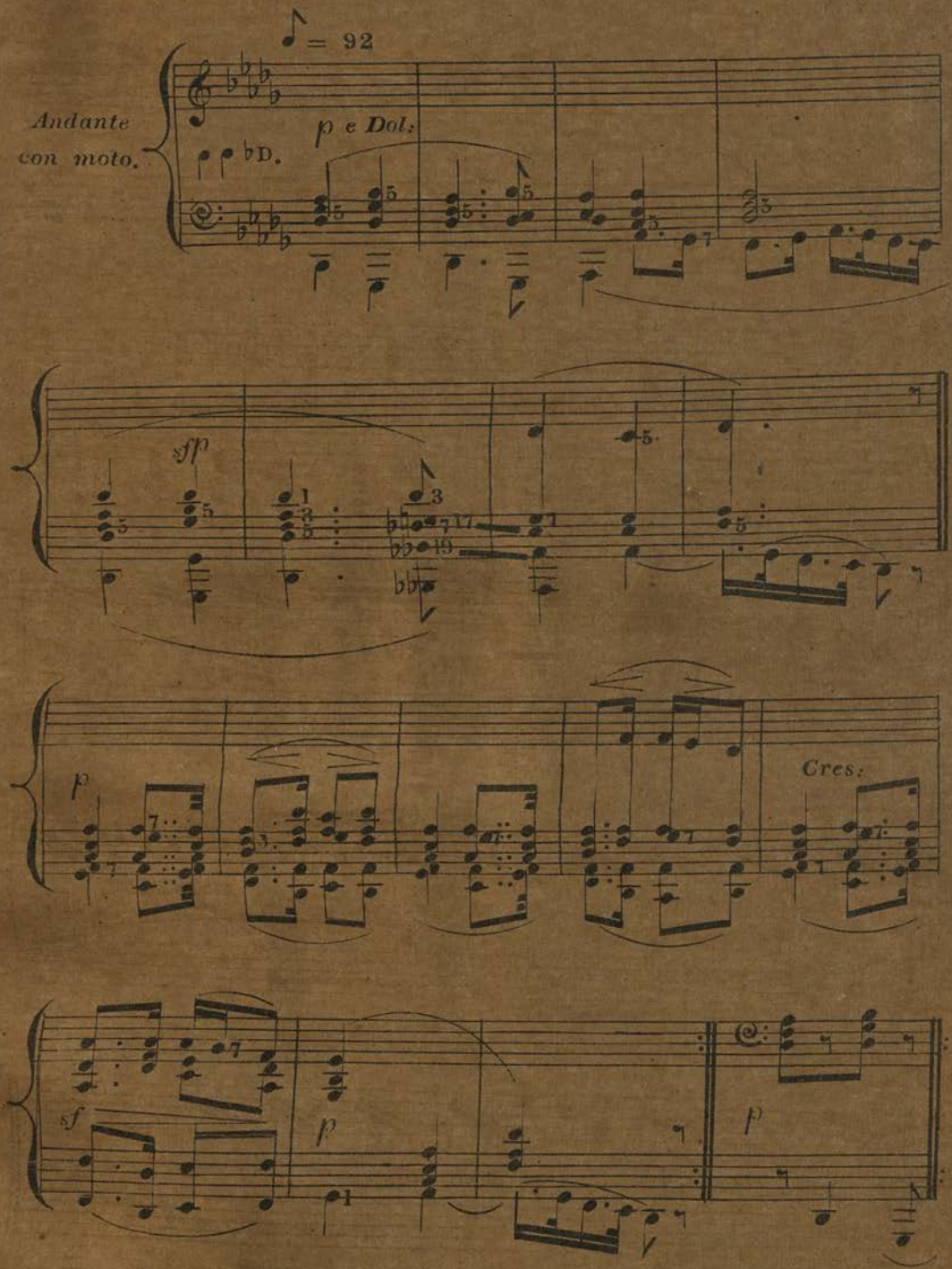

F
Ped
19
8va
8va
17
Ped:
8va
8va
19
Ped

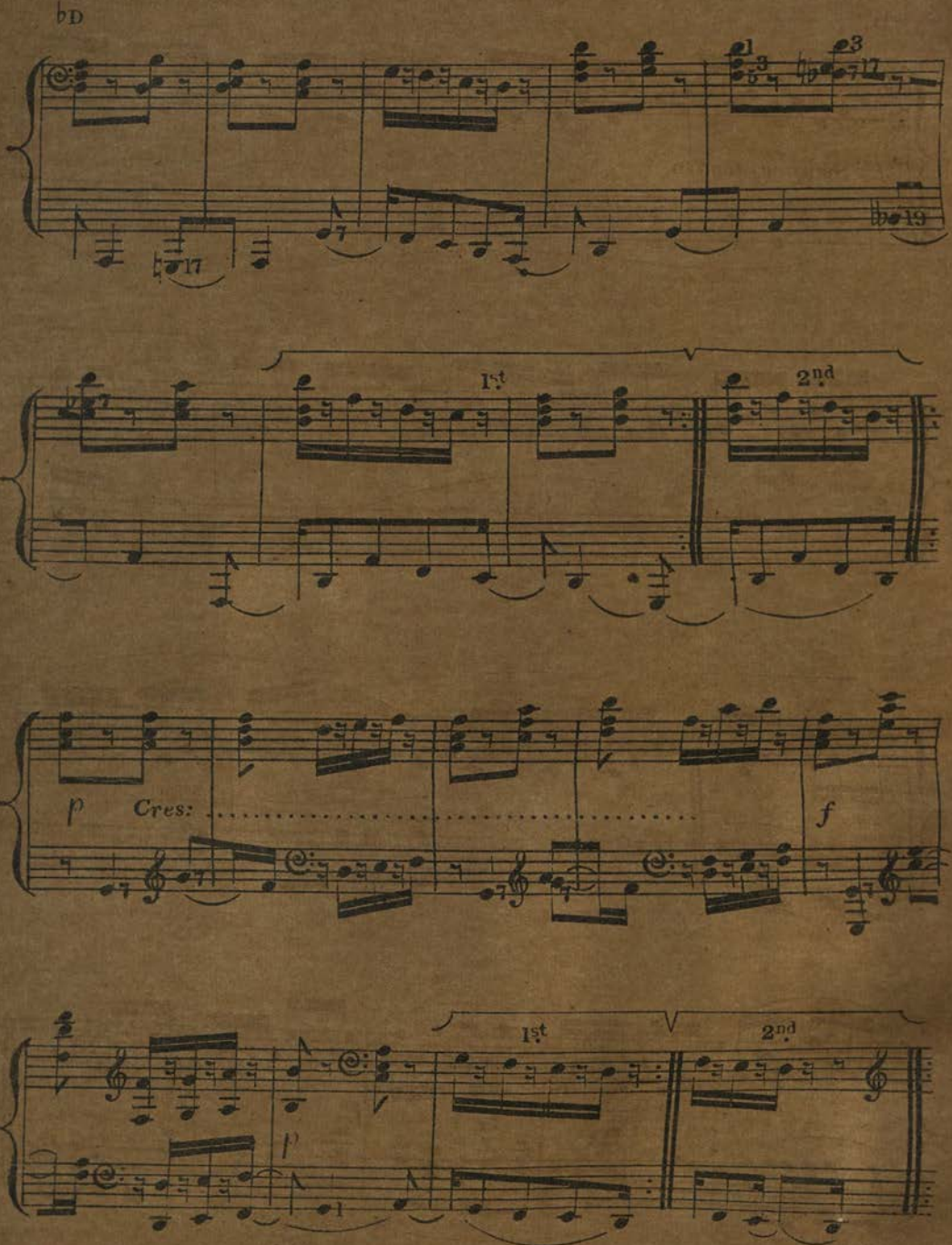

8va
8va
Sempre Ped:
p
Dim:
Rit:
pp
Adagio.
pp
19
ff Piu Allegro ♩. = 160
p
p

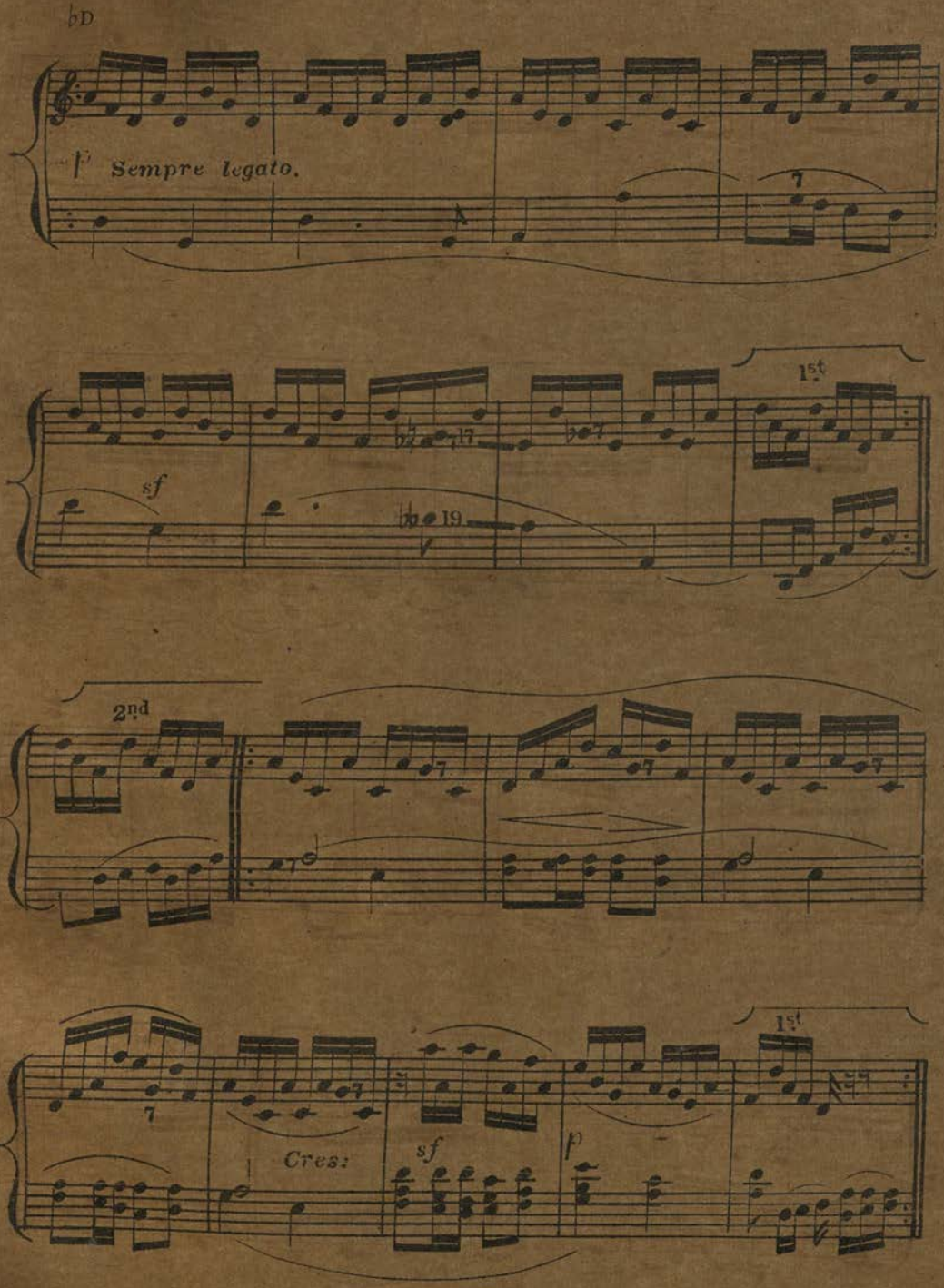

F
Cres:
sf
sf
sf
sf
sf
sf
sf
sf
sf

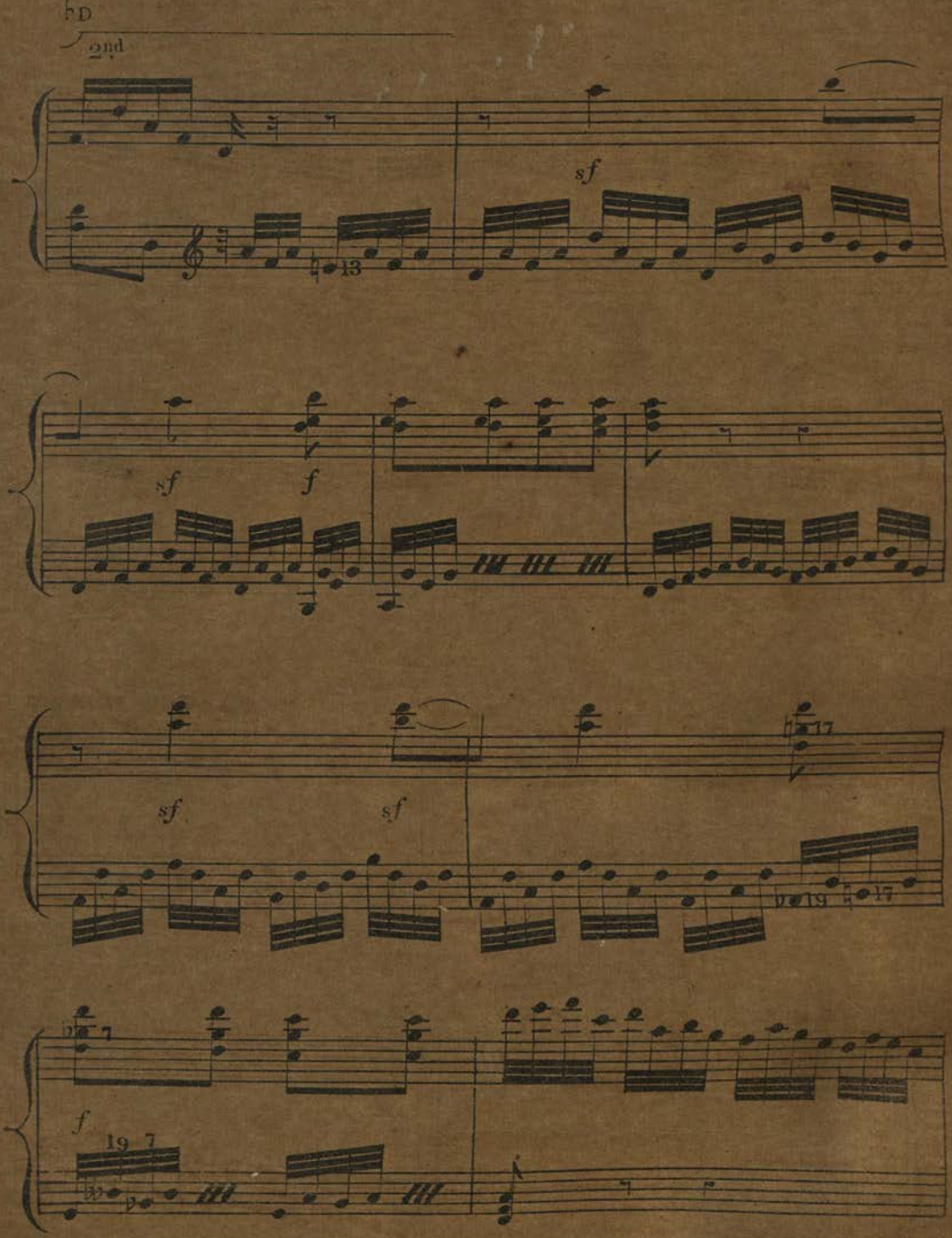
F

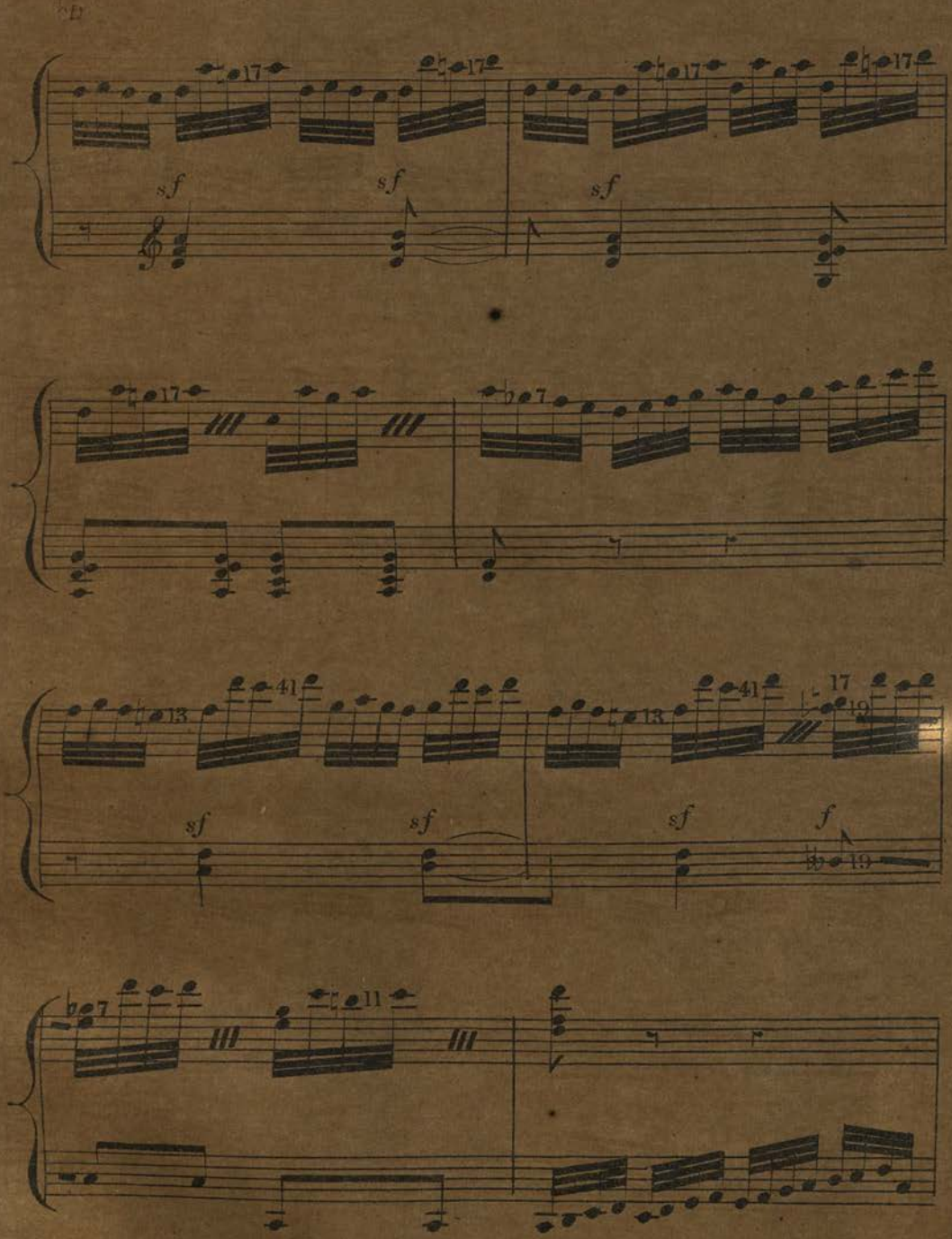


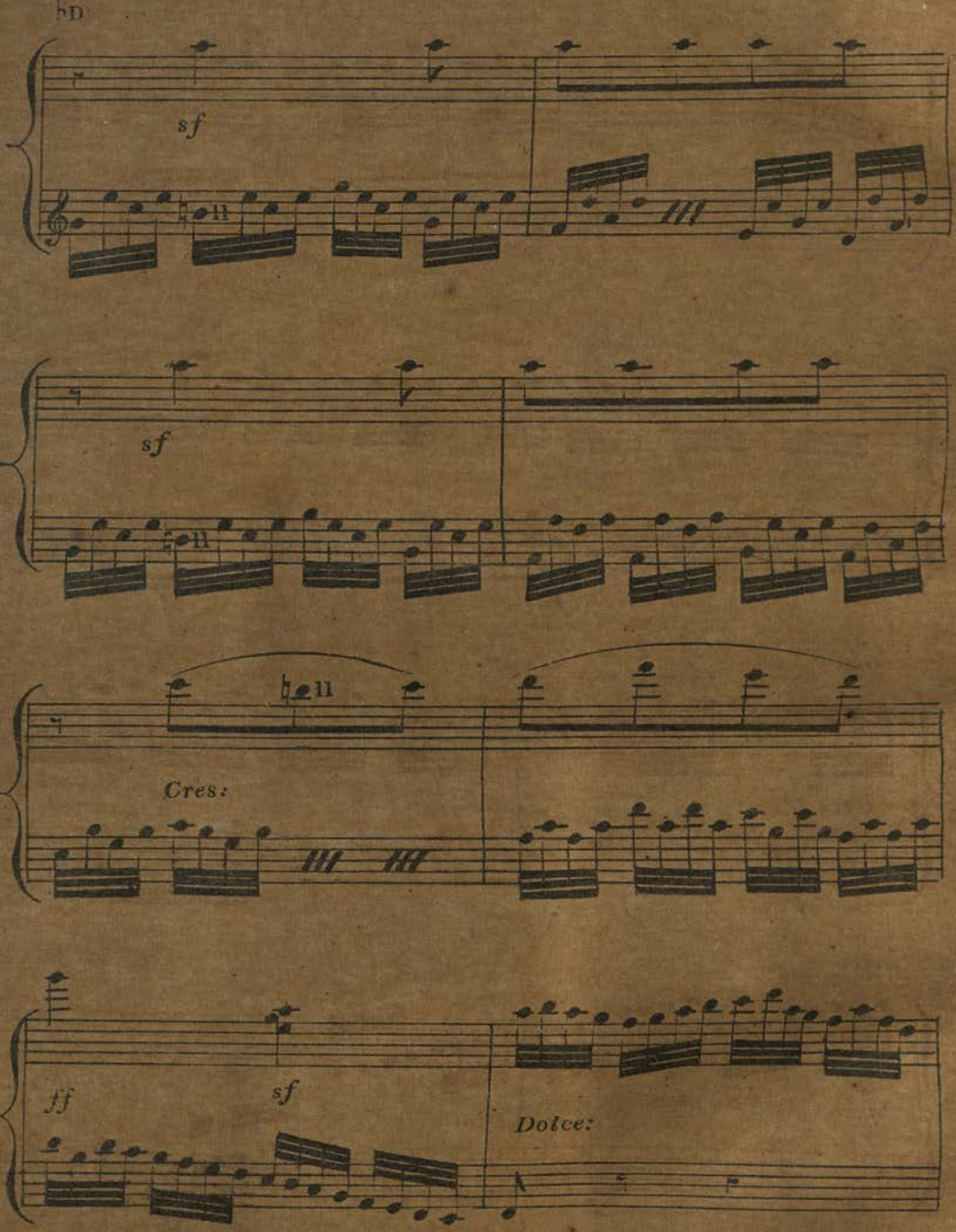

♪ = 92
Andante
con moto.
p e Dol:
♭D.
sfp
p
Cres:
sf
p
p

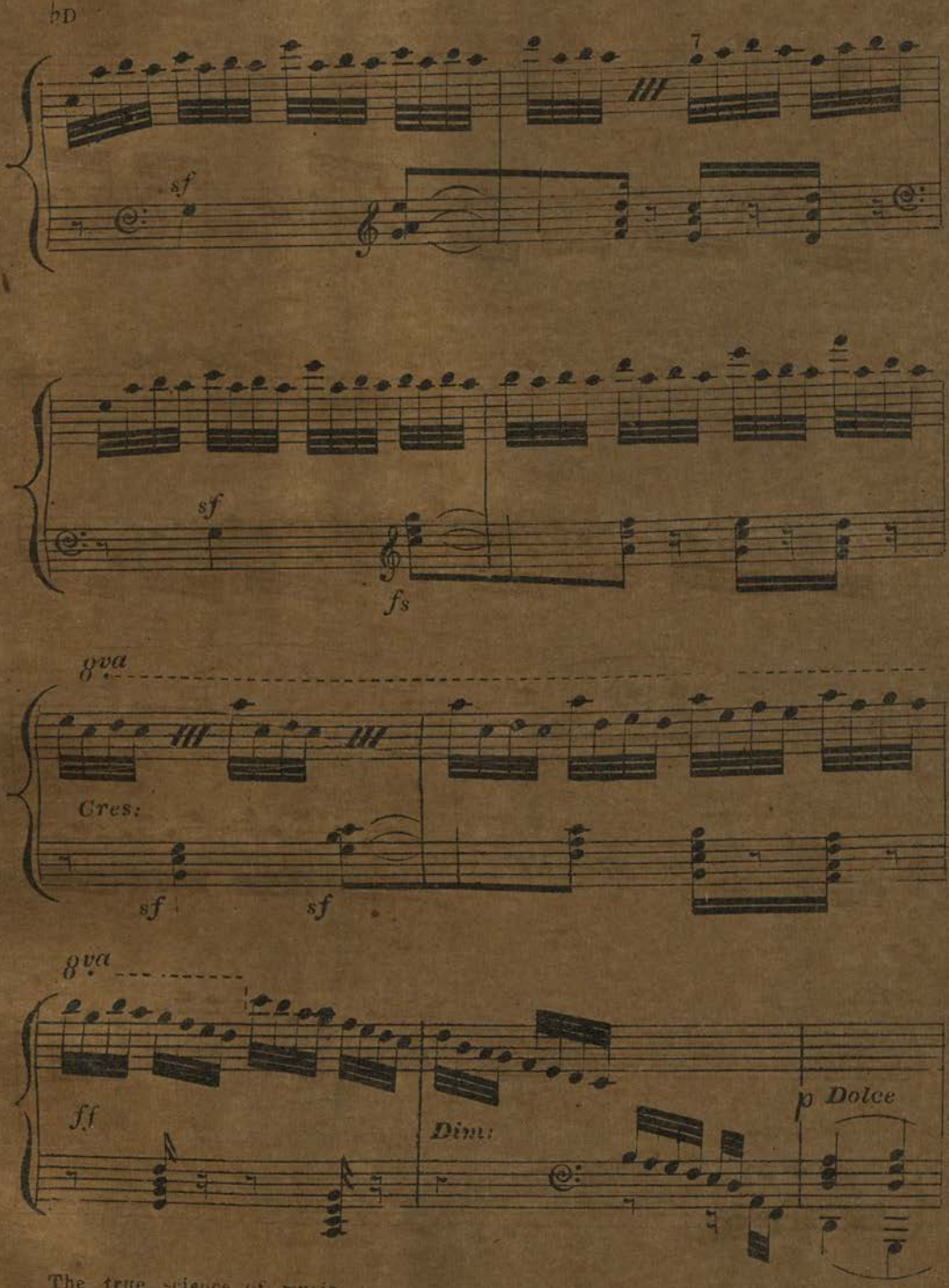

bD
1st
2nd
p
Cres:
f
1st
2nd
p

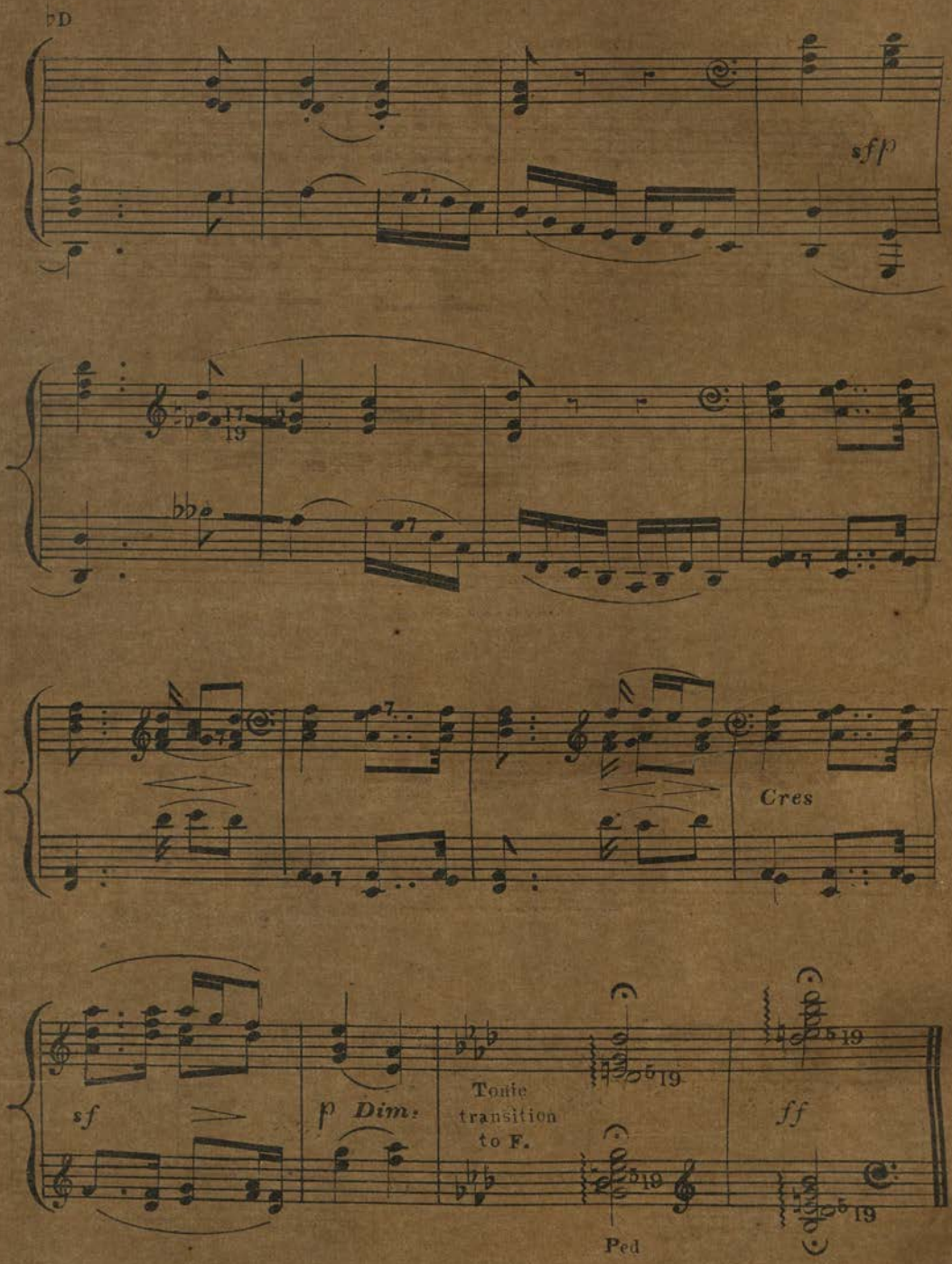

bD
f Sempre legato.
1st
sf
2nd
Cres:
sf
p
1st


2nd
sf
sf
f
sf
sf
f

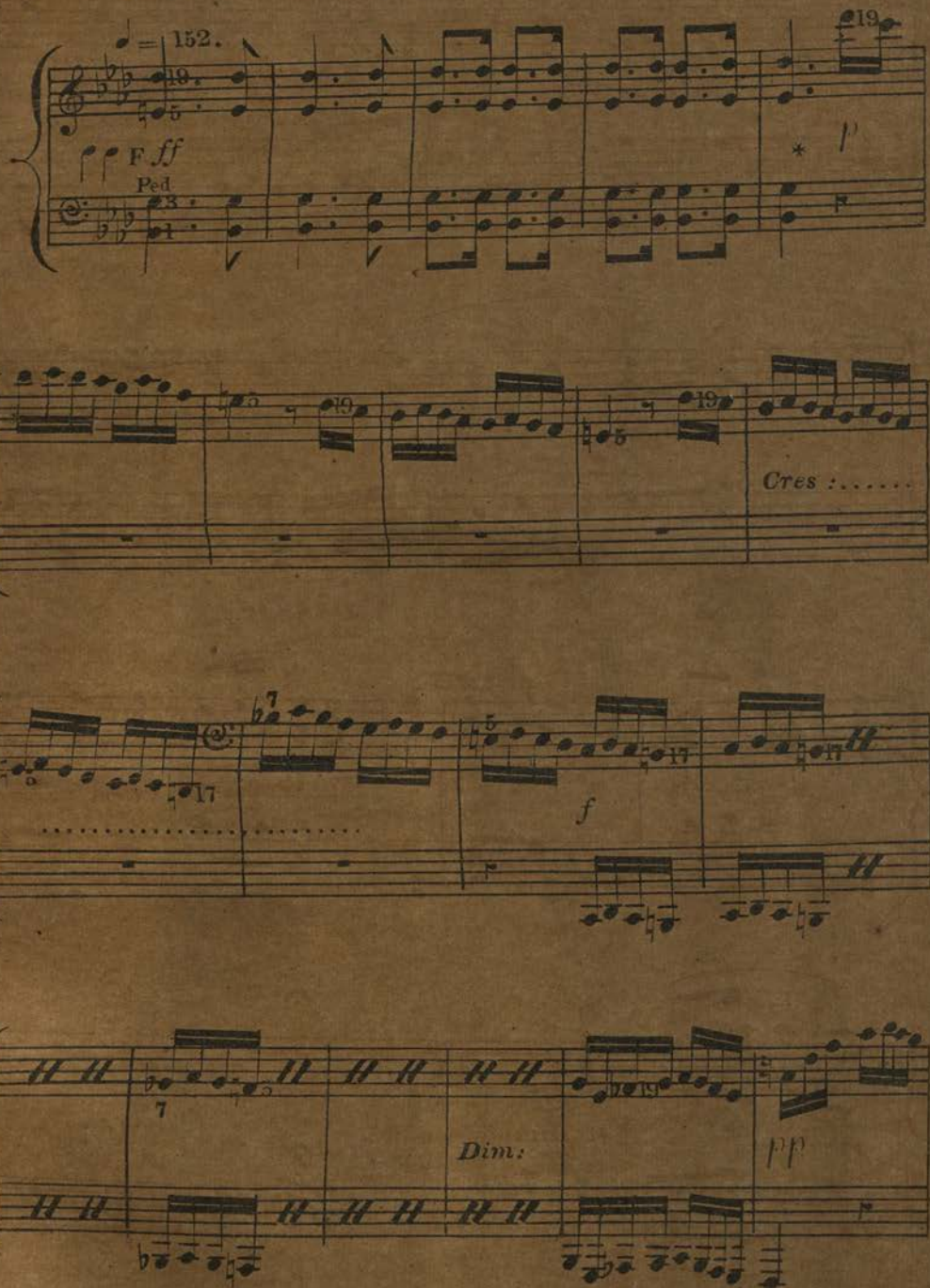

sf
sf
sf
sf
sf
sf
f

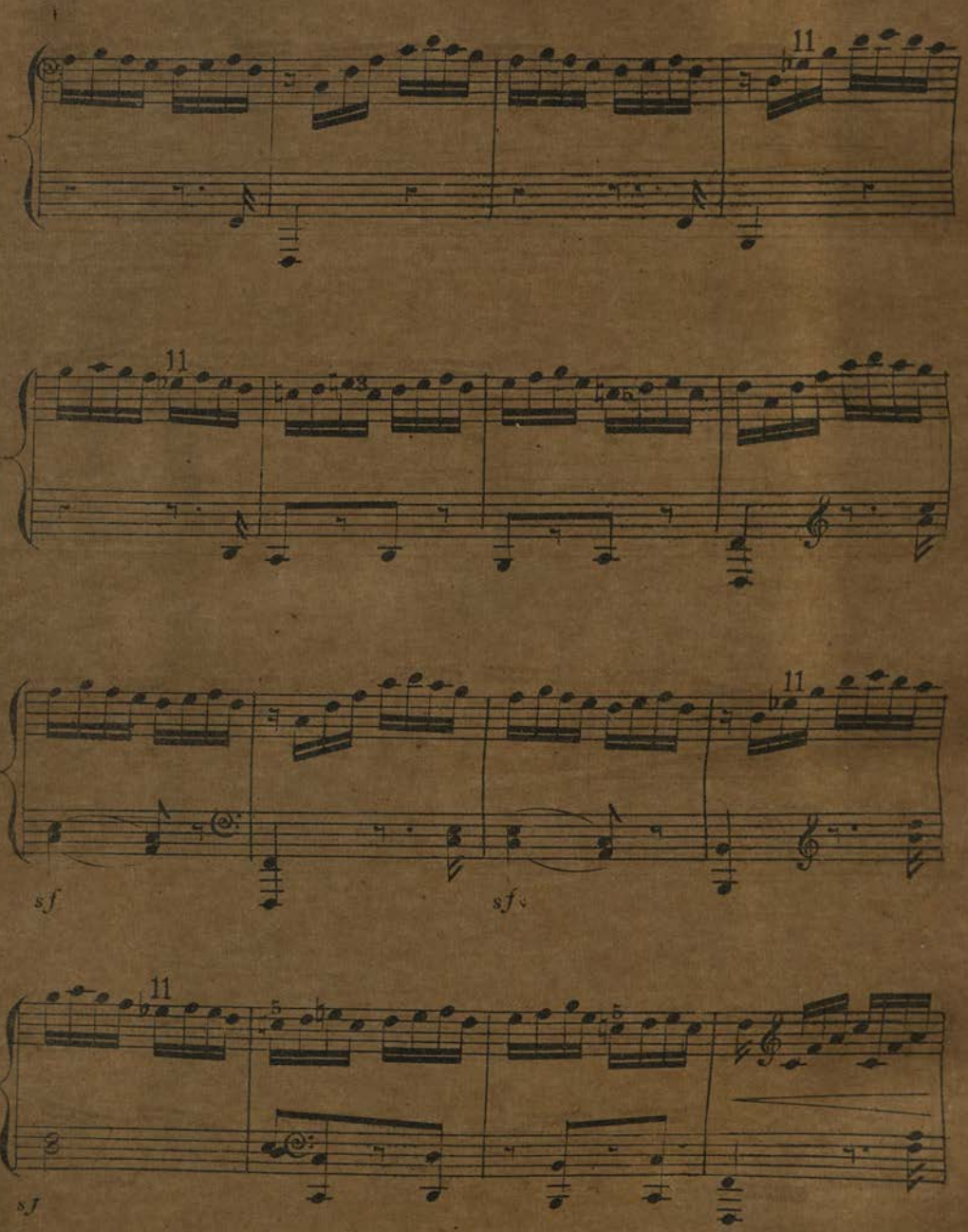

bD
sf
sf
Cres:
ff
sf
Dolce:

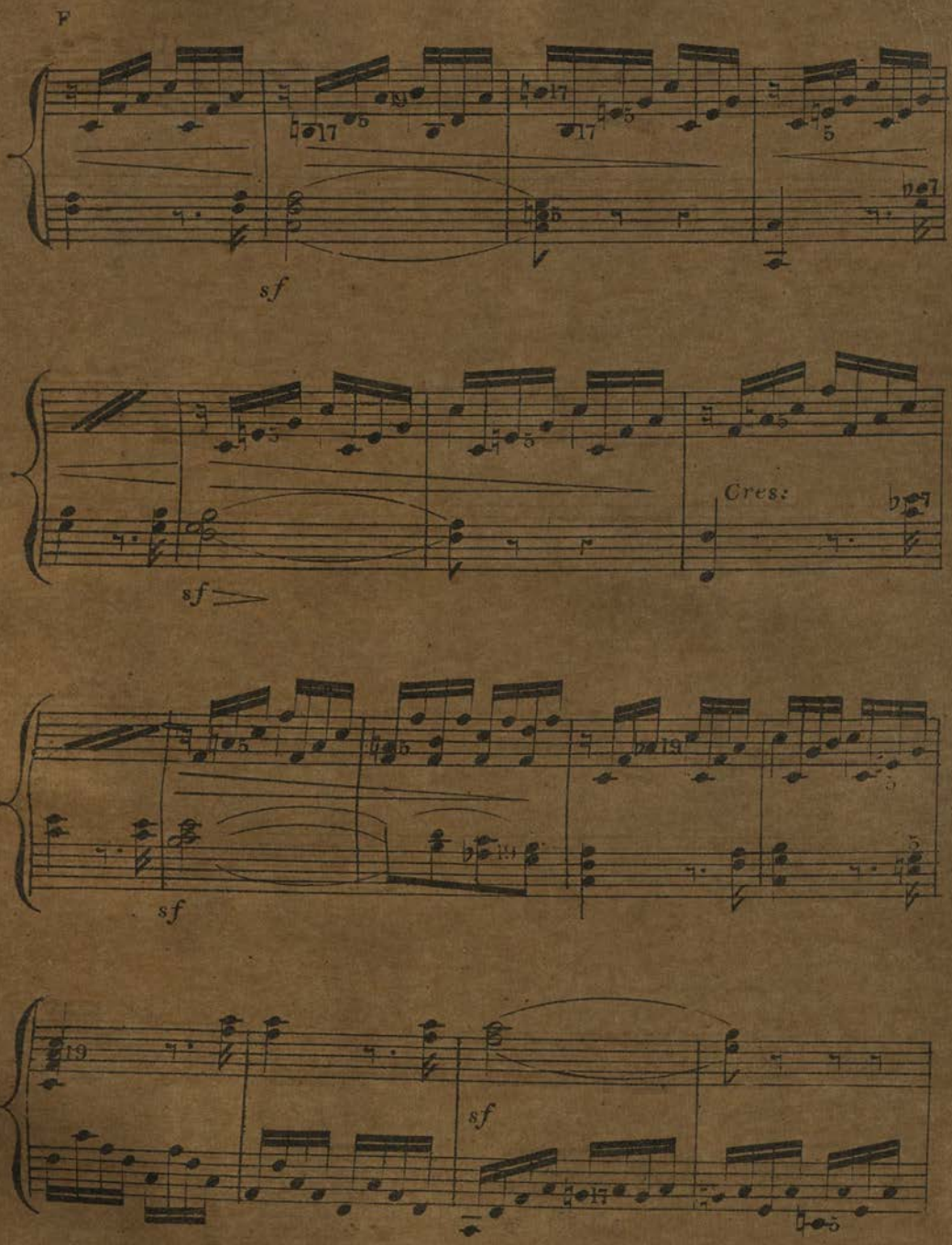

bD
sf
sf
fs
8va
Cres:
sf
sf
8va
ff
Dim:
p Dolce


♭D
sfp
Cres
sf
p Dim:
Tonic
transition
to F.
ff
Ped

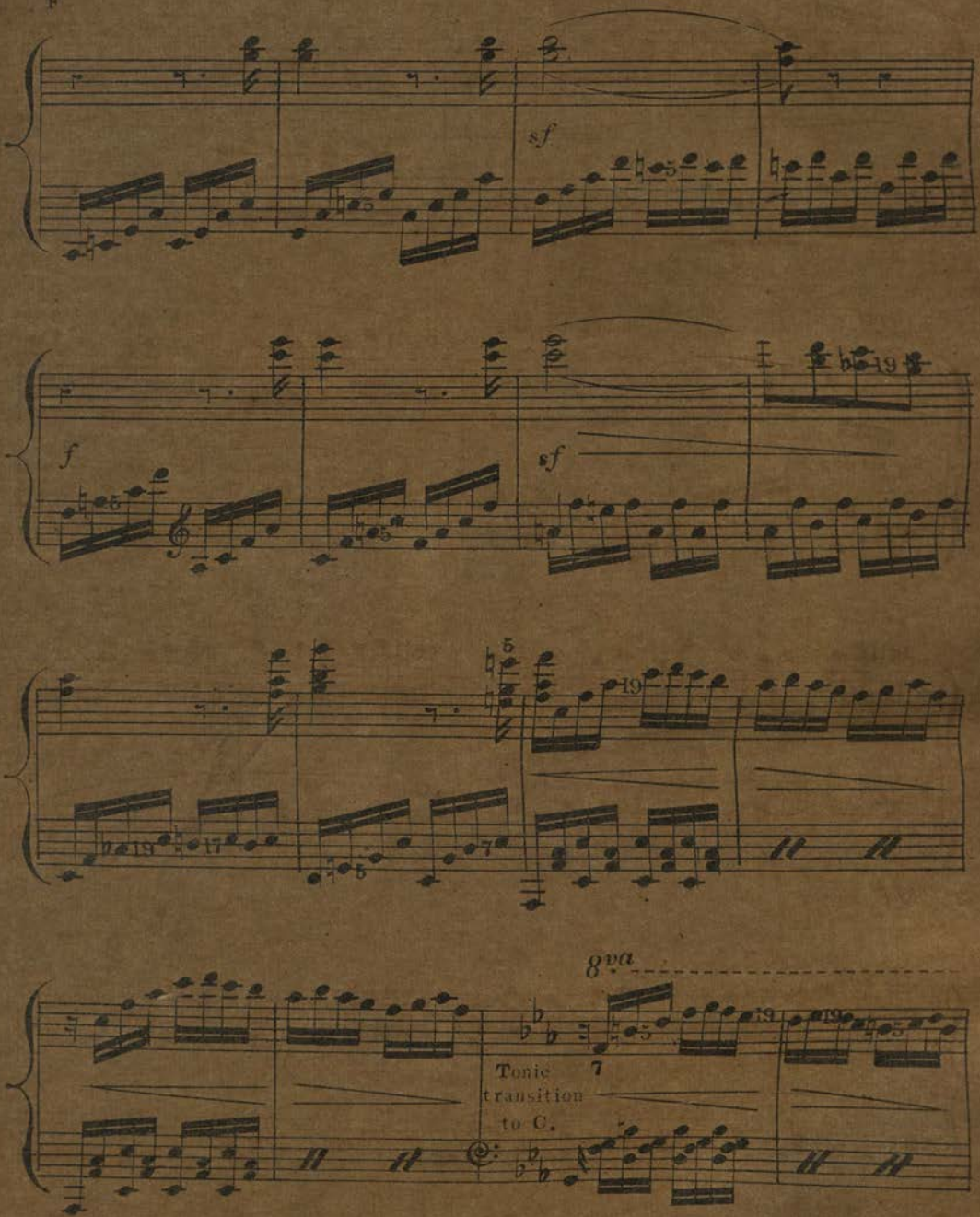
Allegro ma non troppo.

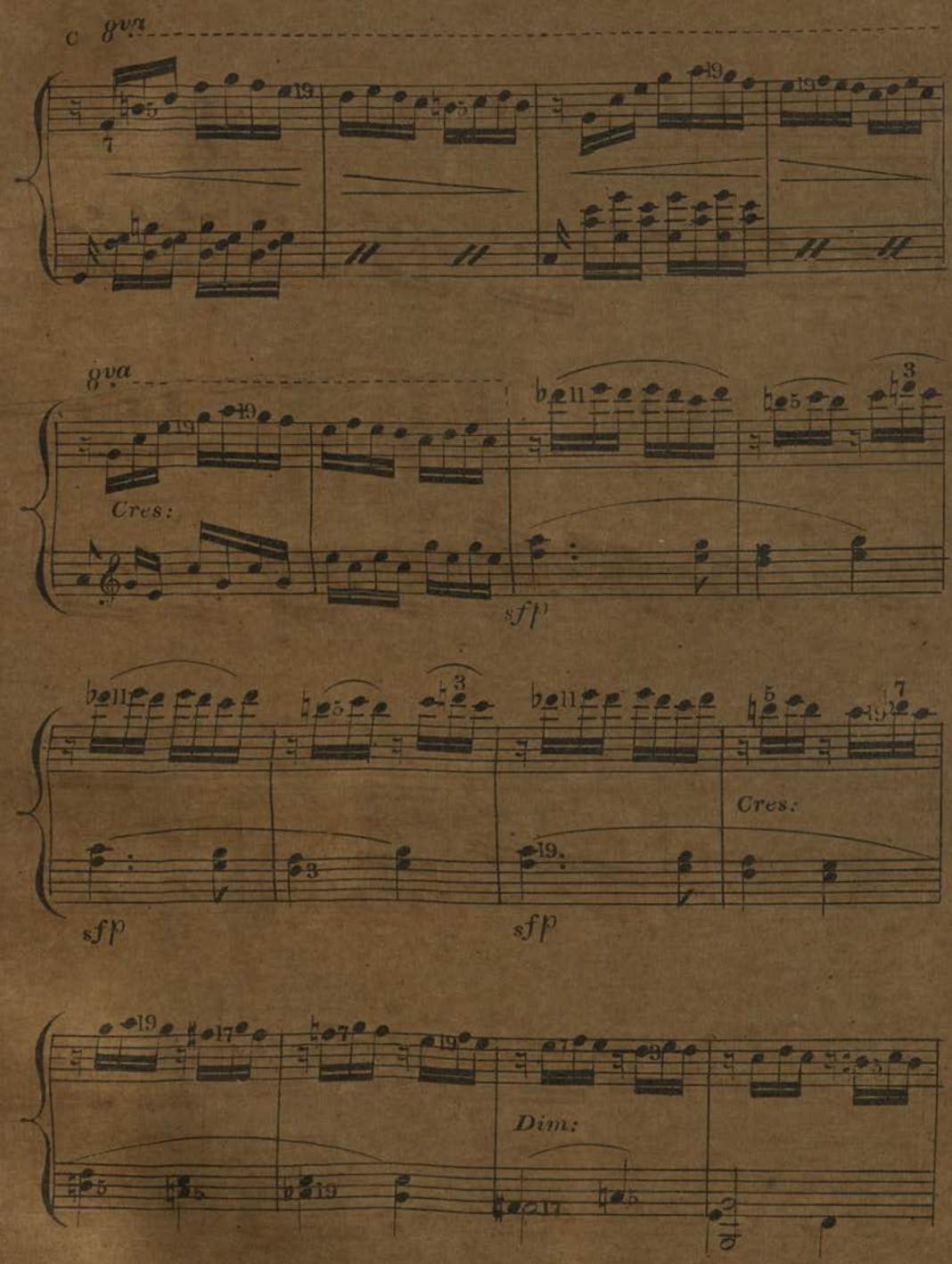


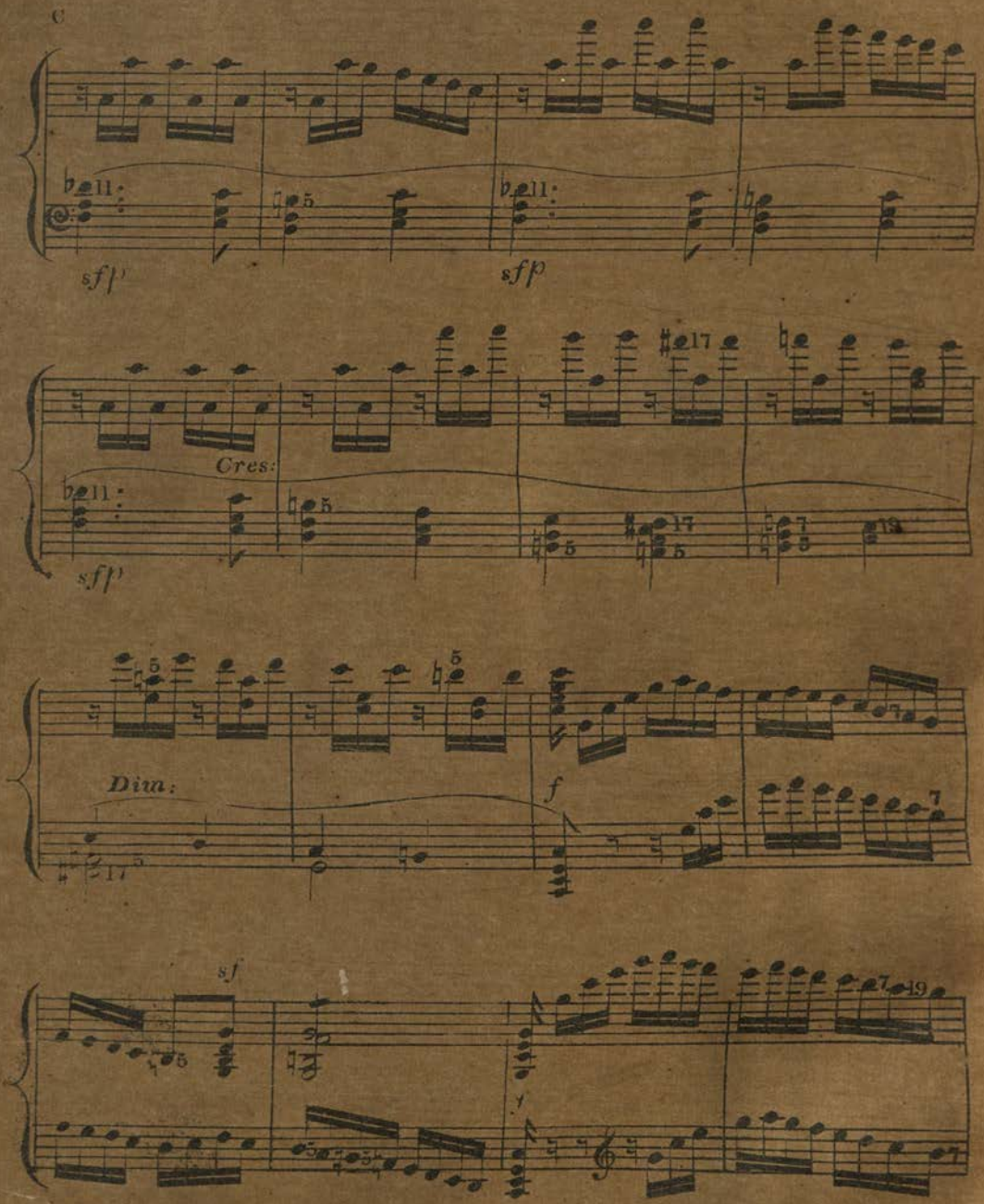

11
11
11
sf
sf
11
sf


F
sf
Cres:
sf
sf
sf

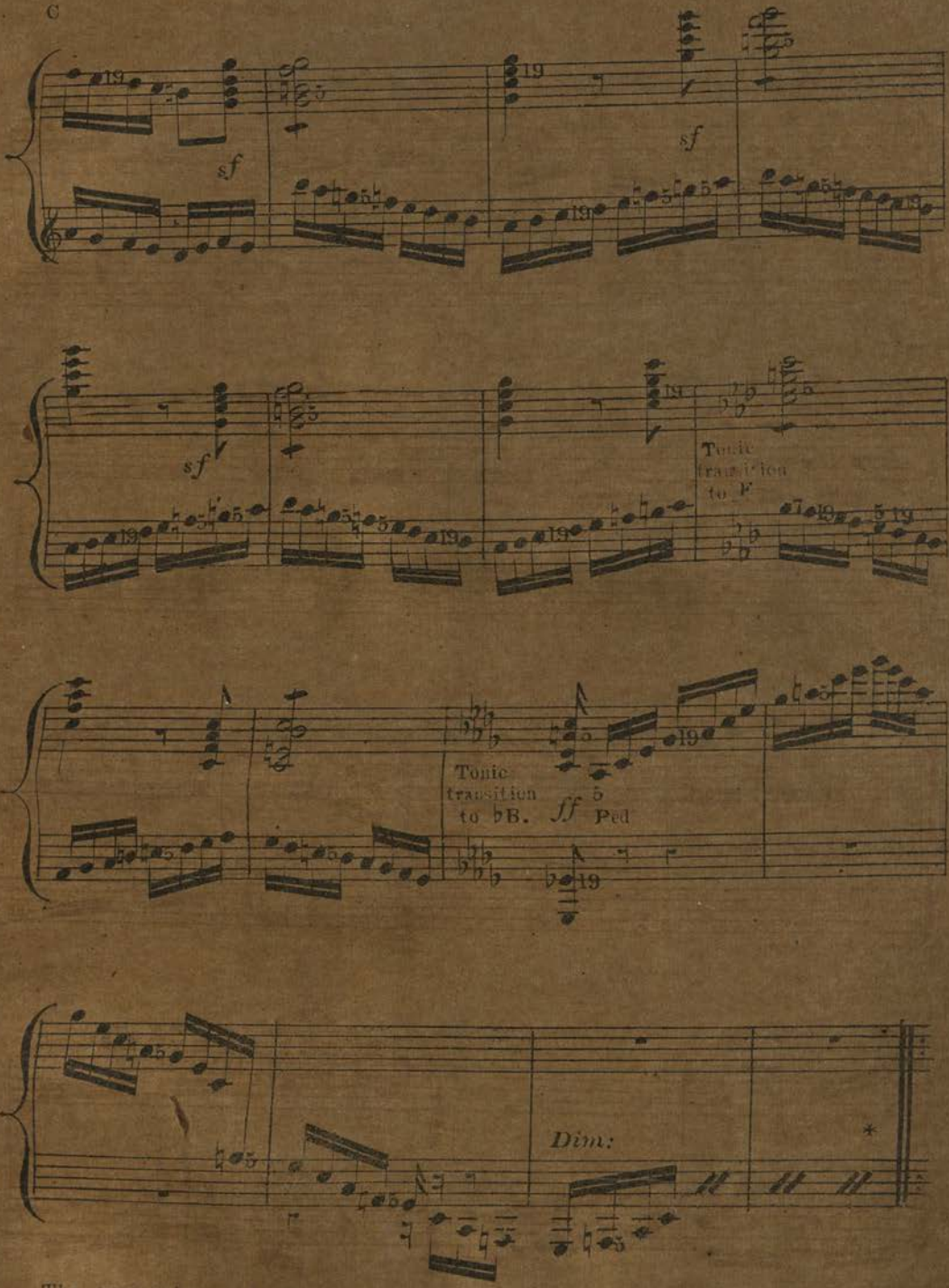
F

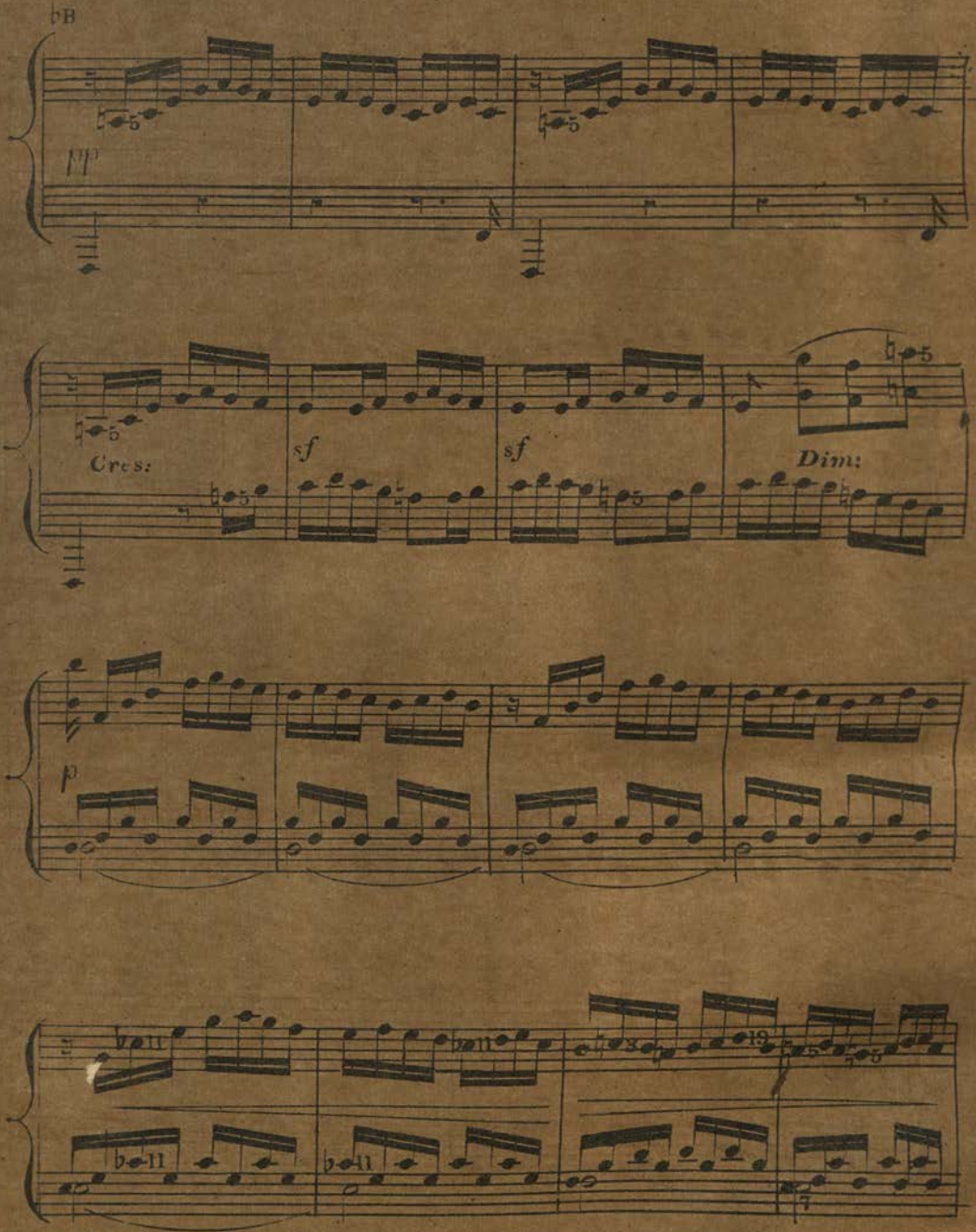


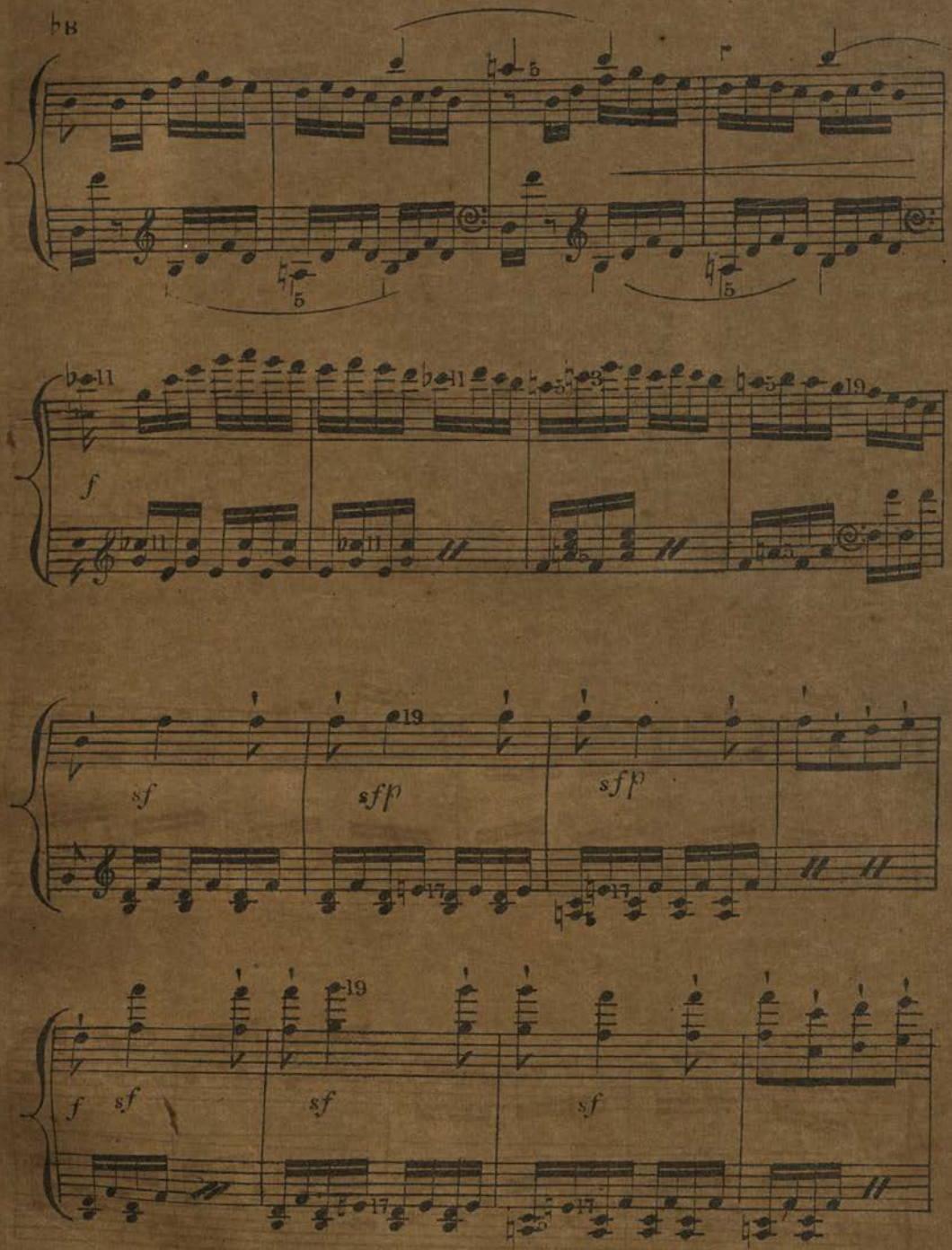

8va
8va
Cres:
sfp
Cres:
sfp
sfp
Dim:

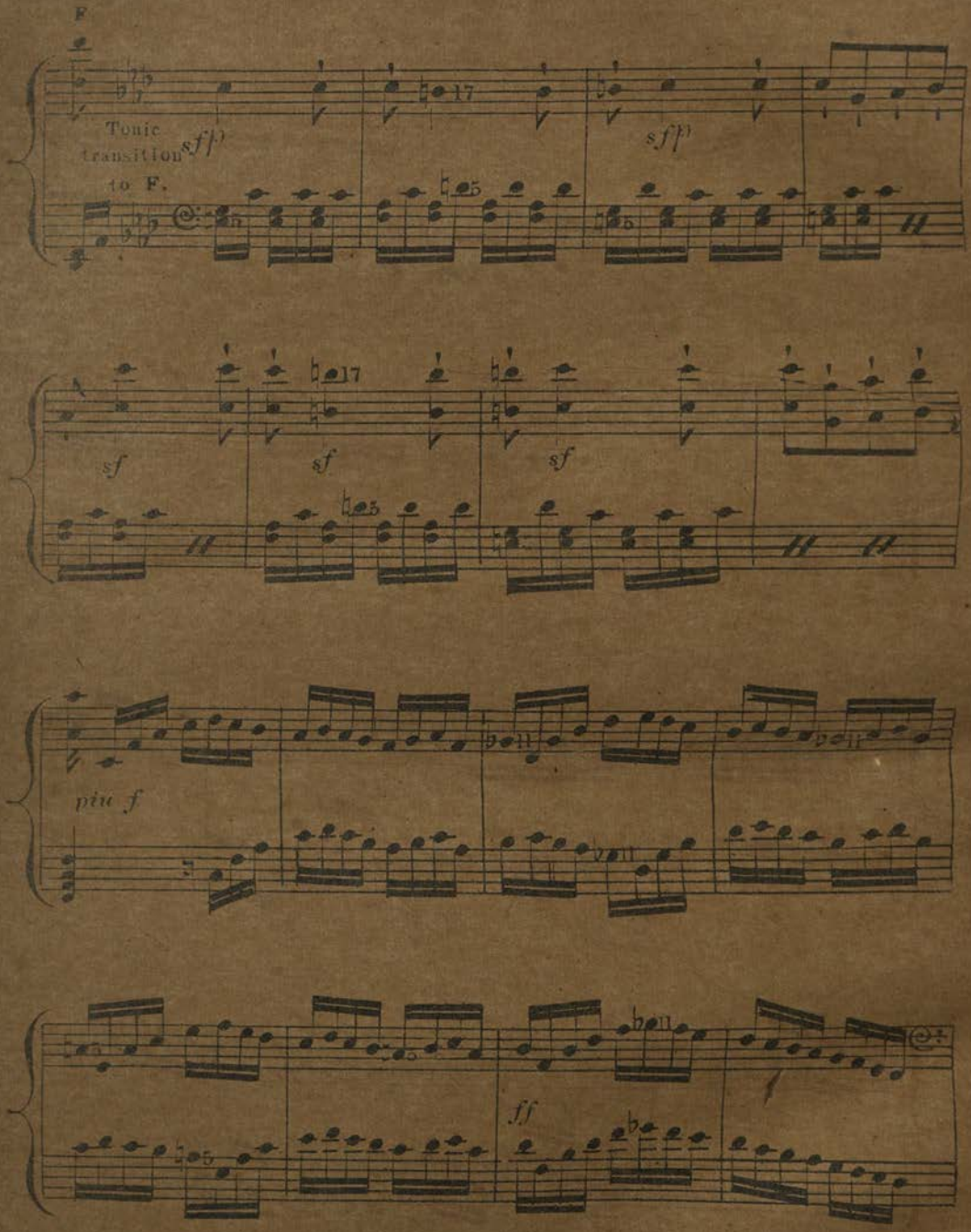

C
sfp
sfp
Cres:
sfp
Dim:
f
sf

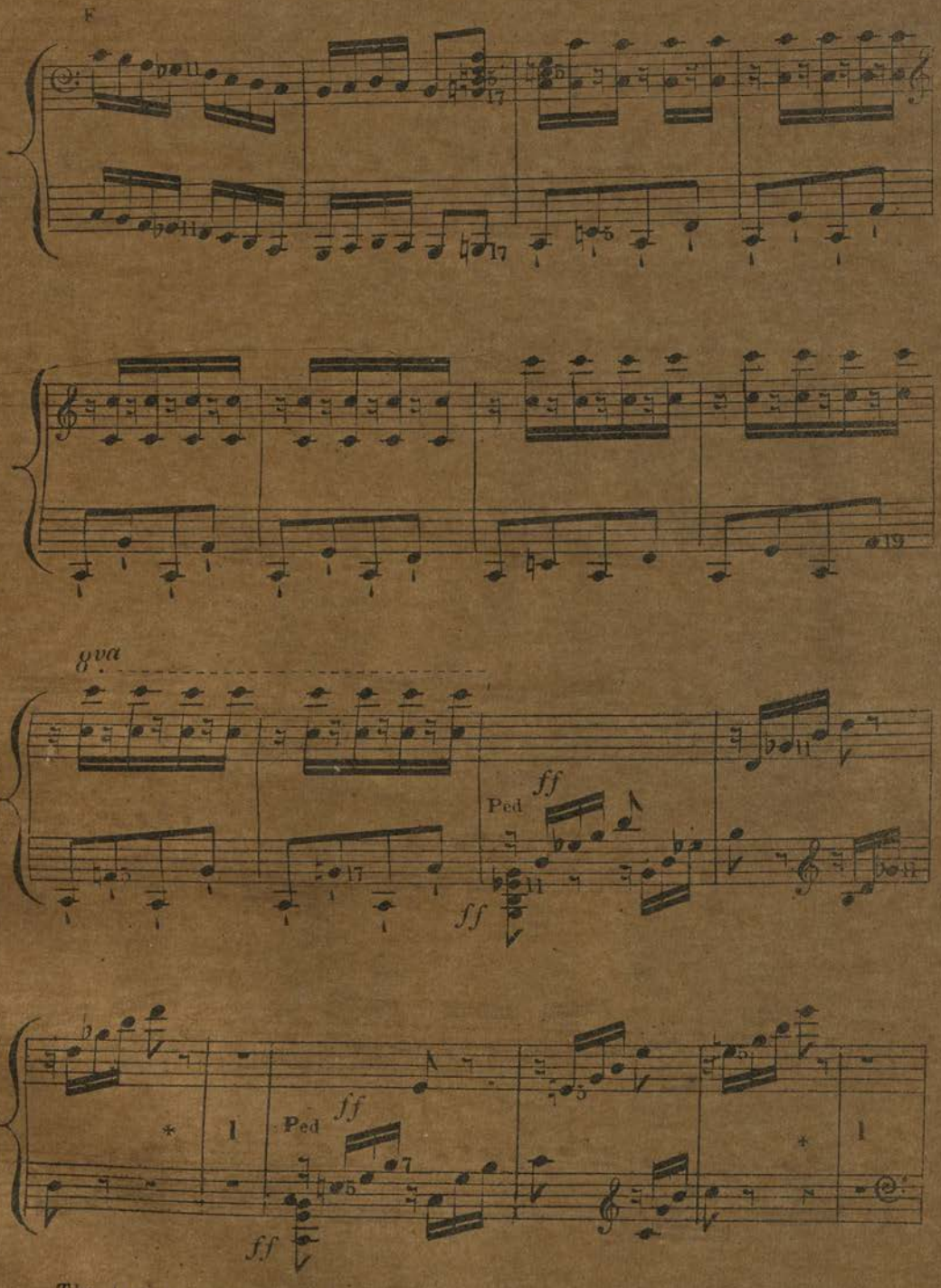
C

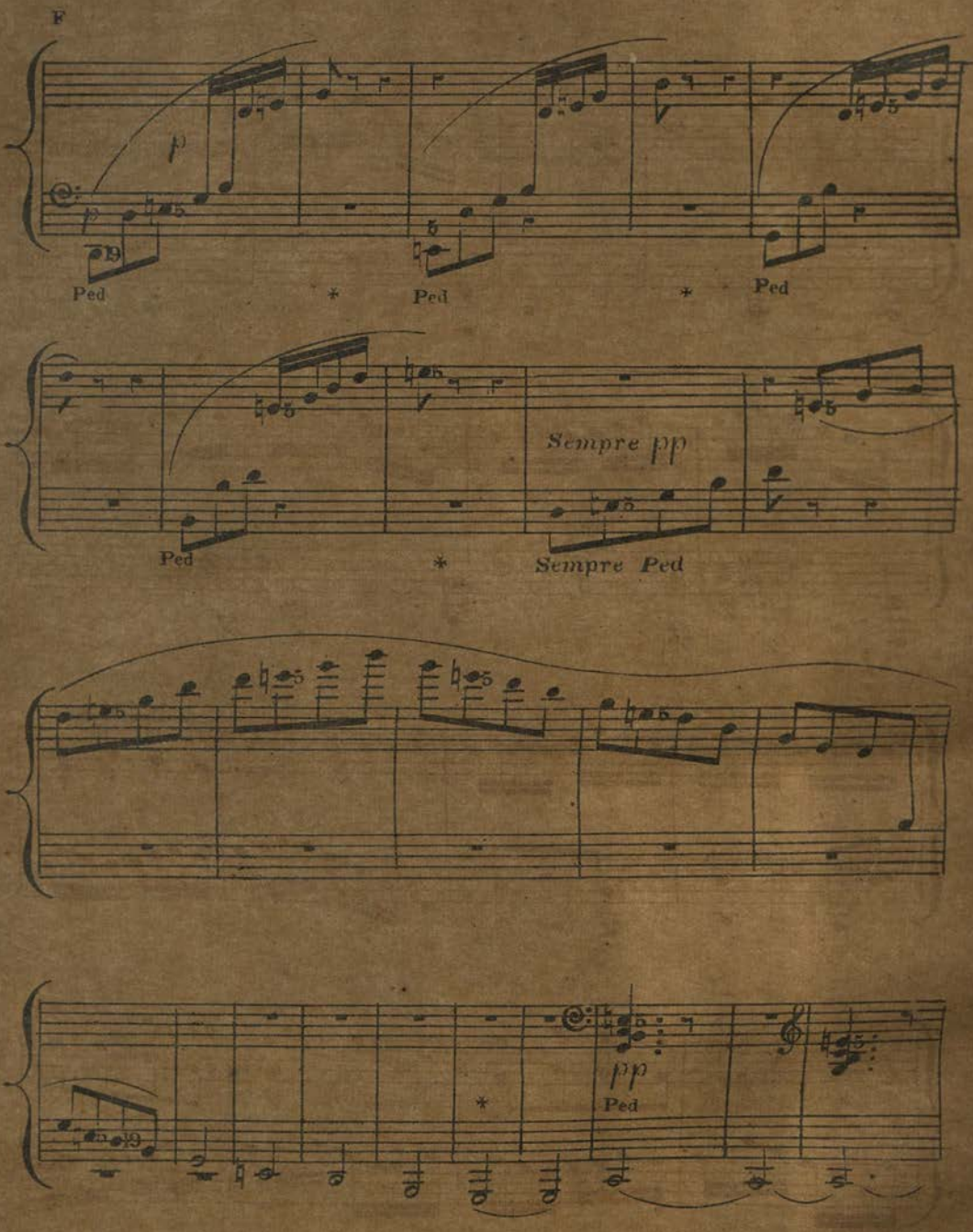

sf
sf
sf
Tonic transition to F
Tonic transition to bB.
ff
Ped
Dim:

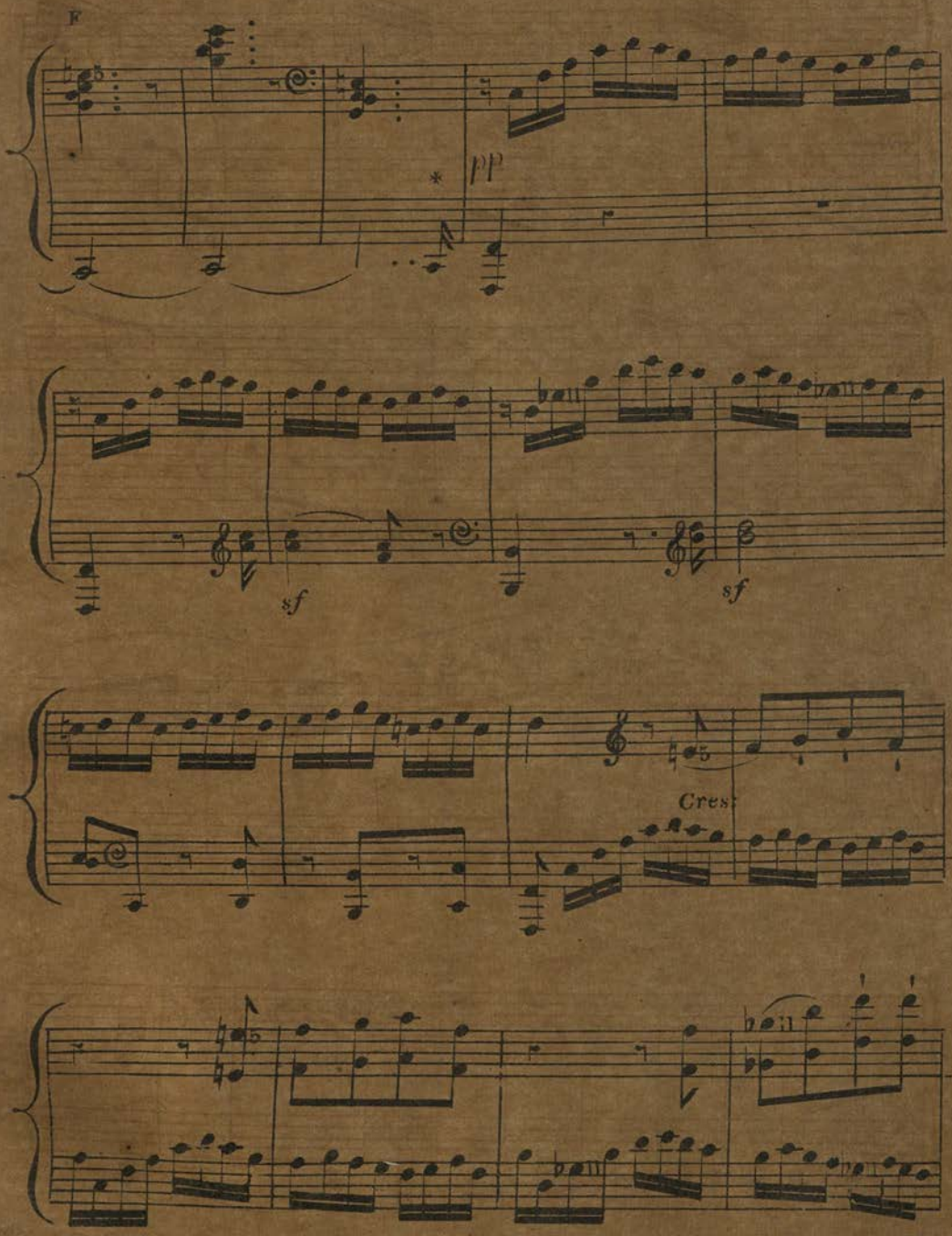

bB
mp
Cres:
sf
sf
Dim:
p

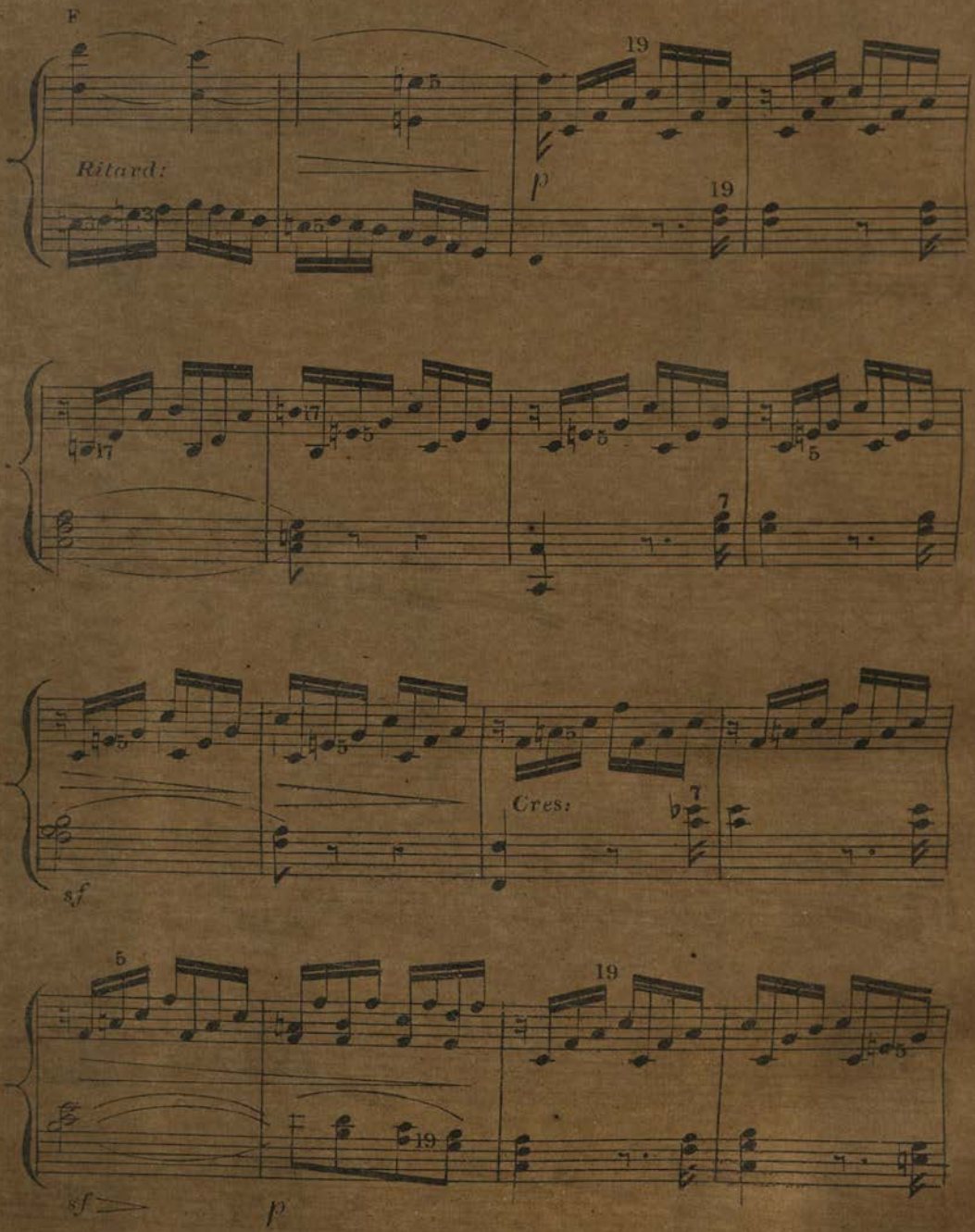

f
sf
sfp
sfp
f
sf
sf
sf


F
Tonic
transition
to F.
sfp
sfp
sfp
sf
sf
sf
più f
ff

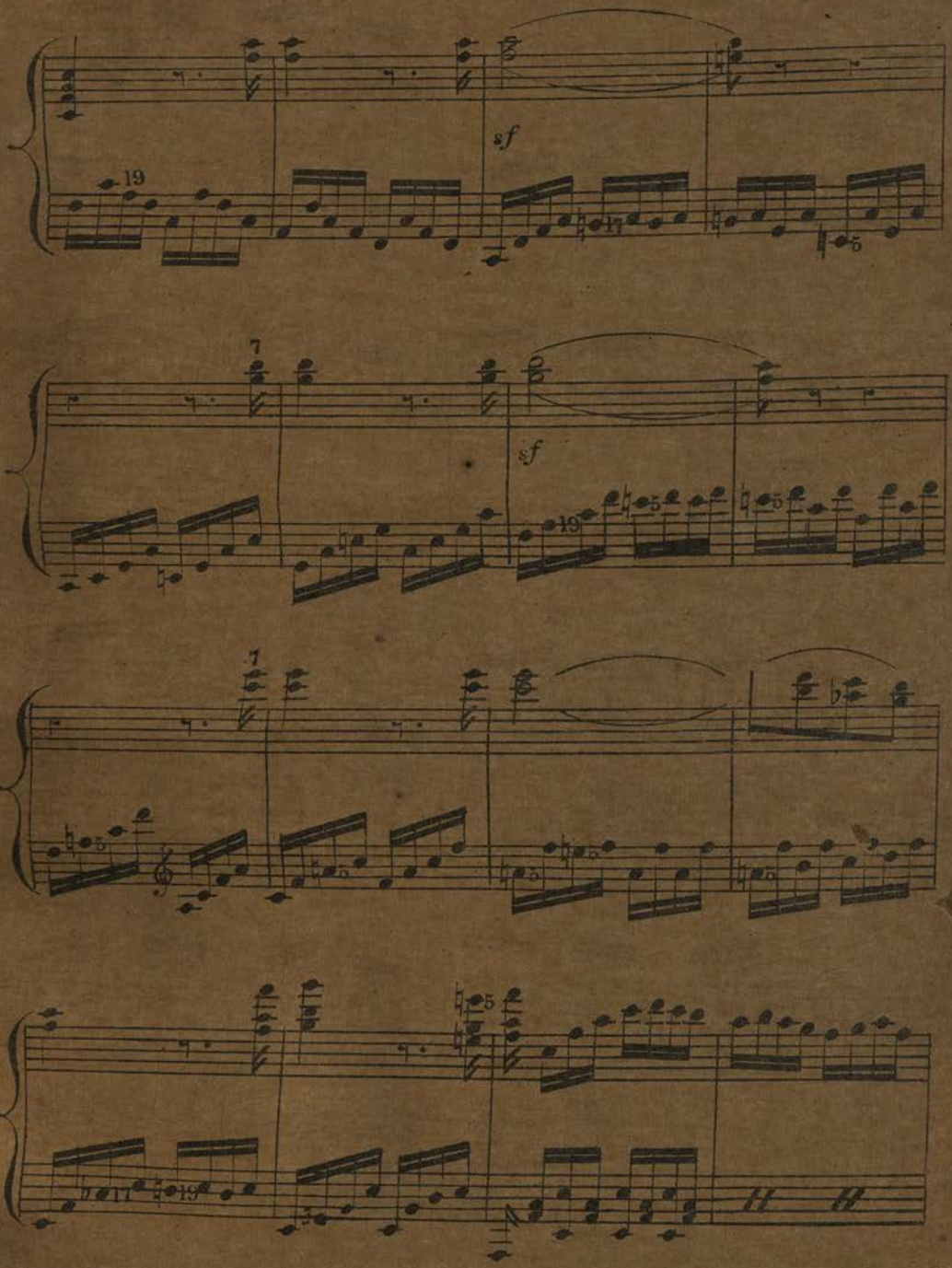

F
8va
Ped
ff
ff
Ped
ff
ff

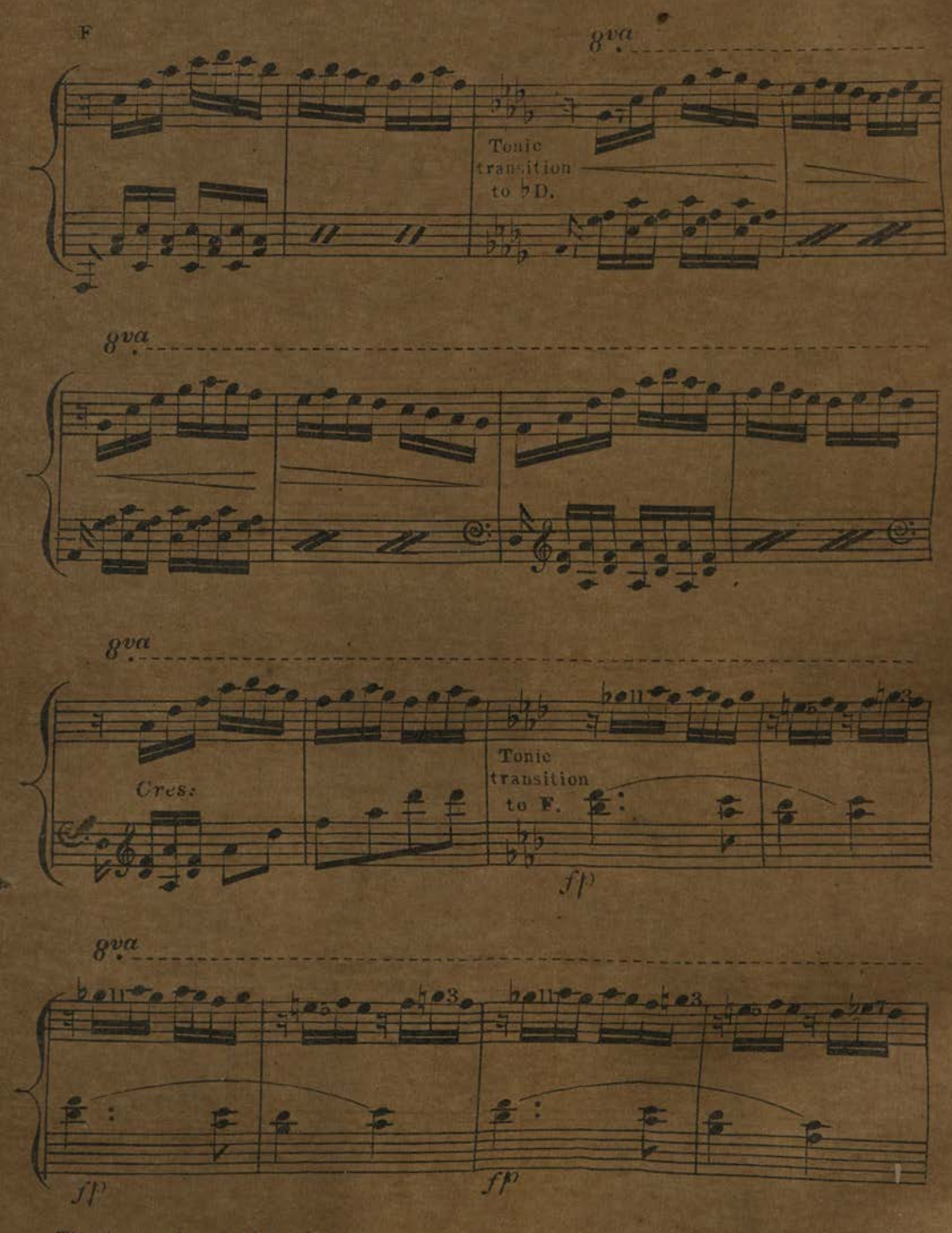

F
p
Ped
*
Ped
*
Ped
Sempre pp
Ped
*
Sempre Ped
pp
Ped
*

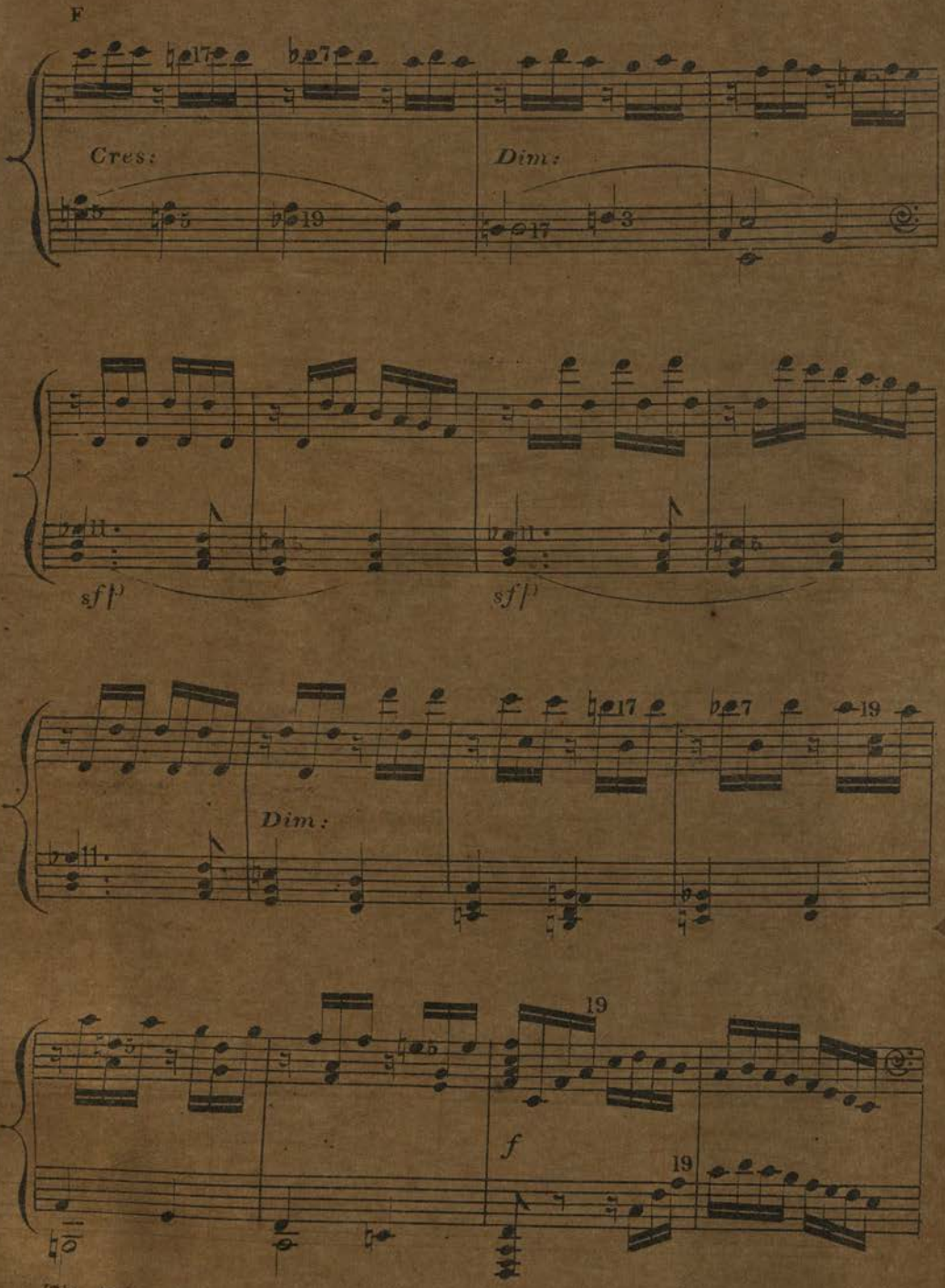

F
pp
sf
sf
Cres:

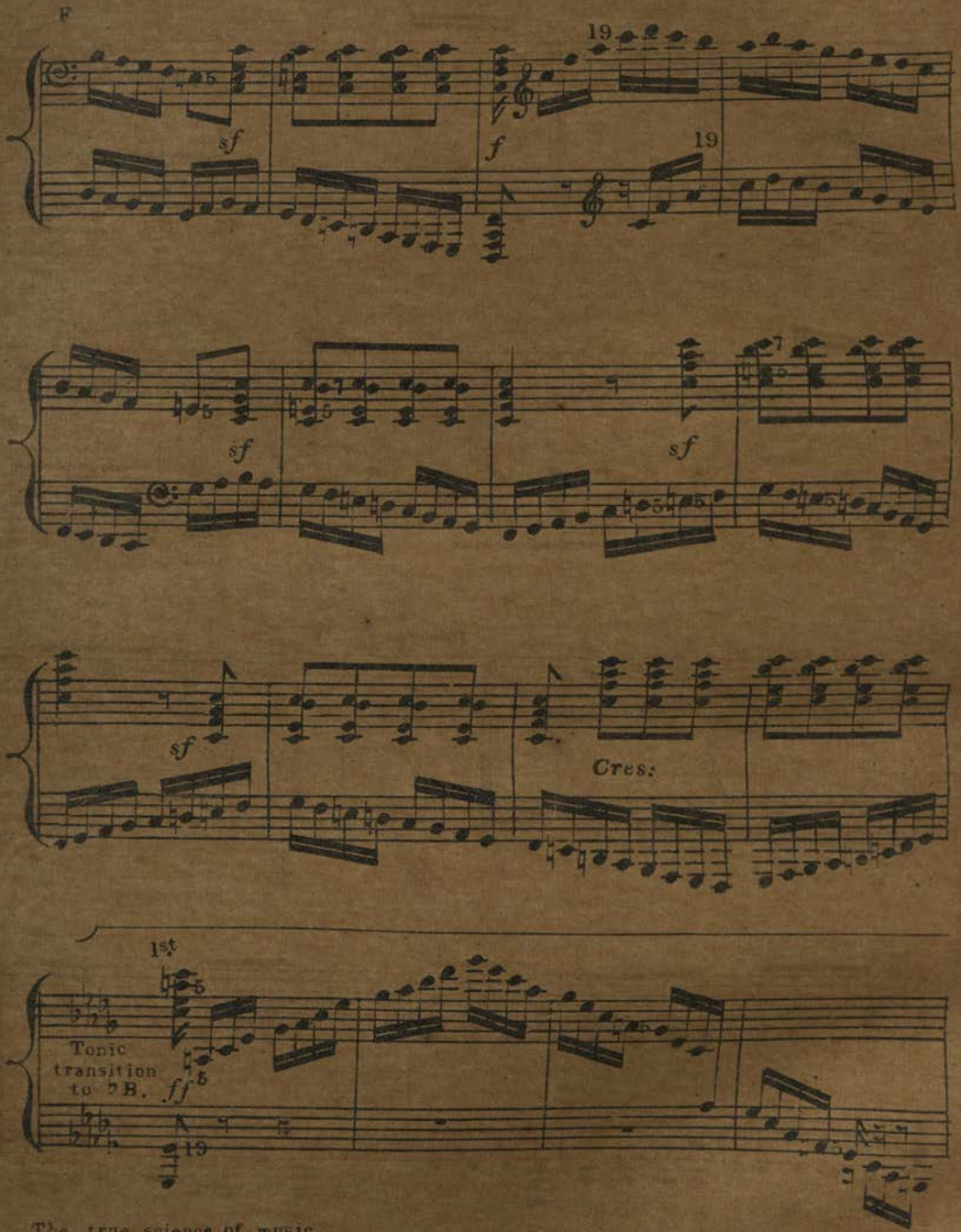

F
Ritard:
p
Cres:
sf
sf
p

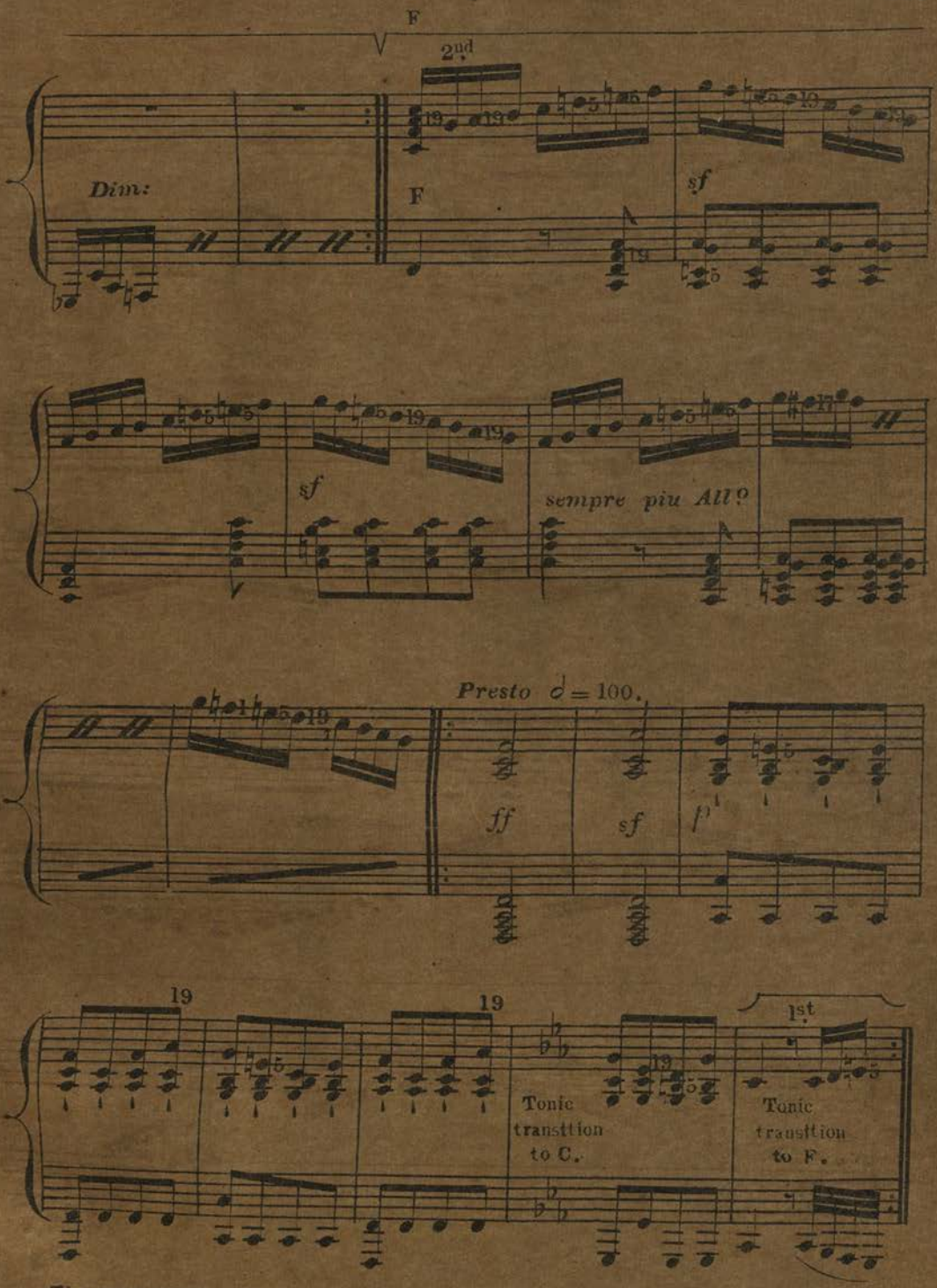
F

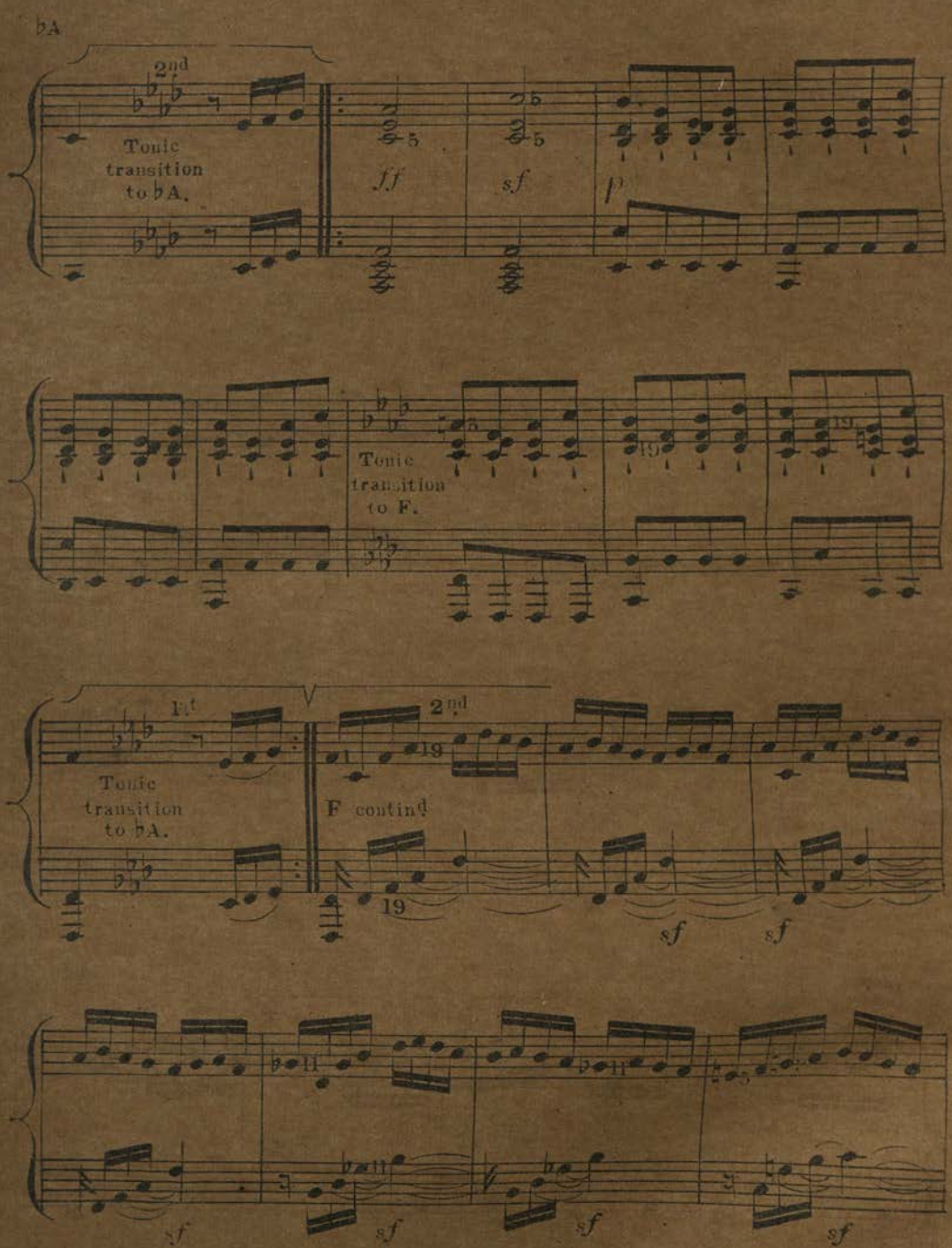


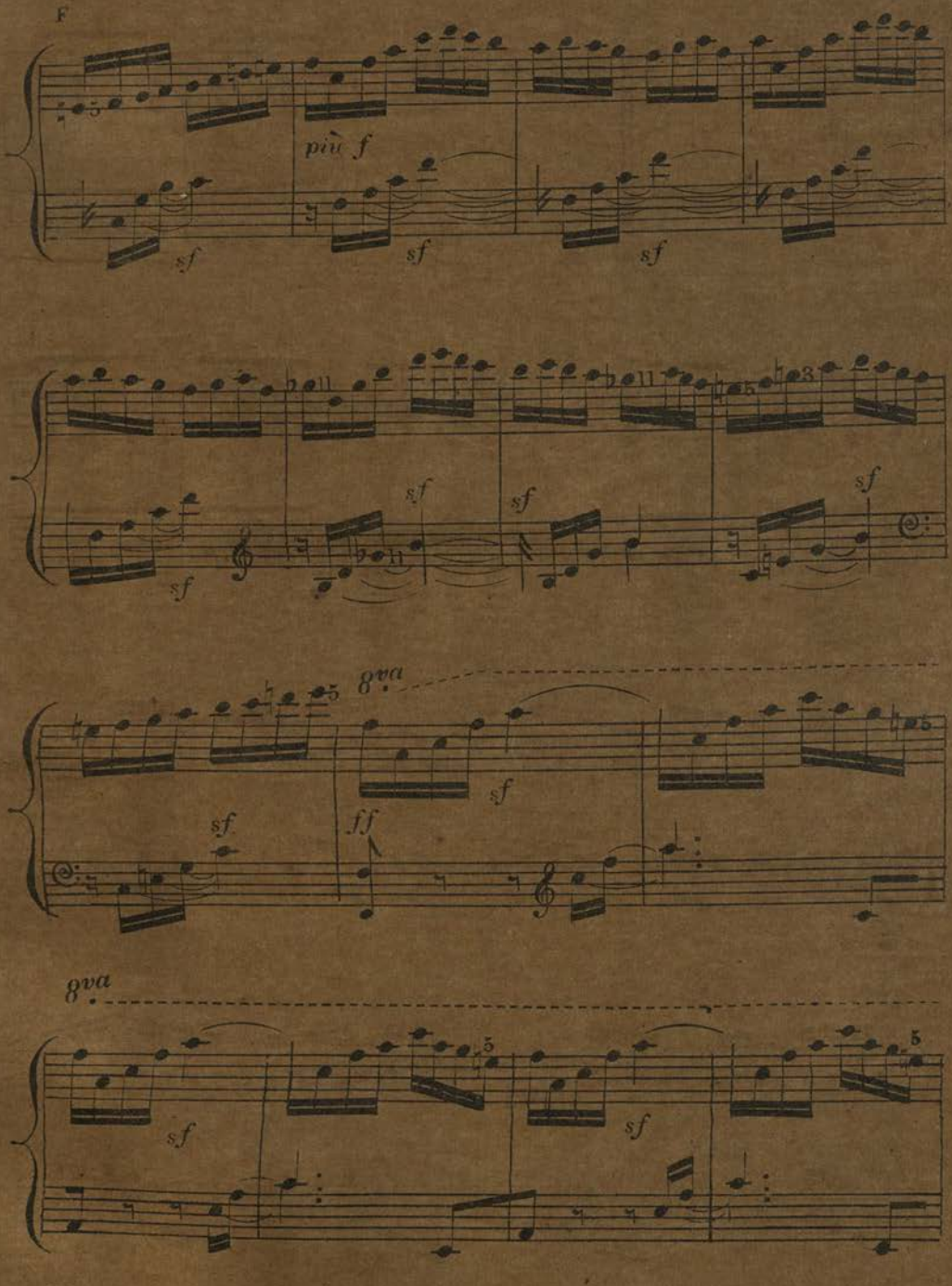

F
8va
Tonic transition to ♭D.
8va
8va
Cres:
Tonic transition to F.
fp
8va
fp
fp


F
Cres:
Dim:
sfp
sfp
Dim:
f


F
19
sf
f
19
sf
sf
sf
Cres:
1st
Tonic
transition
to 2B.
ff
19


F
2nd
Dim:
F
sf
sf
sempre piu All?
Presto ♩= 100.
ff
sf
p
19
19
1st
Tonic transition to C.
Tonic transition to F.


bA
2nd
Tonic transition to bA.
ff
sf
p
Tonic transition to F.
1st
2nd
Tonic transition to bA.
F continu
19
sf
sf
sf
sf
sf
sf


F
più f
sf
sf
sf
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8va
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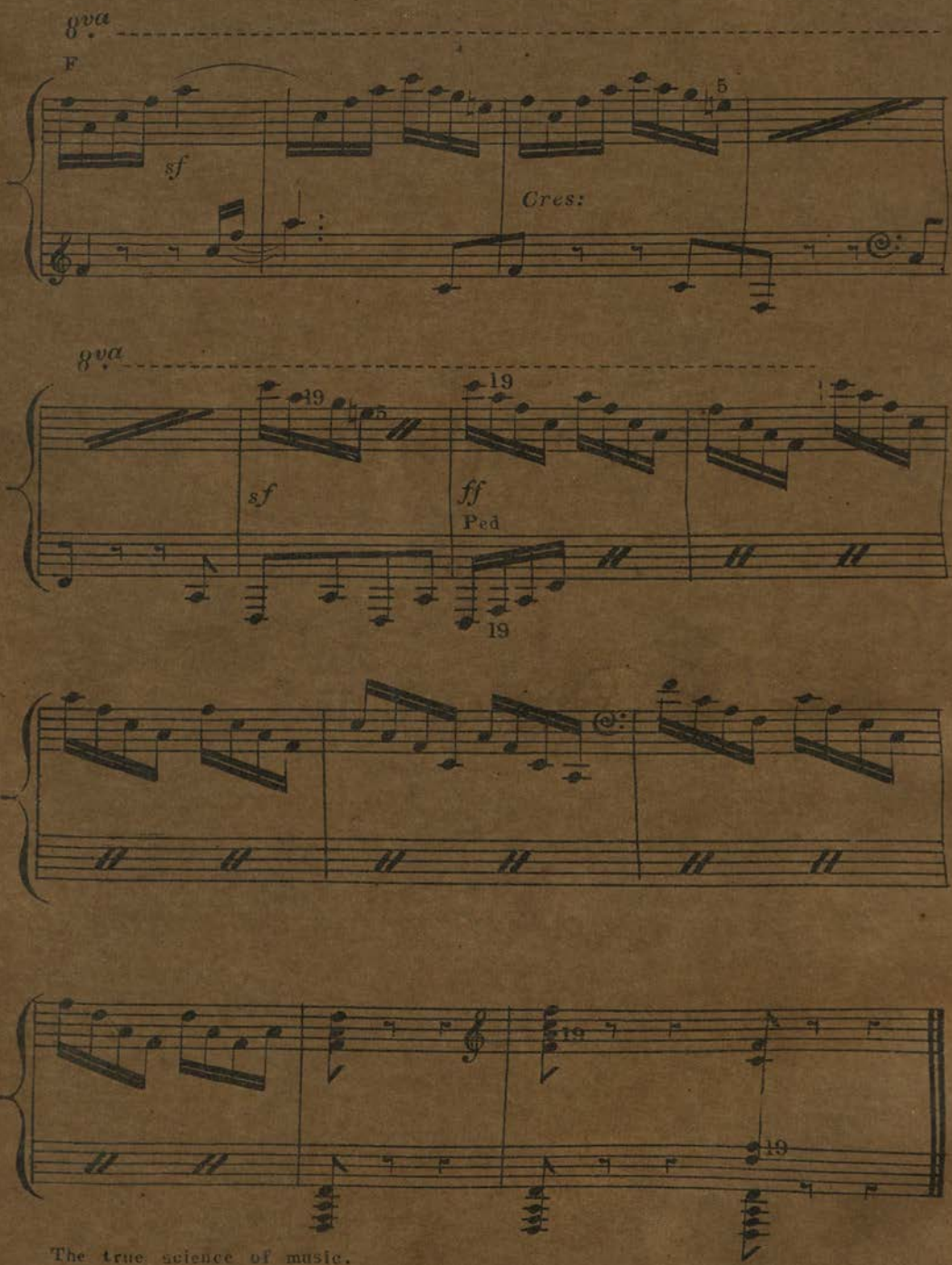
ff
8va
sf
sf

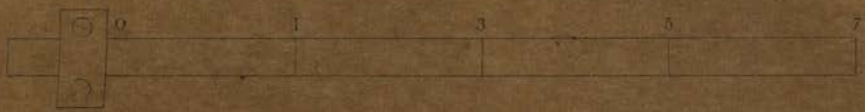

8va
F
sf
Cres:
8va
sf
ff
Ped

Figures and Diagrams.

The sections (§) on the left of the figures refer to the text where the subject is treated.

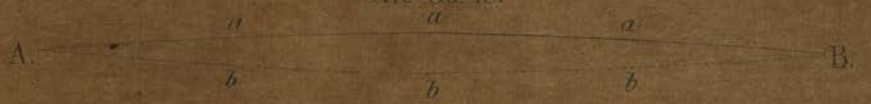
Fig. 1
§ 18.

Fig. 2.
§ 32.



The Basis.

Fig. 3.
§ 36.



The Octave.

Fig. 4.
§ 37.

The Double Octave.

Fig. 5.
§ 38.

The 12th or Octave 5th.

Fig. 6.
§ 39.

The Major 3rd.

Fig. 7.
§ 39.

The ♭7th.

Fig. 8.
§ 39.

The ♭5th or ♯3rd.

Fig. 9.
§ 39.

The harmonic 9, as composed of 3^2

Fig. 10. § 42.

The harmonic 15, as composed of 3×5.

Fig. 11. § 42.

The harmonic 15, as composed of 5×3.

Fig. 12. § 42.

The Unison.

Fig. 13.
§ 51.

Imperfect Unison.

Fig. 14.
§ 52.

Basis and Octave.

Fig. 15.
§ 53.

Basis, Octave and 12th.

Fig. 16.
§ 54.

Major Common Chord

Fig. 17.
§ 55.

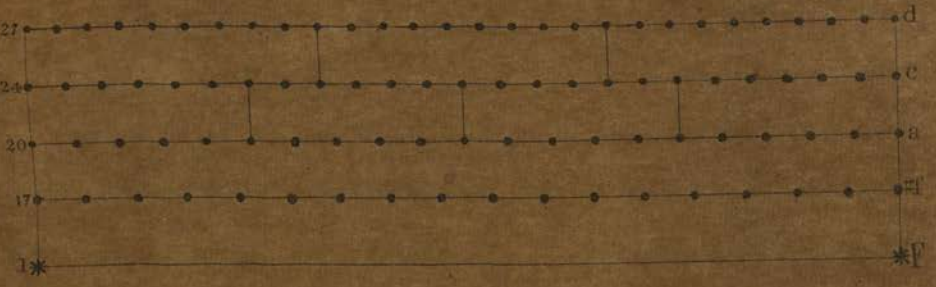
Chord of the ♭7th.

Fig. 18.
§ 56.

Chord of the added 6th Key C.

Fig. 19.

§ 178.



Chord of the ♭7th Key G.

Fig. 20

§ 178.

Same as above.

As 63 to 64.

As 81 to 80.

As 135 to 136

Real Major common chord. Key ♭A. 1st Mode.

Fig. 21.

§ 180.

Apparent Major common chord. Key C. 2nd or 3rd Mode.

Fig. 22.

1 & 19

§ 180.

Real minor common chord. Key A. 2nd or 3rd Mode.

Fig. 23.

§ 180.

Apparent Minor Common chord. Key C. 1st Mode.

Fig. 24.

§ 180.

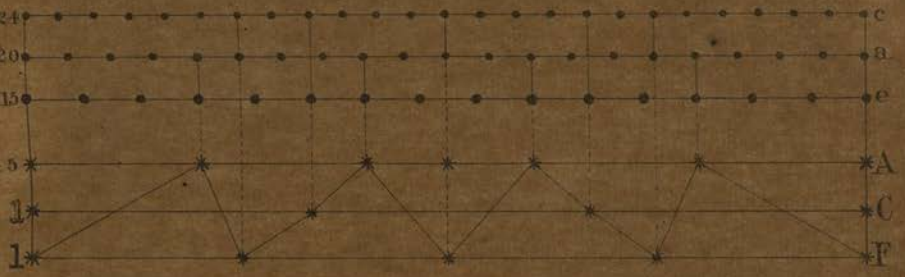


Fig. 25.

§ 184.

Fig. 26.

§ 214.

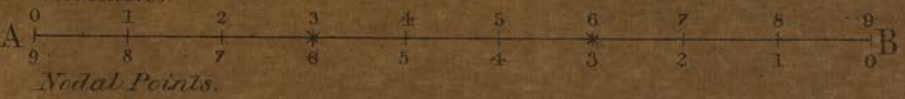


Fig. 27.

§ 219.

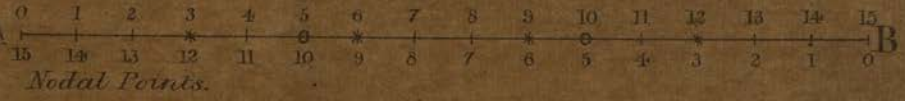


Fig. 28.

§ 284.

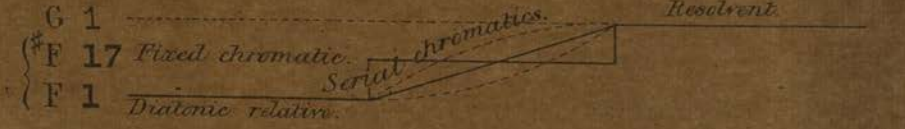


Fig. 29.

§ 284.

Fig. 30

§ 405.

Fig. 31.

§ 405.

Fig. 32

§ 405.

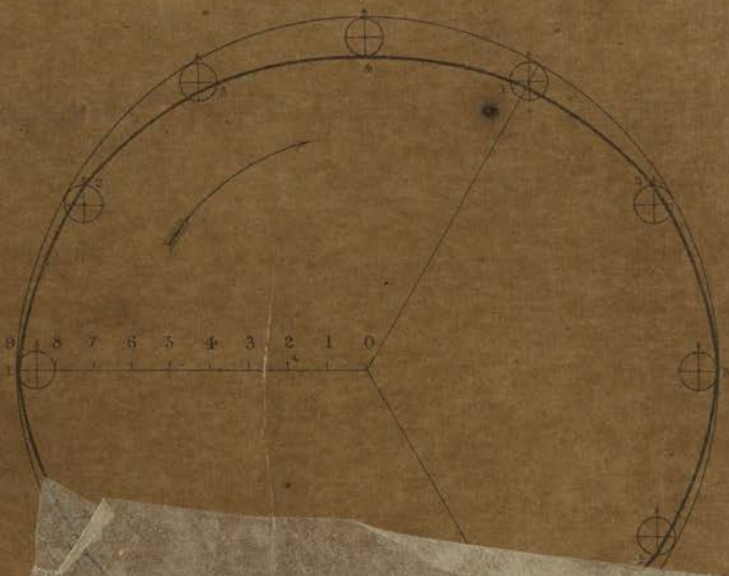


Fig. 34.
§ 410.

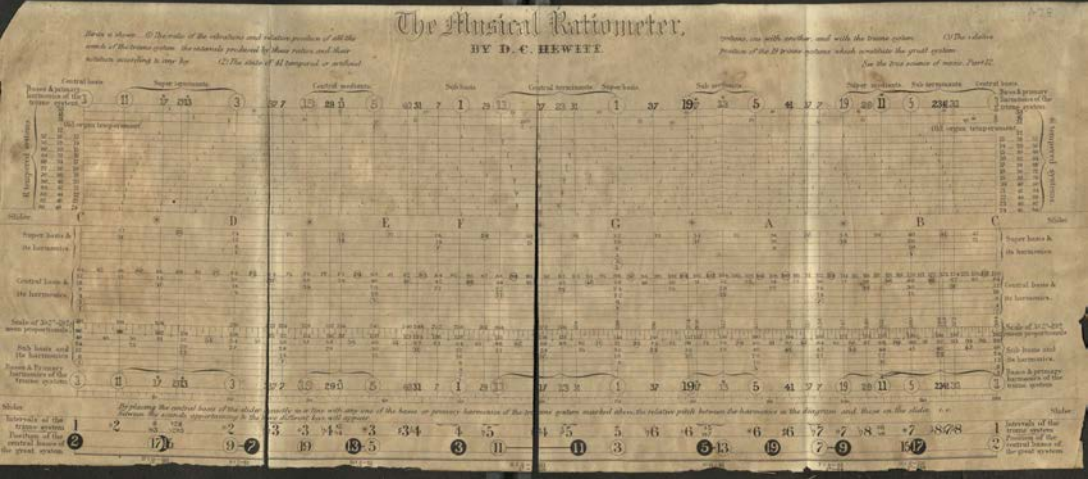
Fig. 33.

§ 404.

Fig. 35.
§ 411.

The Musical Rationometer.

BY D. C. HEWITT.



REPRESENTATION OF THE VIBRATIONS, COINCIDENT VIBRATI
EXTREME FLAT SEVENTH. ACCORDING TO THE RATI

Fig 36. § 583.

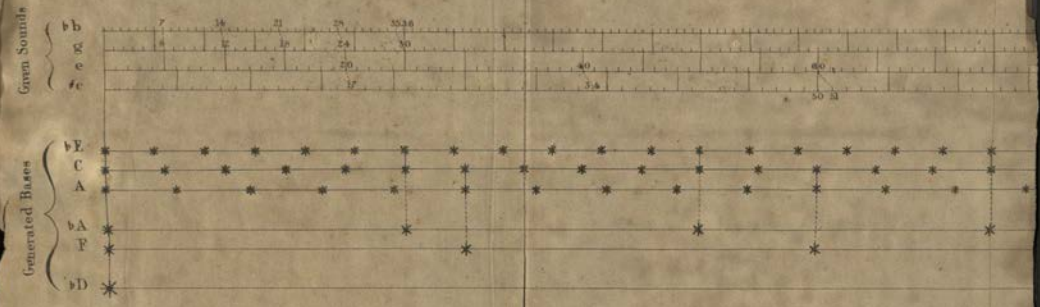
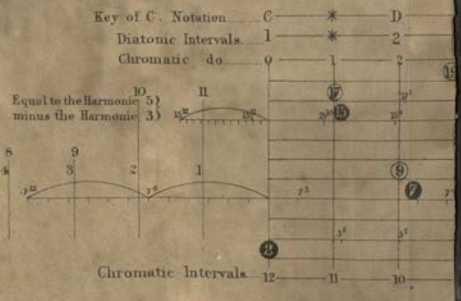


Fig 37. § 583.

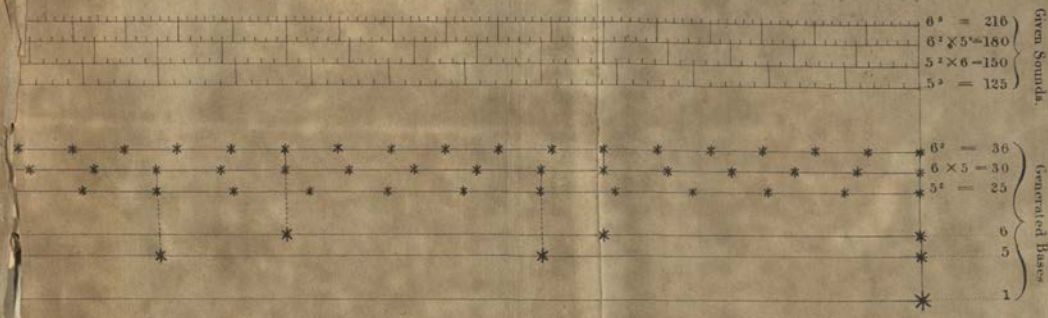
Representation of the true ratios of the vibrations, coincident vibrations and generated Bases, of the chord of the extreme flat seventh.

Fig 38. § 602.

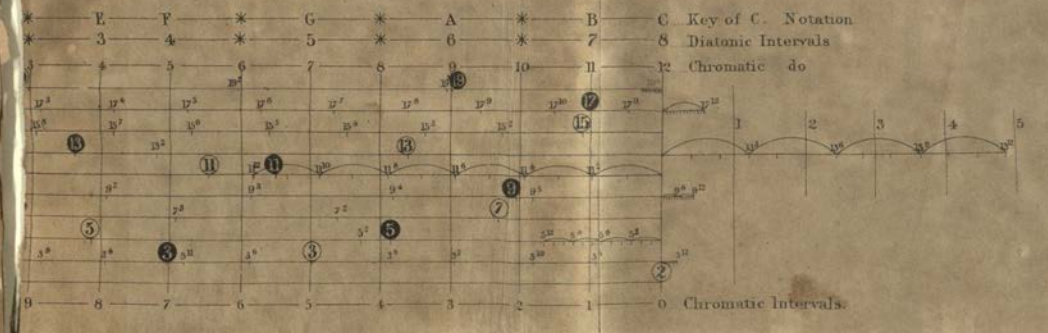
Representation of the discrepancies between the ratios 2^m & 3^m — 2^n powers involved in their respective series, and with the semitone



NS AND GENERATED BASES OF THE CHORD OF THE
OF THE HYPOTHETICAL THEORIES.



5^3 — $2^{10} \wedge 7^6$ — $2^{15} \wedge 9^6$ — $2^7 \wedge 11^3$ — $2^{12} \wedge 13^3$ — $2^{16} \wedge 15^{12}$ — $2^{20} \wedge 17^{12}$ — $2^{27} \wedge 19^6$, and one with another — also the various other scale of the equal temperament — and the retrograde series of these same ratios as bases to the central unit or key-note.



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